STRUCTURAL DESIGN

HALE SUTHERLAND AND HARRY LAKE BOWMAN

STRUCTURAL THEORY, *Fourth Edition*

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HALE SUTHERLAND AND RAYMOND C. REESE

INTRODUCTION TO REINFORCED CONCRETE DESIGN, *Second Edition*

STRUCTURAL DESIGN

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Ninth Printing

NEW YORK
JOHN WILEY & SONS, INC.
LONDON: CHAPMAN & HALL, LIMITED

NINTH PRINTING, NOVEMBER, 1955

PRINTED IN THE UNITED STATES OF AMERICA

PREFACE

This book is intended to contain all the material essential for introducing the student of structural engineering to the fundamentals of the theory and practice of design in steel and timber, excepting only the data in the handbook of the steel manufacturer. The examples of actual structures have been chosen in the field of civil engineering, and, as is necessary in an introductory textbook, both structures and their details are rather simple in character, involving only the more elementary of theoretical considerations.

The first four and the eleventh chapters together with Appendix A deal with matters quite commonly taken up in our American schools in the junior year, preceding a design course in the senior year. The first chapter goes somewhat more deeply into the common beam theory than is usually possible in the preceding course in strength of materials, something which is highly desirable for the civil engineering student. Specializing so largely as he does in structures he should be familiar with moments and products of inertia to the extent given in Appendix A and their practical utility in computations relating to unsymmetrical bending, the S-line and the S-polygon and to torsion, a subject introduced rather sketchily as is inevitable with so complicated a problem which has not yet been completely worked out. If time permits it is probably desirable to go beyond this chapter and introduce the student to the circle of stress, concerning which reference only is given. The second chapter reviews the theory of columns, it is hoped without too much repetition of the excellent summaries given in good texts in strength of materials, and definitely advancing beyond these texts by introducing the student to current technical discussion and specifications. Rivets and bolts are taken up briefly in the third chapter, the treatment of brackets being the only topic not previously studied by the student probably. In the fourth chapter the plate girder is treated at some length in its simpler forms, leaving out such details as complicated web splices, flange splices and heavy flange arrangements, these being unsuited for a first course in design in the authors' opinion. Riveting is the only mode of connection of parts used in this chapter and in the five following which give rather completely worked out design computations for five bridges and a mill building bent, together with discussion of the general considerations relating to these structures. Welding, which

bids fair largely to replace riveting in the not distant future, is given rather general treatment in the eleventh chapter. As pointed out in the text, the usual problems in tier building construction present no difficulty to one conversant with the five design chapters above listed. The difficult problems with building frames involve indeterminate stress analysis largely and lie beyond the scope of a first course. The twelfth chapter gives a brief introduction to design in wood which should enable the student to attack other problems with confidence.

An unusual feature of this book is the number of different design specifications which are reproduced in part in the Appendix and used. This is due in the first instance to the rather long time over which the preparation of this book extended but on consideration of the matter the authors came to the opinion that this multiplicity of standards is in every way an advantage. It is to be hoped that someone will write at length and in detail, some day, concerning the evolution of some of our common specifications since the subject is of great importance in itself and exceedingly illuminating in revealing the progress of structural insight and theory. These several specifications enable the instructor easily to introduce the student to some of these changing ways of answering common problems and so to rid him of the idea that finality has been reached in either practical or theoretical structural standards. The number of these specifications will compel the student to make frequent references to them, a habit which he will have to acquire eventually and might as well gain early in his career. The use of specifications of different ages means that little emphasis has been placed on current practice. Since practice differs between different companies and between different plants of the same company, the authors do not consider that there is a last word in practice which it is essential that the student learn. The last word of today is obsolete tomorrow.

The authors express their gratitude to the many friends who have assisted in the preparation of this volume and in particular to Professor William J. Eney of Lehigh University who placed the "design sheets" in final form; to Mr. F. L. Castleman of the American Bridge Co., to Mr. H. M. Priest of the Carnegie Steel Co., and to Mr. Jonathon Jones, Mr. E. L. Durkee and others of the Bethlehem Steel Co. for the use of valuable design data and for wise counsel; and to the several associations sponsoring design specifications who have permitted their reproduction here. Also, they are grateful to the American Bridge Co., and the Bethlehem Steel Co. for permission which was given to reproduce drawings.

Hale Sutherland
H. L. Bowman

June, 1938

TO THE STUDENT

This book has been written for you if structural engineering is one of your major interests. By way of preparation on your part it presupposes a knowledge — more than superficial — of statics, strength of materials, and structural theory.

The book may be divided roughly into three parts: (1) Chapters I to IV and Appendix A which deal with fundamental material underlying any design problem; (2) Chapters V to XII which apply these fundamentals to common structures; and (3) Appendices B to G which give specifications and design data. All of these have a place but no structural designer is really ready for his job until the first part has been mastered in such a manner that its use becomes a matter of second nature.

The second and third sections as above listed probably contain more material than your instructor can find time for in his course. You will find it useful to supplement your study of assigned structures by comparisons with similar situations in other chapters. Skill in structural design involves among other things ready visualization of framing elements, paths of stress, and systems of forces brought into play in the supporting of any given load. This facility is gained only by practice; the apparent repetitions in this book are designed to furnish you with opportunities for such practice.

The many so-called "design sheets" found in Chapters V to XII fairly represent the form in which an experienced designer presents his work. You will see that the parts and results have been clearly marked and that rather copious references and notes have been included. While studying these sheets you are not only a student learning to design, but you are, in some measure, in the position of the checker in a design office whose function is to verify the computations of other engineers. Whenever you are preparing design computations remember that the checker (at the moment your instructor) appreciates notes and references which facilitate his work. As regards neatness and system, these design sheets are no better than the work of many designers in practice. You cannot afford to be satisfied with you own work until it is at least as orderly and legible as that given here for your study.

Your instructor may have his own ideas as to the form of your computations. If, however, you are left to your own devices the following

may be of assistance to you. Most engineering firms have their designers work in pencil on $8\frac{1}{2}$ by 11-in. sheets, ruled in $\frac{1}{4}$-in. squares. Usually the paper is thin enough and the pencil work heavy enough that prints may be made if desired. You will find a well-sharpened HB, or F, pencil about right for this kind of work.

In the matter of numerical accuracy you will see that results have been obtained in general with the aid of a 10-in. slide rule. When you remember that loads and impact factors are never actually known to three-place accuracy and when you recall from your work in the testing materials laboratory that supposedly identical specimens of materials vary in strength over a considerable range you will realize that the accuracy thus obtained is sufficient.

A major mental hazard which you must surmount is the delusion that since the principles of structural engineering are absolute the application of these principles is absolute also — that is, the delusion that there is only one correct design procedure in any case and that you must learn this procedure by rote. It is true that some common situations have been "standardized" but there is a truly astonishing amount of flexibility of procedure in most cases. In general, the best answer is that one which most truly sizes up the situation, the degree of approximation of the usually simple theory, the nature and degree of indeterminateness involved, the kind of distortions inevitable, and their effects. One reason for presenting several specifications is that you may realize that more than one procedure for the same problem would be followed by different engineers. Another reason is that the presenting of many rules forces you to make systematic reference to them and discourages you from making improper use of your memory in a vain attempt to learn relative and changing rules as though they were eternal principles.

You should realize that specifications are constantly in a state of evolution and change, consequent upon advances in design and construction. More and more, rule-of-thumb regulations are giving way to scientifically founded rules based on research and analytical advance. In following through the details of these computations the attempt should be constantly to understand the why and wherefore of the rule governing each step. No attempt has been made in these pages to explain the basis of rules which are very simple or of those which are complicated. These latter are often discussed in the publications of the association sponsoring the specification and you should form the habit of searching this literature as you carry forward your study of structural design. For example, the Bulletins of the American Railway Engineering Association contain much valuable information regarding the reasons lying back of the rules of their specifications.

You will not find it possible to follow and understand these design computations unless you know the rules of study for engineering subjects. Random reading will leave you helpless and befogged. The most efficient procedure for studying material in this field may be summarized thus:

(*a*) Read quickly through a convenient portion of the assignment to get a comprehensive picture of the whole, *making no attempt to verify details.* If any special reference material is to be required assemble it for use.

(*b*) Proceed slowly through the lesson, verifying details carefully. Remember that the specification governs and read the appropriate articles therein *before* attempting the checking of any item. Always work with pencil and scratch pad at hand. Make generous use of free body force diagrams to ensure accurate understanding of the forces involved. On the margins of your textbooks make *neat, careful* sketches and notes amplifying the text.

(*c*) Review, once, twice, as many times as may be necessary to make the whole picture clear to you. The basic rule of learning is review, REVIEW, REVIEW.

You will perceive that this kind of study calls for an *active mind,* and has no place for a blind staring at a computation item and expecting it to explain itself by some magic of perception. Faced with a "snag" the active mind gets to work at once with pencil and paper, setting and answering these questions: definitely stated, what is this problem; what forces are involved; exactly what is wanted; what relationships exist between the elements of the problem; what principles govern? In answering such questions light will come. Inspiration flows to the busy, not to the stagnant, mind.

CONTENTS

STRUCTURAL DESIGN

CHAPTER I

BEAMS

1-1. A companion volume of this textbook, "Structural Theory," deals with the determination of stresses in structural frames, that is, the finding of the external forces — shear, bending moment, axial stress — which structural pieces are called upon to sustain as load-carrying members of frames. The present volume is concerned chiefly with the proportioning of members to carry their external loads and so deals largely with the internal stresses set up in the members. Accordingly, this volume may be considered to be a textbook on applied strength of materials, carrying forward the subject to slightly more advanced theoretical levels in certain matters than are usually reached in the first undergraduate course on this topic. It deals with the basic parts — and in some respects the simplest parts — of structural design, the proportioning of members of known length to carry their loads adequately as part of a frame of given dimensions. As has been pointed out in the previous volume, the determination of the type, shape, and dimensions of a frame for a given purpose is a difficult matter demanding professional competency, something which is achieved only as arduous experience is added to schooling.

There are four basic types of load-carrying members: **ties** subjected to axial tension; **columns** subjected to axial compression; **beams** carrying transverse loads; **shafts** subjected to torsion. In most structures the members have as their primary loading axial or transverse forces: torsion is less common, and a shaft may be regarded as a machine, and not a structural, element. As we shall see, torsion is also much less clearly understood than the other stress conditions when it occurs with structural shapes. Many structural members have as their primary loading some combination of these basic types, and probably most of them have secondary stress loading of some other type than their primary loading. It is common usage to name a piece according to its major primary loading, thus ignoring the fact that it may be a com-

bination of two or more of these basic types. It is important to note that the nominal designation of a piece does not give at all a complete account of the type of loading to which it is subjected.

In working with this book, it is necessary that the student have constantly available a handbook of data pertaining to structural steel. Here will be found not only full information concerning all the sections rolled (weights, areas, dimensions, moments of inertia, etc.) but also the load-carrying capacity of beam and column sections as computed by the current specifications, data required in the detailing of structures, sundry mathematical tables, and a considerable amount of general information. Each student should become familiar with the contents of his handbook early in his study of structural design. Recently the Carnegie Steel Company and the Bethlehem Steel Company have discontinued publication of their well-known manuals, "Carnegie Pocket Companion" and "Bethlehem Manual of Steel Construction," which leaves "Steel Construction," published by the American Institute of Steel Construction (200 Madison Avenue, New York, N. Y.), the only handbook generally available. The A.I.S.C. is a service organization maintained by a group of steel fabricators and rolling mill operators.

1–2. Common Beam Theory. All elementary textbooks on strength of materials introduce the subject of bending stress in straight beams by consideration of the special case of the beam with a symmetrical cross section, with the (or an) axis of symmetry lying in the plane of the loads. Before proceeding further, the student should review this theory carefully in detail. For this case, it was noted that the neutral axis passes through the centroid of the section and is perpendicular to the plane of loads, and that the unit stress, s, in any fiber distant y from the neutral axis, equals My/I, where M is the resisting moment, the numerical equal of the bending moment at the section, and I the moment of inertia of the section about the neutral axis. The maximum unit stress occurs in the extreme fibers, distant c from the neutral axis, and is given by

$$s = \frac{Mc}{I} \qquad \left(\frac{\text{in-lb}\ \ \text{in.}}{\text{in.}^4} = \frac{\text{lb}}{\text{in.}^2}\right) \tag{1-1}$$

This formula may be written

$$s = \frac{M}{I/c} = \frac{M}{S} \tag{1-1A}$$

where $S = I/c$, a quantity given in the handbooks for steel sections, and called the *section modulus* (measure of a section, that is, of its resistance to bending).

It is important to note that the symmetrical loading of a symmetrical beam section, that is, the placing of the loads and supporting forces in a plane of symmetry of the cross section, results in bending with shear and without torsion. Pure bending is obtained when the beam is loaded by equal end couples in a plane of symmetry, and is not at all a common case. In order to determine the internal stress conditions in a beam for all loading conditions, we must be provided with an analysis which will give the relation between stress and load when the loading, transverse or consisting of couples, alone or in combination, causes bending with or without transverse shear, and with or without torsion. The torsional stress effect will be shear throughout with direct stress over a limited length. A complete treatment of all these cases belongs to a treatise on the theory of elasticity or advanced strength of materials, and only an elementary discussion is here attempted. It is evident that the first necessity of analysis is to secure a criterion for the position of loads which ensures bending without torsion. Since this involves the action of transverse shearing stress, it is necessary to precede discussion of bending by consideration of shearing stress in beams.

Problem 1–1. (*a*) Prove, for a rectangular section of breadth b and depth d parallel to the load, that Eq. 1–1 takes the form $M = \frac{1}{6}\, sbd^2$.

(*b*) Is there need for mathematical proof for the statement that a beam with plane of loading containing the longitudinal axis and an axis of symmetry of the cross section bends without twisting? Assuming that the internal stresses set up on one side of the load plane tend to cause twisting, what can be said of the corresponding tendency on the other side in view of the symmetrical state of beam and load?

1–3. Shear in Beams. The intensity of transverse shearing stress in a beam is given by the familiar equation (whose derivation should be checked by the student at this time)

$$s_s = \frac{VQ}{bI} \quad \left(\frac{\text{lb in.}^3}{\text{in. in.}^4} = \frac{\text{lb}}{\text{in.}^2}\right) \qquad 1\text{–}2$$

where s_s = unit shearing stress on longitudinal plane parallel to the neutral plane,

= unit shearing stress at the same point on a plane section perpendicular to the longitudinal axis,

V = external shear at the section,

Q = statical moment about the centroidal axis of the portion of the section beyond the plane of shear,

I = moment of inertia of the cross section about the centroidal axis,

b = lateral breadth of the cross section at the plane of shear.

Problem 1–2. (*a*) Demonstrate that for a rectangular cross section of width b and depth d the maximum intensity of shearing stress equals 1.5 times the average stress.

(*b*) Demonstrate that, when a shearing stress exists on some plane at any point in a body, a shear of equal intensity must exist at the same point on a plane perpendicular both to the direction of the first shear and to the plane on which it acts.

Problem 1–3. A 24-in. wide-flanged beam weighing 100 lb per ft (abbreviated as 24 WF 100) is subjected to an external shear of 112,000 lb at a certain section. Plot the variation of unit shearing stress from top to bottom of the section.

Ans. The shear will vary nearly as a straight line from zero at the extreme fiber to 337 lb per sq in. in the flange at the junction with the web, will change abruptly to 8650 lb per sq in. in the web at the junction with the flange, and then will increase along a parabola to a maximum of 11,000 lb per sq in., at mid-depth of beam. Sketch this curve to scale.

A study of the variation of shear intensity through the web of a rolled beam shows that the variation at different heights is not great. Consequently, the common method for specifying unit shearing stress is to give an allowable value based on dividing the external shear at the section by the area of the web — taken as the product of web thickness and beam depth. Applying this method to Prob. 1–3 gives a shearing stress of 10,000 lb per sq in., which is nearly 10 per cent less than the true maximum. The values allowed by specifications (10,000 to 13,500 lb per sq in.) were determined by consideration of the average maximum values attained.

Problem 1–4. A cast-iron lintel is shaped like an inverted T; it is 7 in. deep, flange 5 in. wide, with all metal 1 in. thick. What total uniformly distributed load can this lintel carry on an 8-ft span? Allowable unit stresses are

tension	3,000 lb per sq in.
compression	15,000 " " " "
shear	3,000 " " " "

Discussion. Compute the load permitted by shear as well as that by bending.

Ans. 5400 lb

1–4. The Shear Center.[1]

Until quite recently, it was supposed that for a beam to carry load without twisting, it was necessary that the plane of loading should contain the axis which pierces each cross section in its centroid. However, it has been found that the axis of no twist is not that through the centroids except in a cross section with two axes

[1] This discussion is based upon Chapter V of SEELY's "Advanced Mechanics of Materials" (John Wiley & Sons, 1932). The matter is gone into more deeply in *Bulletin* 211, University of Illinois Engineering Experiment Station, "The Torsional Effect of Transverse Bending Loads on Channel Beams," by SEELY, PUTNAM, and SCHWALBE. The earliest reference given by these authors for the solution of this problem is a thesis presented at the University of Zurich in 1920 by H. SCHWYZER

of symmetry or with that variety of symmetry possessed by the Z-bar. The longitudinal axis of a straight prismatic beam, through which the transverse loads must act if there is no torsion, is called the *bending axis,* and its trace in a cross section, the *center of twist* or *shear center.* The reason for these names will be made apparent by consideration of the action of a structural channel.

In Fig. 1–1*a* is shown a portion, *AB*, of a channel, used as a cantilever

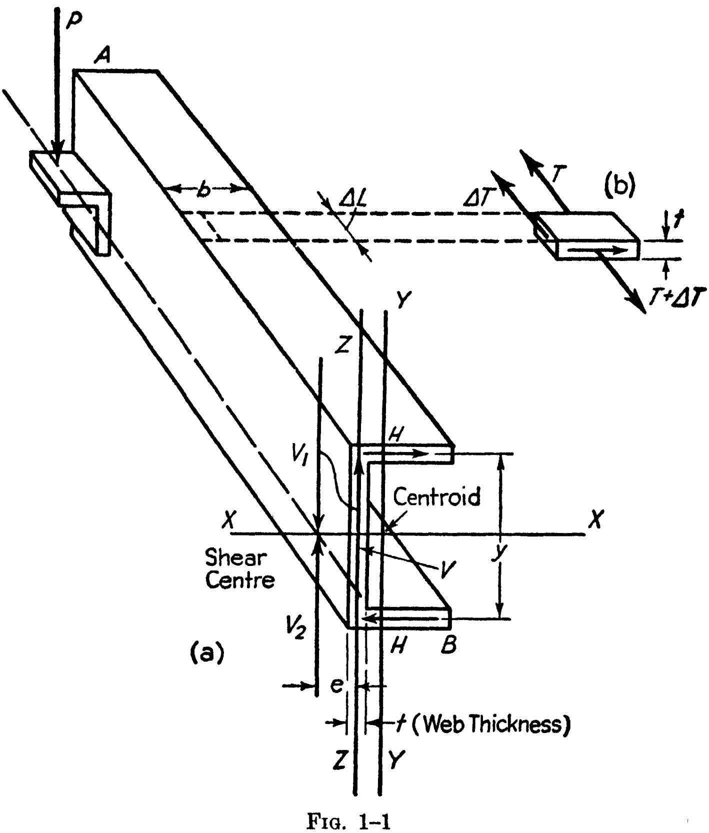


Fig. 1–1

beam, loaded with load P at the free end, A, the line of action of the load cutting the bending axis. At section B the resisting shear, $V = P$, is shown acting vertically on the web. There will be shear, H, acting horizontally on the flanges, as may be seen by consideration of the free body formed by a portion of the top flange (Fig. 1–1*b*), between two nearby sections, with vertical plane of junction with the web. The increment of flange stress, ΔT, constitutes a shear on this plane of

junction, and accordingly there must be shear on the faces of the free body perpendicular to this first shear and the junction plane on which it is developed. There will be no change in the equilibrium nor in the deformation of free body AB by applying two equal and opposite forces, V_1, V_2, equal in magnitude to the load P, at the *shear center* of section B. The couple composed by P at section A and V_2 at B causes bending about the X axis; the couple V_1e tends to twist the channel in one direction; and the couple Hh, in the other. Since there is no twisting, these two couples are equal.

Were the load P in Fig. 1–1a applied at the center of the web, it is evident that the free body AB could not be in equilibrium as shown, since the torsional couple Hh would not be balanced. Consequently, by hypothesis, this channel deflects without twisting. Analysis and experiment demonstrate that any lateral position of the load P other than that shown will result in twisting until the torsional shear develops sufficient resistance to maintain equilibrium.

An approximate expression locating the shear center of a channel may be derived as follows. The intensity of the shear at the junction of the web and the flange is, by Eq. 1–2,

$$s_s = \frac{V(b - t)\,(t')\,(y/2)}{t'I}$$

Assuming that the shear in the flange varies uniformly from a maximum at the junction with the web to zero at the edge, we have, since $Ve = Hy$,

$$e = \frac{Hy}{V} = \frac{s_s\,(b - t)\,(t')y}{2\,V}$$

Substituting the value of s_s, and noting that $I = 2(b - t)\,(t')\,(y/2)^2 + ty^3/12$, gives

$$e = \frac{\dfrac{b - t}{2}}{1 + \dfrac{ty}{6\,(b - t)\,t'}}$$

The location of the shear center is thus seen to be a function of the cross section and independent of the load. It is to be noted here that e is always less than one-half of the width of the outstanding flange and that it decreases as the ratio of the web area to the area of the outstanding flange increases.

Consideration of the equilibrium of free body AB, Fig. 1–1a, makes plain that the resultant shear on the cross section shown acts through the shear center, thus justifying the term.

If the plane of forces on a channel beam cuts all cross sections on the axis of symmetry, bending without torsion results. If the plane of loading passes through the shear center, making any angle with the web, again no torsion results, for the loads may be divided into components along the axis of symmetry and normal to it through the shear center, and neither component causes twisting.

Problem 1–5. Show that the shear center lies at the centroid of the cross section in

(*a*) an I-beam.
(*b*) a rectangular beam.
(*c*) a Z-bar.

Problem 1–6. Show that the shear center for an angle section lies at the intersection of the long axes of the two rectangles which make up the angle.

1–5. General (or Unsymmetrical) Bending.[1] Although the majority of straight beams are symmetrical of section and loading, many are not, notably *purlins*, the horizontal beams supporting sloping roofs, pictured in Fig. 1–2. It is important that the designer's primary thinking of

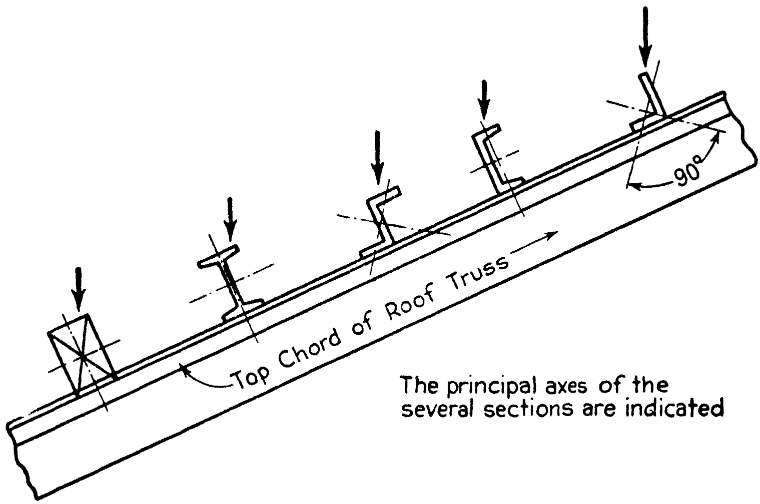
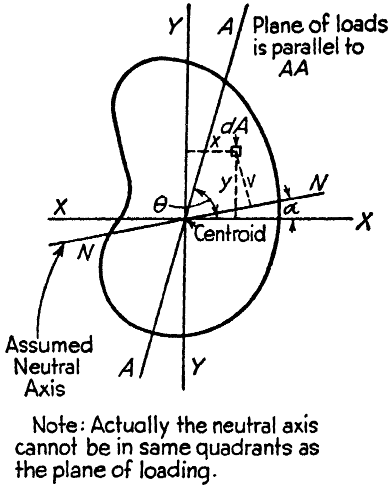


FIG. 1–2

beams shall be in terms of the general (unsymmetrical) case. The importance of this fundamental theory justifies in the following paragraphs a repetition of the derivation which will be made from a standpoint somewhat different from that adopted in a first course in strength of materials.

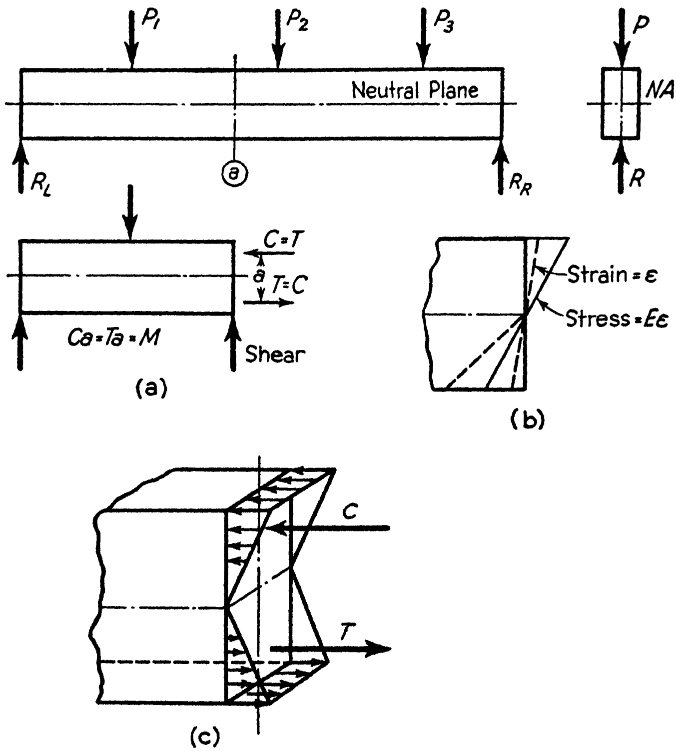
In Fig. 1–3 there is shown any cross section of a straight beam of homogeneous material, of any shape consistent with freedom from local buckling

[1] Probably the introduction of this theory in this country was by these articles: " A General Formula for the Normal Stress in Beams of Any Shape," by Professor George F. SWAIN, *Van Nostrand's Engineering Magazine*, 1880; " The Determination of Unit Stresses in the General Case of Flexure," by Professor L. J. JOHNSON, *Journal* of the Associated Engineering Societies, May, 1902; " An Analysis of General Flexure in a Straight Bar of Uniform Cross Section," also by Professor JOHNSON, *Transactions*, American Society of Civil Engineers, 1906.



Y
A
Plane of loads is parallel to AA
dA
x
y
v
N
X
θ
α
X
Centroid
N
Assumed Neutral Axis
A
Y
Note: Actually the neutral axis cannot be in same quadrants as the plane of loading.

FIG. 1–3



P_1
P_2
P_3
P
Neutral Plane
NA
R_L
a
R_R
R
C=T
a
T=C
Ca=Ta=M
Shear
(a)
Strain = ε
Stress = Eε
(b)
C
T
(c)

FIG. 1–4

and other weakness, uniform in section, loaded through the shear center by transverse loads so that only shear and bending stresses are brought into play on the cross section, with no torsion. Assuming this to be a simple end-supported beam in a horizontal position for simplicity in discussion, like, for example, the rectangular beam of Fig. 1–4, observation would show that the upper longitudinal fibers are shortened as by a compressive stress, that the lower fibers are elongated in tension, and an intermediate layer of fibers is unchanged in length. Since there is no twist, these unstressed fibers must lie in a plane (when the beam is unstressed) called the *neutral plane*, and the trace of this plane on the cross section is the line NN, the *neutral axis*.

Structural beams are made of materials which obey HOOKE's law (1678) that stress (force per unit area) is proportional to strain (deformation per unit of length) up to the elastic limit of the material. This obedience either is precise or is approximate to such a degree that exact agreement may be reasonably assumed. Observation of beams shows that a straight line scribed on the observed surface of the unloaded beam perpendicular to the longitudinal axis remains a straight line when the beam is loaded. Since this straight line is a trace of a plane section, this observation demonstrates NAVIER's (1785–1836) hypothesis that plane sections before bending remain plane after bending. Actually, there must be some deviation from straightness due to the action of shear, but since this deviation is not easily observable in beams of usual proportions, it may be disregarded. Consideration of two closely spaced normal plane sections of a beam before and after loading will show that strain is proportional to distance from the neutral plane. Assuming that HOOKE's law holds for the material in beam form as well as for the axially loaded member used in demonstrating the law (which is equivalent to assuming that each longitudinal fiber acts without constraint from adjoining fibers) it results that stress is also proportional to distance from the neutral plane.

Application of the laws of static equilibrium of a non-concurrent coplanar force system, to the free body consisting of one end of a horizontal simple beam, up to any normal cross section taken between supports, will give additional limited information concerning the internal stresses; Fig. 1–4a. The internal stresses acting on the free body at the section are those actually exerted by the portion of the beam removed. Since $\Sigma V = 0$, the tangential or shearing component of these stresses must equal the external shear at the section (the resultant of all the transverse forces acting on the free body beyond the section); since $\Sigma H = 0$, the resultant of the normal compressive forces must equal the total normal tensile force; since $\Sigma M = 0$, this couple thus constituted must equal in magnitude and oppose in direction the moment of the external forces acting on the free body about any point in the section (resisting moment equals bending moment).

A point of the neutral axis in Fig. 1–3 may be located by observing the expression set up to give the total normal stress, N, on the section, which, as already noted, equals zero. Letting s_1 equal the intensity of normal stress a unit distance from the neutral axis along the plane of the section, and v the distance of any elementary area, dA, from that axis, results in the expression $N = \int s_1 v\,dA = 0$; whence $\int v\,dA = 0$, which can be true only when the axis from which the distances are measured is the centroidal axis.

The important principle results that **in simple bending the neutral axis passes through the centroid of the cross section.**

The centroid of the section thus becomes the reference origin for our analysis, and for reference axes we shall take X and Y, the principal axes[1] of the section for that point in Fig. 1–3. Through the origin is drawn the line AA, parallel to the plane of loads, which plane must pass through the shear center. The plane of the external bending moment, then, is perpendicular to the plane of the paper and parallel to AA.

Consider the free body consisting of a portion of the beam from the end up to any cross section, acted upon by a force system consisting of external forces and of the internal fiber stresses at the section. The application of the three equations of equilibrium has already been discussed. The detailed application of the equation $\Sigma M = 0$ will give the relations sought between stress, bending moment, and section properties. From the condition $\Sigma M = 0$ about the X axis, Fig. 1–3, we may write

$$M \sin \theta = \int s_1 v \, dA \cdot y$$

the coordinates of elementary area dA being x and y referred to axes X and Y. Since $v = y \cos \alpha - x \sin \alpha$, this expression becomes

$$M \sin \theta = \int s_1 (y^2 \cos \alpha - xy \sin \alpha) \, dA$$

On noting that $\int y^2 \, dA = I_X$ and that $\int xy \, dA = K_{XY} = 0$, this takes the form

$$M \sin \theta = s_1 I_X \cos \alpha \tag{A}$$

In similar fashion, taking $\Sigma M = 0$ about the Y axis, we obtain

$$M \cos \theta = -s_1 I_Y \sin \alpha \tag{B}$$

Combining these two expressions gives

$$\tan \alpha = -\frac{I_X}{I_Y} \cot \theta \tag{1–3}$$

which completes the location of the neutral axis for any slope of the plane of loading. Since both θ and α were assumed as positive angles the negative sign in Eq. 1–3 signifies that these two angles actually are always of opposite sign, never lying in the same quadrants. For the position of the plane of loading shown in Fig. 1–3 the neutral axis must lie in the second and fourth quadrants with a negative angle.

The intensity of fiber stress on the area dA is

$$\begin{aligned} s &= s_1 v \\ &= s_1 (y \cos \alpha - x \sin \alpha) \end{aligned}$$

Substituting the values of $s_1 \cos \alpha$ and $s_1 \sin \alpha$ from equations (A) and (B) above gives

$$s = \frac{(M \sin \theta) y}{I_X} + \frac{(M \cos \theta) x}{I_Y} \tag{1–4}$$

Inspection of Eq. 1–4 shows at once that it is simply the repetition of the common beam formula (Eq. 1–1), applied separately to each component

[1] See Appendix A, p. 305.

of loading along a principal centroidal axis; that the fiber stress at any given point in a cross section for this case is simply the algebraic sum of the stresses due to each component of bending moment along a principal axis[1] as determined by the common beam formula. Eq. 1–1 is seen to be the form taken by Eq. 1–4 when $\theta = 0°$ or $90°$, the coordinates being for an extreme fiber. Eq. 1–3 shows that when the plane of loads includes one principal axis the neutral axis is the other; and that *for all other cases the trace of the loading plane does* **not** *make an angle of* 90° *with the neutral axis.* When dealing with symmetrical sections it is remembered that an axis of symmetry is a principal axis.

In substituting actual values in Eq. 1–4 it is not necessary to think of the expression mathematically in terms of the signs of x and y. It is simpler to consider the kind of stress at the point in question, caused by each component of moment in turn, and to combine them algebraically, with the sign chosen to denote the resulting stress.

Problem 1–7. (*a*) Demonstrate that the conservation of plane sections in a beam results in strain being proportional to distance from neutral plane.

(*b*) Is it essential to planar distribution of stress (s varies with y) that plane sections be conserved (remain plane after bending)? Is distortion of plane sections on bending inconsistent necessarily with strain being proportional to distance from neutral plane?

(*c*) Sketch the elevation of a beam with two parallel plane sections close together, in the region of positive shear. Sketch the shape taken by the traces of these two planes when the beam bends, considering the action of shear alone. (Divide the area between the sections into a series of parallelograms, and consider the distorted shape of each parallelogram, beginning at the neutral plane.)

(*d*) Will the center cross section of a simple beam with loading symmetrical about the center of the span be distorted by bending? by shear? If there is planar distribution of stress on every cross section what does this imply regarding the distortion of every other plane section? Is this consistent with the presence of shear in the end portions of the beams?[2]

(*e*) Derive Eq. 1–1.

(*f*) Are the forces acting on beams actually applied at points, and do their lines of action lie in a plane?

(*g*) Sketch an isometric view of a horizontal simple end-supported rectangular timber loaded with a single concentrated vertical load at the center with lines of action of load and reactions determining a vertical plane parallel to, but not containing, the longitudinal axis. Show that this piece is subjected to torsion. *Suggestion.* Remember that the application of two equal and opposite forces at a point does not change the state of stress of a member. Place such a vertical pair opposite each force on the vertical axis of symmetry.

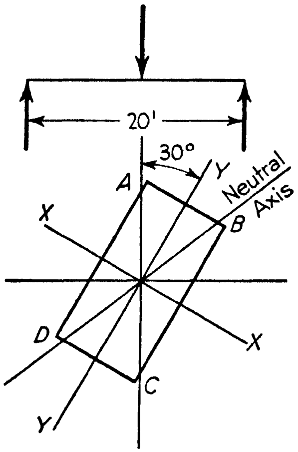
Ans. (*b*) No. No. (*c*) Remember variation of shear intensity and keep horizontal sides of parallelograms horizontal, giving a reversed curve, concave to the right

[1] It should be understood that, whenever principal axes are referred to in any discussion of stress analysis, centroidal axes are indicated. There is a pair of principal axes for each point in a cross section.

[2] The inconsistency here pointed out is considered by SWAIN on p. 219 of his "Strength of Materials." The approximation of the usual theory is thus indicated. More exact theories will be found in textbooks on theory of elasticity, but they are not practical or entirely satisfactory in themselves.

above and to the left below. (*d*) No. All other sections must remain plane if strain is to be proportional to distance from neutral plane. No; these end sections must be somewhat distorted by shear. This action is small except in very short and deep beams.

Example 1-1. A timber beam is loaded to produce a maximum bending moment of 150,000 in-lb without torsion. The beam has a section 6 in. $\times$ 12 in. and the plane of loading makes an angle of 30° with the long side as shown. Determine the fiber stresses at the corners of the section, and find the angle between the plane of loading and the neutral plane.



Ex. 1-1

Solution. The component of moment parallel to the Y axis, that is, the component producing bending about the X axis, is 150,000 cos 30° = 130,000 in-lb. Owing to this moment there will be a compressive stress of

$$\frac{130{,}000}{\frac{6 \times 12^2}{6}} = 900 \text{ lb per sq in.}$$

along the line AB and a tensile stress of the same amount along CD.

The moment causing bending about the Y axis is 150,000 sin 30° = 75,000 in-lb. The corresponding fiber stresses will be

$$\frac{75{,}000}{\frac{12 \times 6^2}{6}} = 1040 \text{ lb per sq in.}$$

compression along line AD and the same amount of tension along BC.

Combining these values there is obtained:

Compression at A	= 1 940 lb per sq in.
Tension at B	= 140 " " " "
Tension at C	= 1 940 " " " "
Compression at D	= 140 " " " "

The stress changes sign between B and A, and, therefore, some point on this line is also a point on the neutral axis. The distance from B to this point will be, by similar triangles, with straight-line stress variations, $6 \times 140/(140 + 1940) = 0.40$ in., and the distance from the Y axis to the point will be $3 - 0.40 = 2.60$ in. Hence, the angle between the Y axis and the neutral axis is $\tan^{-1} = 2.6/6$, or 23.5°. The angle between the plane of loading and the neutral plane is 30° + 23.5° = 53.5°.

Suggestion. Check by means of Eq. 1-3.

Example 1-2. Compute the center deflection of the beam of Ex. 1-1. Does the bent longitudinal axis of this beam lie in a vertical plane? If not, what is indicated regarding torsion? E = 1,500,000 lb per sq in.

Solution. Expressed in terms of bending moment the expression for deflection is

$$\frac{WL^3}{48\,EI} = \frac{ML^2}{12\,EI}, \quad M \text{ being } \frac{WL}{4}$$

The Y deflection is

$$y = \frac{130{,}000 \times 20^2 \times 144}{12 \times 1{,}500{,}000 \times 864} = 0.48 \text{ in.}$$

The X deflection is

$$x = \frac{75{,}000 \times 20^2 \times 144}{12 \times 1{,}500{,}000 \times 216} = 1.11 \text{ in.}$$

The vertical deflection is 0.98 in., and the horizontal, 0.72 in. to the right. (Prove that the resultant is normal to the neutral axis!)

Since the line of action of the load moves out of the vertical plane through the end supports, twisting must result if the ends are held without tipping. This indicates that the general theory is not consistent or complete. An exact theory is very complicated, involving as it does an account of the side deflection.

Problem 1–8. Find the position of the neutral axis in the beam of Ex. 1–1 above if the plane of loading coincides with a diagonal of the section. Determine the fiber stresses at the corners. Locate the resultants of the tensile and compressive forces. Do they satisfy the condition that a couple can be balanced only by another couple in the same or a parallel plane? Compute the magnitudes of these resultants, and show that the moment of the couple they form equals the external moment at the section.

Problem 1–9. Using Eq. 1–3 prove that, for any rectangular beam of height h and width b, the neutral axis lies in one diagonal when the plane of loading cuts the section in the other diagonal.

Eq. 1–4 may be obtained by equating the component of external moment perpendicular to the neutral axis, to the similar component of resisting moment: $M \sin(\theta - \alpha) = \int s_1 v^2\, dA = s_1 I_N = s I_N/v$, which gives Eq. 1–4 when the I_N is replaced by its equivalent in terms of the moments of inertia about the X and Y axes, and v replaced by its equivalent in terms of the x,y coordinates of dA. Another possible procedure is to equate the components of external and internal moment parallel to the neutral axis; $M \cos(\theta - \alpha) = \int s_1 vw\, dA = sK/v$; etc., w being the abscissa of dA corresponding to ordinate v, and K being the product of inertia for axis NN and another perpendicular to it through the centroid. The two expressions given above implicitly for s may be used for its determination, evidently, when the neutral axis has been located. This is illustrated in Ex. 1–4, p. 16.

If the centroidal axes, X and Y, used in the derivation of Eq. 1–3 and 1–4, had been any general pair of rectangular axes instead of the principal centroidal axes, the resulting equations would have taken the forms

$$\tan \alpha = \frac{I_X - K_{XY} \tan \theta}{K_{XY} - I_Y \tan \theta} \qquad \text{1–3}A$$

$$s = \frac{M \sin \theta\, (I_Y \cdot y - K_{XY} \cdot x)}{I_X I_Y - K_{XY}^2} + \frac{M \cos \theta\, (I_X \cdot x - K_{XY} \cdot y)}{I_X I_Y - K_{XY}^2} \qquad \text{1–4}A$$

In order to use these forms it is not necessary to locate the principal axes and they are, therefore, sometimes easier to apply than (1–3) and (1–4). However, (1–4A) has the disadvantage that its use is bound to be mechanical while (1–4), referred to principal axes, involves familiar operations easily visualized.

Remembering that K_{XY} equals zero for principal axes, (1–3) and (1–4) may be readily derived from (1–3A) and (1–4A).

A useful form for the unsymmetrical bending equation is that suggested by Professor CROSS[1] which is obtained from Eq. 1–4A as follows.

Let

$$M_X = M \sin\theta$$
$$M_Y = M \cos\theta$$
$$M'_X = M_X - M_Y \frac{K_{XY}}{I_Y}$$
$$M'_Y = M_Y - M_X \frac{K_{XY}}{I_X}$$
$$I'_X = I_X - \frac{K^2_{XY}}{I_Y}$$
$$I'_Y = I_Y - \frac{K^2_{XY}}{I_X}$$

As usual, each subscript indicates the axis about which the moment or for which the moment of inertia is computed. Multiplying both numerator and denominator of Eq. 1–4A by $1/I_X I_Y$ and introducing the "skew" terms, the designation Professor CROSS employs for the prime values of M and I above, gives Professor CROSS's equation

$$s = \frac{M'_X y}{I'_X} + \frac{M'_Y x}{I'_Y} \qquad \text{1–4B}$$

This expression has the advantage of being in the simple form of Eq. 1–4 for all centroidal axes. It is identical with Eq. 1–4 when the principal axes are used, the skew terms with K being zero. Its advantage for cases where the principal axes are not given comes from the fact that the procedure is identical with that where the principal axes are used, except at the very end when the corrections embodied in the skew terms are added. The propriety of the designation " skew " terms is evident since they come into the problem only when the reference axes chosen make an angle with — that is, are skew to — the principal axes.

Example 1–3. Determine by means of Eq. 1–4 the fiber stresses at all exterior corners of an angle $8 \times 6 \times \frac{1}{2}$ which is stressed by a moment of 80,000 in-lb acting parallel to the long leg, without torsion.

Solution. It is known from Ex. A–1, p. 306, that the principal axes make an angle of $\phi = 29° \, 10'$ ($\sin\phi = 0.487$, $\cos\phi = 0.873$) with the sides and that the values of $I_{\text{max.}}$ and $I_{\text{min.}}$ are 54.49 and 11.50 in.[4] KETCHUM'S "Structural Engineers' Handbook " gives the angle between the principal centroidal axes and those parallel to the legs of the section, which facilitates the solution of problems of this sort.

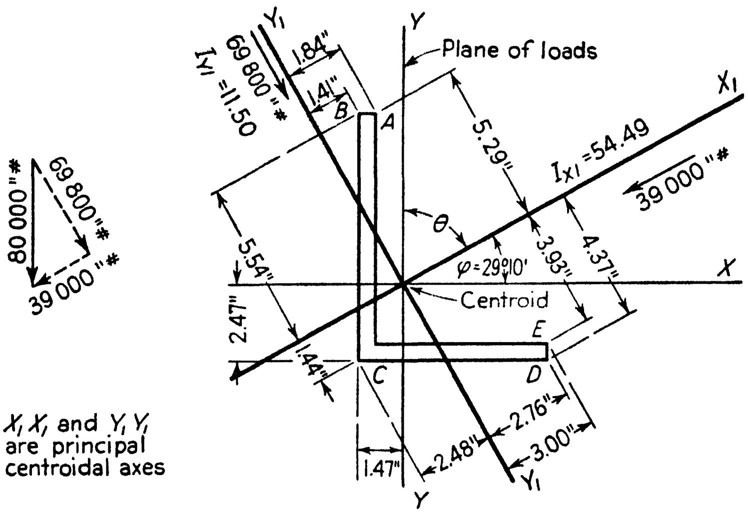
[1] *Bulletin* 215, University of Illinois Engineering Experiment Station, "The Column Analogy," by Hardy CROSS. Professor CROSS uses a different notation from that here given.

The coordinates of the corners of the figure with respect to the principal axes may be scaled from a carefully made sketch or may be computed by means of the relation

$$x_1 = x \cos \phi + y \sin \phi$$
$$y_1 = y \cos \phi - x \sin \phi$$

In the present solution this latter method was used, the computations being given in the table which follows.

The components of bending moment parallel to the principal centroidal axes were found to be 39,000 and 69,800 in-lb as recorded on the diagram. The tabulation is self-explaining. There is no particular need of trying to identify the angle θ (which is $90° - \phi$) for use in Eq. 1–4; it is just as easy to use the angle given, 29° 10′, remembering that the bending moment is being resolved into components along the principal axes.



Ex. 1–3

Point	x	y	$x(0.873)$	$y(0.487)$	$x_1 =$ $x \cos \phi$ $+ y \sin \phi$	$y(0.873)$	$x(0.487)$	$y_1 =$ $y \cos \phi$ $- x \sin \phi$
A	−0.97	+5.53	−0.85	+2.69	+1.84	+4.82	−0.47	+5.29
B	−1.47	+5.53	−1.28	+2.69	+1.41	+4.82	−0.72	+5.54
C	−1.47	−2.47	−1.28	−1.20	−2.48	−2.16	−0.72	−1.44
D	+4.53	−2.47	+3.96	−1.20	+2.76	−2.16	+2.21	−4.37
E	+4.53	−1.97	+3.96	−0.96	+3.00	−1.72	+2.21	−3.93

$$s = \frac{(M \sin \theta)y}{I_X} + \frac{(M \cos \theta)x}{I_Y} \qquad 1\text{–}4$$

The signs may be determined by inspection. ($-$ Compression, $+$ Tension.)

$$s_A = -\frac{69{,}800 \times 5.29}{54.49} - \frac{39{,}000 \times 1.84}{11.50} = -13{,}010 \text{ lb per sq in}$$

$$s_B = -\frac{69{,}800 \times 5.54}{54.49} - \frac{39{,}000 \times 1.41}{11.50} = -11{,}870$$

$$s_C = +\frac{69{,}800 \times 1.44}{54.49} + \frac{39{,}000 \times 2.48}{11.50} = +10{,}250$$

$$s_D = +\frac{69{,}800 \times 4.37}{54.49} - \frac{39{,}000 \times 2.76}{11.50} = -3{,}760$$

$$s_E = +\frac{69{,}800 \times 3.93}{54.49} - \frac{39{,}000 \times 3.00}{11.50} = -5{,}130$$

Note. If the fiber stresses were computed by the somewhat common method of using I about the X axis, and measuring y from that axis, the following results would be obtained:

$$s_A = s_B = \frac{80{,}000 \times 5.53}{44.31} = -9980 \text{ lb per sq in.}$$

$$\text{Error for } A\text{: } \frac{13{,}010 - 9980}{13{,}010} = 23\%$$

$$s_C = s_D = \frac{80{,}000 \times 2.47}{44.31} = +4450 \text{ lb per sq in.}$$

$$\text{Error for } C\text{: } \frac{10{,}250 - 4450}{10{,}250} = 57\%$$

Evidently this slovenly application of the flexure formula grossly overestimates the strength of the angle.

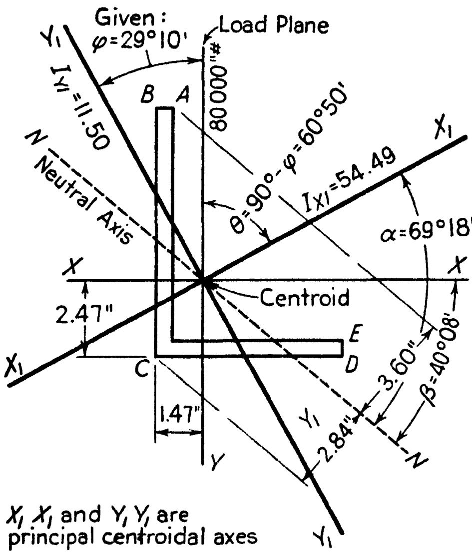
Example 1–4. Same problem as Ex. 1–3. Result to be obtained by locating the neutral axis, and the component of moment and I about the neutral axis.

Solution. As before, note that Ex. A–1, p. 306, has located the principal axes and determined the maximum and minimum moments of inertia; these data are recorded on the figure. Using Eq. 1–3

$$\tan\alpha = -\frac{I_X}{I_Y}\cot\theta = -\frac{I_X}{I_Y}\tan\phi = -\frac{54.49}{11.50}(0.558) = -2.645$$

$$\alpha = -69^\circ\ 18'$$

After the neutral axis is located it is seen that A and C are the most stressed points. The distances of these points from the neutral axis were obtained by means of the tabulation below, starting with the known coordinates of the points related to the X and Y axes. The angle between the X and N axes is $\beta = \alpha - \phi = 40^\circ\ 08'$.



Ex. 1–4

Point	x	y	$y(0.765)$	$x(-0.645)$	$y_2 = y\cos\beta - x\sin\beta$
A	−0.97	+5.53	+4.23	+0.63	+3.60
C	−1.47	−2.47	−1.89	+0.95	−2.84

$$\text{Moment about neutral axis} = 80{,}000 \cos 40^\circ\, 08' = 61{,}200 \text{ in-lb}$$

$$I_{na} = \cos^2 \alpha I_{X1} + \sin^2 \alpha I_{Y1} = (0.353)^2\, 54.49 + (-0.935)^2\, 11.50 = 6.80 + 10.05 = 16.85 \text{ in.}^4$$

$$s_A = -\frac{61{,}200 \times 3.60}{16.85} = -13{,}070 \text{ lb per sq in.}$$

$$s_C = +\frac{61{,}200 \times 2.84}{16.85} = +10{,}320 \text{ lb per sq in.}$$

Problem 1–10. (*a*) Derive Eq. 1–3*A* and 1–4*A*. Follow the procedure already employed.

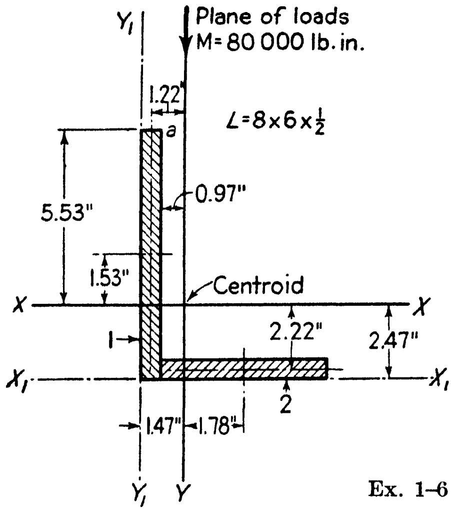
(*b*) Derive Eq. 1–4*B* from 1–4*A*. Note the directions given in the text.

Example 1–5. Same problem as Ex. 1–3 and 1–4, solved by Eq. 1–4*A*.
Solution. From Ex. A–1, p. 306.

$$K_{XY} = -18.34$$
$$I_X = 44.31$$
$$I_Y = 21.67$$

Since $\theta = 90°$, the second term on the right side of Eq. 1–4A equals zero.

$$s_a = \frac{M[(21.67)\,(5.53) - (-18.34)\,(-0.97)]}{(44.31)\,(21.67) - (-18.34)^2}$$
$$= 13{,}070 \text{ lb per sq in. compression.}$$



Ex. 1–6

Example 1–6. Same as Ex. 1–3, p. 14. Solution by Eq. 1–4B. The data are completely shown in the figure.

Note. The form of tabulation is closely that recommended by Professor Cross.

Properties of Section										
Given					Computed					
					Area	Statical Moments		Products of Inertia		
Member	Length	Width	x	y				I_Y	I_X	K
					a	ax	ay	$ax^2 + I_0'$	$ay^2 + I_0''$	$axy + K_0$
1	8.0	0.5	+0.25	+4.00	4.00	+1.00	+16.0	0.25 0.08	64.0 21.3	+4.0 0
2	5.5	0.5	+3.25	+0.25	2.75	+8.94	+ 0.7	29.0 6.93	0.2 0.1	+2.24 0
Correct to Centroid			+1.47	+2.47	6.75	+9.94	+16.7	36.26 −14.6	85.6 −41.2	6.24 −24.5
Product of inertia: Centroidal axes.............								21.7	44.4	−18.3

Skew terms:

$$M_X = 80{,}000 \text{ in-lb}$$
$$M_Y = 0$$
$$M'_X = M_X - M_Y \frac{K}{I_Y} = 80{,}000 \text{ in-lb}$$
$$M'_Y = M_Y - M_X \frac{K}{I_X}$$
$$= -80{,}000 \times \frac{-18.3}{44.4} = +33{,}000 \text{ in-lb}$$
$$I'_X = I_X - \frac{K^2}{I_Y} = 44.4 - \frac{18.3^2}{21.7} = 28.9$$
$$I'_Y = I_Y - \frac{K^2}{I_X} = 21.7 - \frac{18.3^2}{44.4} = 14.2$$
$$s_a = \frac{80{,}000 \times 5.53}{28.9} + \frac{33{,}000\,(-0.97)}{14.2}$$
$$= 15{,}300 - 2{,}250$$
$$= 13{,}050 \text{ lb per sq in. compression.}$$

Discussion. In order to illustrate the procedure in the general case of a built-up section where the centroid is not given, the solution starts with reference axes X_1 and Y_1 along the outer edges of the section with origin at the vertex of the angle. The tabulation proceeds first with the location of the centroid, the section being divided into the two parts shown. Next comes the computation of the moments of inertia and the product of inertia under the general heading of "Products of Inertia." The term I'_0 indicates the moments of inertia of the two divisions about a vertical axis through their centroids; I''_0, the moments of inertia of the divisions about a horizontal axis through their centroids; the products of inertia, K_0, for the divisions for these axes are zero.

In the horizontal division headed "Correct to Centroid" appear the terms Ad^2 and abA in the familiar expressions

$$I_{X1} = I_X + Ad^2 \qquad \text{(A–3, p. 303)}$$

$$K_1 = K_0 + abA \qquad \text{(A–4, p. 304)}$$

For clearness, the skew corrections were added separately below the table; it is plain that the corrections could have been entered without a record of their computation in the table, giving I'_X and I'_Y as the result of the tabulation, a more workmanlike computation than that given. Note that the signs are automatic, the positive result indicating compression as the nature of the stress for point A. Here x and y were measured as usual, positive above and to the right of the origin. The moment was taken as positive, causing compression in the top fiber; acting in the same plane and causing tension in the top fiber, it would have been negative. Had the moment been in the plane of the X axis, causing compression in the extreme fiber to the right of the origin, it would have been positive moment; causing tension in that fiber, negative moment.

1–6. The S-Line. It is possible to construct a line which will make simple the computation of fiber stress at any point in a cross section for any possible position of the plane of bending. The following is the method of obtaining this line.

Eq. 1–4 may be rewritten

$$s = M \frac{I_Y y \sin \theta + I_X x \cos \theta}{I_X I_Y}$$

$$= M \div \frac{I_X I_Y}{I_Y y \sin \theta + I_X x \cos \theta}$$

The divisor of M may be called the section modulus, S, of the given point for any value of θ, the angle which the plane of loading makes with the X axis, the X and Y axes being principal axes. That is,

$$S = \frac{I_X I_Y}{I_Y y \sin \theta + I_X x \cos \theta} \qquad 1\text{-}5$$

In this expression θ is the only variable since x and y are the coordinates of the point whose stress is being computed and I_X and I_Y are the values of the moments of inertia about the principal axes. The student will note that this is the polar equation of a straight line.

For any given point whose stress is desired it is possible to plot on the lines representing the various positions of the plane of bending a series of points whose distances from the origin, the centroid of the section, are equal to the value of S for the corresponding position of the bending plane. The locus of these points for all possible positions of the plane of bending is the section-modulus line, the S-line for the point. The practical value of this line is enhanced by the fact that it is a *straight* line (see Prob. 1–11) and is easily plotted by drawing a line through the intercepts on the X and Y axes (the values of S for $\theta = 0°$ and $\theta = 90°$).

Problem 1–11. For a point x, y, in the first quadrant of any cross section, as in Fig. 1–3, plot the value S for $\theta = 0$, Oa to the right on the X axis; for $\theta = 90°$, Ob upward on the Y axis; for $\theta > 90°$, $< 180°$, a distance Oc upward to the left on a line of the given inclination. Prove that a, b, and c are on a single straight line (a) by identifying Eq. 1–5 as the equation of a straight line in polar form; (b) by demonstrating that triangle aOb equals in area the difference between triangles bOc and aOc.

Since fiber stress equals M/S it is seen that stress varies inversely as S. Clearly the value of S for any point will be least when the plane of loading cuts the section at right angles to the S-line for the point. This position of the plane of loading makes the stress at the point a maximum.

Similarly, the value of S will be greatest, and the stress least, when the plane of loading cuts the section in a line parallel to the S-line. For this position of the plane of loading the neutral axis must pass through the point since its stress has zero value.

Reference to Prob. 1-11 will show that the S-line, at the point where it is nearest to the origin, passes through the quadrant in which the point itself lies. The character of stress may best be determined by inspection. For example, since the neutral axis in simple bending passes through O, the portion of the load line on one side of O will lie in a region of one kind of stress, that on the other side of O in a region of stress of the opposite character. The stress at the point for which the S-line is constructed will be of the same character as the stress along that portion of the load line which cuts the S-line.

1-7. The S-Polygon. When the cross section of a beam in simple bending is angular, it is evident that one of the corners — or two if the neutral axis is parallel to an adjacent boundary line — will be farthest from the neutral axis and therefore most stressed. The S-lines drawn for the extreme corners of the cross section give all the S values needed for analysis. Later it will be clear that only those parts of the S-lines which form a closed polygon — the S-polygon — are required.

This S-polygon might be constructed by computing for each S-line the intercepts on the principal axes after the manner of the last article. Usually, however, greater accuracy may be obtained by computing the coordinates of the points where the S-lines intersect, that is, the apices of the S-polygon. For any point A — coordinates x_A, y_A — the S-line intersects the Y axis at the point I_X/y_A and the X axis at the point I_Y/x_A. The equation of the straight line passing through these points[1] is

$$y = -\frac{I_X x_A}{y_A I_Y} x + \frac{I_X}{y_A}$$

The equation of the S-line corresponding to another point B is

$$y = -\frac{I_X x_B}{y_B I_Y} x + \frac{I_X}{y_B}$$

These equations may be solved simultaneously to get the coordinates x_{ab}, y_{ab} of the apex of the S-polygon at which the lines meet. (Fig. 1-5.)

[1] Obtained from the slope form of the equation of a straight line

$$y = mx + b$$

where m is the slope of the line — positive upward to the right — and b is the distance from the origin to the intersection of the line and the Y axis.

Making this solution

$$x_{ab} = -\frac{(y_A - y_B)I_Y}{x_A y_B - x_B y_A} \qquad \text{1-6}A$$

$$y_{ab} = \frac{(x_A - x_B)I_X}{x_A y_B - x_B y_A} \qquad \text{1-6}B$$

In these equations all distances are measured from the principal axes and the values of I are computed for these same axes. The plot may be made to any convenient scale.

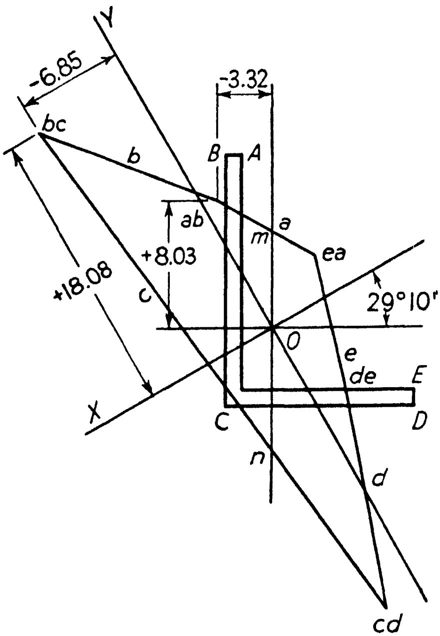


FIG. 1-5

Problem 1-12. Prove that the apex of an S-polygon formed by the intersection of the S-lines for points $A(x_A, y_A)$ and $B(x_B, y_B)$ when referred to axes X' and Y', which make the angle ϕ with the principal axes X and Y, has for its coordinates,

$$x'_{ab} = \frac{-(y'_A - y'_B)I'_Y + (x'_A - x'_B)K'_{XY}}{x'_A y'_B - x'_B y'_A}$$

$$y'_{ab} = \frac{(x'_A - x'_B)I'_X - (y'_A - y'_B)K'_{XY}}{x'_A y'_B - x'_B y'_A}$$

Example 1-7. Plot the S-polygon for an angle $8 \times 6 \times \frac{1}{2}$ by means of Eq. 1-6A and B.

Solution. Fig. 1–5. The numerical work of this example is greatly reduced by using the results of Ex. 1–3, p. 14, and Ex. A–1, p. 306. This fact must be kept in mind in making a comparison of methods.

$$x_{ab} = \frac{-(y_A - y_B)I_Y}{(x_A y_B - x_B y_A} \qquad \text{1–6}A$$

$$= \frac{-[(+5.29) - (+5.54)]\ 11.50}{(+1.84)\ (+5.54) - (+1.41)\ (+5.29)}$$

$$= +1.05$$

$$y_{ab} = \frac{(x_A - x_B)I_X}{(x_A y_B - x_B y_A)} \qquad \text{1–6}B$$

$$= \frac{[(+1.84) - (+1.41)]\ 54.49}{(+1.84)\ (+5.54) - (+1.41)\ (+5.29)}$$

$$= +8.56$$

Similarly

$$x_{bc} = -6.85 \qquad y_{bc} = +18.08$$
$$x_{cd} = -2.27 \qquad y_{cd} = -19.28$$
$$x_{de} = +2.25 \qquad y_{de} = -\ 5.81$$
$$x_{ea} = +4.59 \qquad y_{ea} = +\ 2.74$$

Note. This S-polygon may be used to check the results of Ex. 1–3 and 1–4. It may be seen that for a plane of bending through the centroid and parallel to the long leg points A and C are most stressed. By scale, the distance $Om = 6.12$ in.^{3} and $On = 7.75$ in.^{3}

Therefore,

$$s_A = \frac{80{,}000}{6.12} = -13{,}070 \text{ lb per sq in.}$$

$$s_C = \frac{80{,}000}{7.75} = +10{,}320 \text{ lb per sq in.}$$

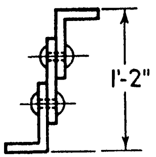
Problem 1–13. Solve Ex. 1–7 using the equation derived in Prob. 1–12 and the centroidal axes parallel to the legs of the angle. *Note.* It will be observed that points bc and de of this S-polygon are on a straight line through the centroid of the section. Why? Does the parallelism of lines BC and DE have any significance here? Does a similar relation hold for any other points?

Reference was made earlier to the fact that only those parts of the S-lines are needed which help form the S-polygon. For example, if the plane of bending in Fig. 1–5 passes through O and ab it is clear that $S_A = S_B$ and, therefore, the stresses at points A and B are equal. (Incidentally, this position of the plane of loading is that which makes the neutral axis parallel to the sides AB and CD. What position of the plane of loading will make the neutral axis parallel to BC?) If the plane of loading passes to the right of point ab, the distance from O to the a line will be less than to the b line, showing that S_A is less than S_B. Therefore $s_B < s_A$, and the part of the b line to the right of ab is not needed if the problem is the usual one of finding maximum stresses.

It is sometimes convenient to use the general expression for S, referred to any pair of rectangular axes X and Y through the centroid of the figure. This expression is

$$S = \frac{I_X I_Y - K_{XY}^2}{(I_X y - K_{XY} x) \sin \theta + (I_X x - K_{XY} y) \cos \theta} \qquad \text{1-5A}$$

See Eq. 1-4*A*, p. 13.



2-∠s 6 x 4 x ½
1-Pl. 8 x ½

Prob. 1-17

Problem 1-14. Plot the S-polygon of an angle $6 \times 6 \times 1$.

Problem 1-15. A hollow cast-iron member has outside dimensions 6 in. $\times$ 12 in. The metal is 1 in. thick. Plot the S-polygon.

Problem 1-16. Plot the S-polygon of a T shape 10 in. high with 10 in. cross-bar, $1\frac{1}{2}$ in. metal, area 27.75 sq in.

Problem 1-17. Compute the maximum and minimum values of the moment of inertia of this section. Plot the S-polygon.

Ans. $I_{\text{max.}} = 312.0 \text{ in.}^4$
$I_{\text{min.}} = 9.8 \text{ in.}^4$

1-8. Torsion. In the previous sections it has been pointed out that a beam will twist as well as deflect if the line of action of the transverse loading does not pass through the shear center at each cross section. Twisting is accompanied by shearing stresses, together with direct normal stresses on sections extending over a relatively small part of the length of the beam. Until recently but little has been known about evaluating these stresses, and torsional loads have been avoided as far as possible. Recent research at the University of Illinois and at Lehigh University has given us information sufficient for the analysis of common load conditions.[1]

The relation between torsional resistance (torque = T in-lb) and unit shearing stress s_s for a circular shaft of radius r is given by the familiar

$$T = \frac{s_s J}{r} \qquad \text{1-7}$$

where J is the polar moment of inertia of the cross section. The torque may also be expressed in terms of the unit angular twist, θ, that is, the angle turned through by a diameter, measured in radians per inch of length of shaft,

$$T = J E_s \theta \qquad \text{1-8}$$

where E_s is the shearing modulus of elasticity of the material. The corresponding formulas for non-circular sections take similar, but much more complicated, forms,

[1] The material in this article is from four sources: the bulletin and the text by Professor Seely of the University of Illinois referred to in the footnote on p. 4; "Structural Beams in Torsion" by Professor Inge Lyse (Ing'-ga, the first rhyming with *king*, Lee'-za) and Mr. Bruce Johnston, Research Fellow, Lehigh University, *Transactions*, Am. Soc. C. E., 1936; "Manual of Steel Construction," Bethlehem Steel Co., 1934.

as is required by the increased complexity of the deformations and the corresponding stresses. As a circular piece twists, all plane cross sections remain plane after twisting. The sections of any other than a circular piece warp with the twisting, that is, they cease to be planes, unless this action is prevented by fixation such as is secured by boxing in the end of a rolled section or by complete welding of the beam to the face of the support, or as is secured by symmetry. The center section of a simple beam uniformly loaded in such a manner as to cause torsion will not warp, since the deforming tendency on one side is balanced by that on the other. Normal stresses, often of considerable magnitude, are set up by torsion in the region adjacent to a fixed section. Beyond this region of normal stress, shearing stress develops running continuously round the periphery of the section as in a shaft. This is illustrated in Fig. 1–6, which for simplicity is drawn without showing the twisting and deflection of the channel. The passage from the stress condition there shown at section B to that at section C is, of course, gradual, intermediate sections carrying a combination of torsional and lateral flange shear.

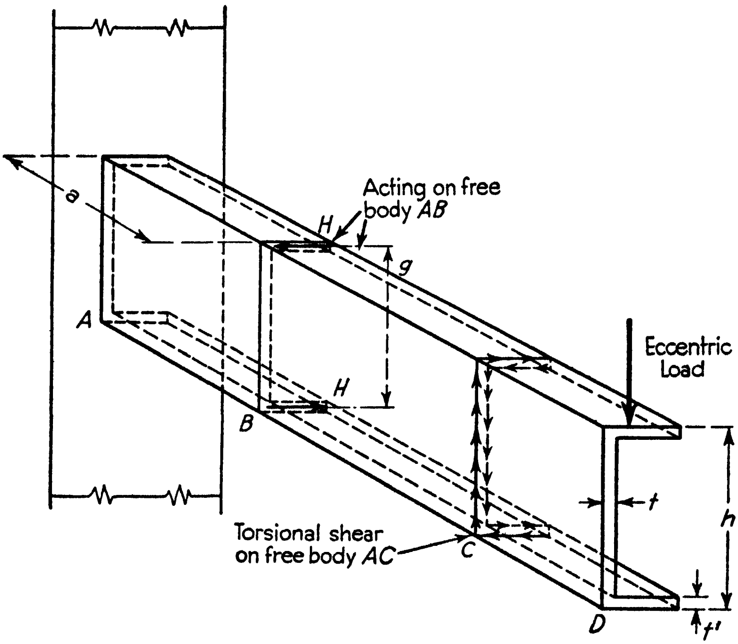


FIG. 1–6

Consideration of the action of the lateral shear in a flange at section B, Fig. 1–6, shows that it must be accompanied by lateral bending of the flange, which, accordingly, acts as a cantilever beam fixed at the unwarped section at A. This bending results in longitudinal fiber stresses in each flange, tensile on one side and compressive on the other, which are a maximum at section A. Professor SEELY[1] suggests the following approximate method for evaluating these normal stresses in a channel.

[1] *Bulletin* 211, University of Illinois. p. 36.

The maximum bending moment in a flange may be taken as Ha (Fig. 1–6), the lateral shear H being considered to exist up to a section distant a from the unwarped section, where for a channel we have, as an approximate value,

$$a = 0.38\,h\sqrt{\frac{EI_y}{E_sK}} = 0.6\,h\sqrt{\frac{I_y}{K}} \qquad \text{1–9}A$$

K being a torsional constant. (*Query.* a being in inches, in what units is K expressed?)

The value of the lateral flange shear is given by the equality of the couple Hh, or more accurately Hg, Fig. 1–6, to the torque. The maximum normal stress due to the lateral moment at the unwarped section is given by

$$s = \frac{Mc}{I} = \frac{(Ha)\frac{b}{2}}{\frac{t'b^3}{12}} = \frac{6\,Ha}{t'b^2}$$

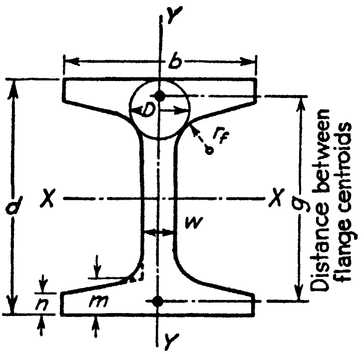
where b is the breadth and t' the thickness of the channel flange.

A similar analysis may be made for wide-flange beam sections. For these sections Messrs. LYSE and JOHNSTON found a somewhat larger value for a,

$$a = 0.806\,g\sqrt{\frac{I_y}{K}} \qquad \text{1–9}B$$

g being the distance between flange centroids. The value of the torsional constant K for beams and channels is given approximately by $\frac{1}{3}\,\Sigma W^3L$ where W and L are the width and length of each of the rectangles into which the cross section may be divided. In the tables below are given values of this and other constants which occur in torsional stress analysis for a few wide-flange and beam shapes.[1]

[1] The "Manual of Steel Construction" defines the various torsional constants as follows, in terms of L (span length), a (Eq. 1–9B), and section properties indicated in the figure. Pages 279ff.



$$Q_u = \tanh\frac{L}{2a} - \frac{2a}{L}\left(1 - \frac{1}{\cosh\frac{L}{2a}}\right).$$

$$Q_c = \tanh\frac{L}{2a}$$

$$R_u = \left[\left(\frac{L}{2a} - \tanh\frac{L}{2a}\right) - \frac{4a}{L}\tanh\frac{L}{2a}\left(\frac{L}{4a} - \tanh\frac{L}{4a}\right)\right]$$

$$R_c = \frac{L}{2a} - \tanh\frac{L}{2a}$$

$$B = \frac{ab}{gI_y} \qquad C = \frac{a}{KE_s}$$

$$N = \frac{D+m}{2K} \qquad Z = \frac{w + 0.3\,r_f}{K}$$

Expressions for K will be found in *Trans.* A.S.C.E., Vol. 101, 1936, pp. 860ff.

$\frac{L}{2a}$	Flange Stress Factors		Angle of Twist Factors	
	Q_u	Q_c	R_u	R_c
0.0	0.0000	0.0000	0.0000	0.0000
0.5	0.2358	0.4621	0.0285	0.0379
1.0	0.4096	0.7616	0.1807	0.2384
1.5	0.5219	0.9052	0.4562	0.5949
1.9	0.5839	0.9562	0.7322	0.9438
2.0	0.5969	0.9640	0.8061	1.0360
2.5	0.6518	0.9866	1.1963	1.5134
3.0	0.6948	0.9951	1.6103	2.0050

From "Manual of Steel Construction," p. 284.

Torsional Properties of Certain Bethlehem Wide-Flange Shapes and American Standard Beams

Section Number	Weight per Foot	Torsional Constants					
		K	a	B	C	N	Z
		in.^{4}	in.	in.$^{-3}$	in-lb^{-1}	in.$^{-3}$	in.$^{-3}$
B 24a	100	5.24	116.5	0.296	0.000 002 0	0.206	0.126
B 21	59	1.67	88.8	0.731	0.000 004 8	0.483	0.323
B 18b	96	6.01	81.82	0.269	0.000 001 2	0.188	0.115
B 16	50	1.62	58.29	0.759	0.000 003 2	0.505	0.308
B 14b	74	3.92	63.04	0.355	0.000 001 4	0.243	0.161
B 12	25	0.30	65.04	2.533	0.000 020	1.775	1.17
B 10b	100	11.05	34.85	0.175	0.000 000 28	0.116	0.076
I 24	100	7.70	46.60	0.303	0.000 000 54	0.170	0.120
I 20a	100	9.70	35.63	0.260	0.000 000 33	0.143	0.112
I 18	70	4.19	33.63	0.498	0.000 000 72	0.261	0.210
I 12	35	1.10	27.75	1.24	0.000 002 3	0.769	0.513

From " Manual of Steel Construction," pp. 285ff.

The Bethlehem " Manual of Steel Construction " gives these formulas for the direct flange stress due to torsion at the center of the span for (a) a simply supported beam loaded with a uniform eccentric load over its whole length and (b) a simply supported beam carrying a single eccentric concentrated load at the center

$$(a) \qquad s = TBQ_u \qquad \text{1–10}A$$

$$(b) \qquad s = TBQ_c \qquad \text{1–10}B$$

where B is a function of the properties of the cross section and Q a variable depending on the span and the properties of the section.

The torsional shear intensity tends to be a maximum at the reentrant angles of the section, increasing in severity with increasing sharpness of curvature of the fillets. However, our greatest interest is in the shear in the flanges and that in the web which combines with the web shear due to beam action. The intensity of the torsional web shear is given approximately (suggested by SEELY) by

$$s_s = \frac{Tt}{K} \qquad \text{1–11}A$$

and more precisely (Bethlehem " Manual ") by

$$s_s = TZ \qquad \text{1–11}B$$

where Z is a function of the cross section. The shear intensity in the flanges is given by

$$s_s = TN \qquad \text{1–11}C$$

It is sometimes important to know the amount of twist in a beam subjected to torsional loading. Where the twisting action is uniform from end to end of a beam, as where the twisting is done by application of opposing couples, one at each end of the beam (the manner of loading used in the Lehigh experiments), it is useful to compute the unit twist (radians per unit length) which is given by a formula similar to that derived for circular sections (see Eq. 1–8 above):

$$T = KE_s\theta \quad \text{or} \quad \theta = \frac{T}{KE_s} \qquad \text{1–8}A$$

For a simple beam with (a) uniform eccentric load and (b) with center eccentric load, the Bethlehem " Manual " gives for the total twist from end to center of beam

$$(a) \qquad \psi = \tfrac{1}{2}TCR_u \qquad \text{1–12}A$$

$$(b) \qquad \psi = TCR_c \qquad \text{1–12}B$$

where as before C and R are constants depending respectively on the cross section alone and upon span and section.

The equations which have been reported are valid only when the member subjected to torsion is free to twist and sway sideways. If this motion is prevented by lateral support, vertical deflection alone may be compelled, with the result that the beam stresses will be as given by the common formulas for bending and shear, in spite of loads acting beyond the bending axis. Since rolled sections offer relatively small resistance to twisting, the required restraint to eliminate torsion is not great except in the case of the deliberate application of a large twisting moment.

The direct stress due to torsion exists at its maximum at the unwarped section only and decreases rapidly away from this section. Where this stress is excessive the result in general would be local yielding and not failure of the whole piece. Torsional stress accordingly may be considered in the nature of a secondary effect.

Problem 1–18. For a beam with one end fixed and the other free, with a uniform eccentric load along its length, Professor J. B. REYNOLDS[1] shows that the total twist is given by the following expression:

$$\psi = \frac{Ta}{KE_s}\left[-\tanh\frac{L}{a} + \frac{a}{L}\operatorname{sech}\frac{L}{a}\left(\cosh\frac{L}{a} - 1\right) + \frac{L}{2a}\right]$$

[1] *Transactions*, A.S.C.E., Vol. 101, 1936, p. 904.

Eq. 1–12A gives the total twist from end to center in a simple beam carrying a uniform eccentric load over the whole length, expressions for C and R being given in the footnote, p. 26. Demonstrate that this equation is simply a rearrangement of that given by Professor REYNOLDS.

Suggestion. Put the expression for R_u in terms of the whole angle, $L/2\,a$: rewrite REYNOLDS' equation, replacing L by $L/2$. The same relations hold between these hyperbolic functions as between the corresponding trigonometric functions except that $\tanh \frac{u}{2} = (\cosh u - 1)/\sinh u$.

Problem 1–19. A 12-in. 40-lb channel 10 ft long, simply supported at the ends without fixation, carries a total uniformly distributed load of 27,000 lb acting in the plane of the web. What is the maximum intensity of compressive stress at the center of the span?

Suggestion. The desired stress is that due to ordinary beam action plus the secondary effect of torsion. Assume the flange to be a rectangle $\frac{1}{2}$ in. thick for an approximate solution.

Ans. s = 9700 (torsion) + 12,350 (bending) = 22,050 lb per sq in.
(e = 0.62 in. a = 13.2 in. T = 8380 in-lb H = 700 lb)

The following example and problems are from the Bethlehem "Manual."

Example 1–8. A 14 WF 74 carries a total uniformly distributed load of 44 kips on a 20-ft span, simply supported. The load has an eccentricity of 2 in. Compute the maximum normal and shearing unit stresses.

Solution. The table of p. 27 gives these values for torsional constants: a = 63.04 in.; B = 0.355 in.$^{-3}$; C = 0.0000014 (in-lb)$^{-1}$; K = 3.92 in.^{4}; N = 0.243 in.$^{-3}$; Z = 0.161 in.$^{-3}$ The value of $L/2\,a$ is 240/126.08 = 1.90. Entering the first table of p. 27 with this value, we find that Q_u = 0.5839. The other section dimensions and properties are found in every structural-steel handbook.

Maximum bending moment: $M = \dfrac{WL}{8} = 1{,}320{,}000$ in-lb

Maximum torque: $T = \dfrac{44 \times 2}{2} = 44{,}000$ in-lb

Maximum end shear: $V = 22{,}000$ lb

Maximum normal stress at center of span:

$$\begin{aligned} s &= \frac{M}{S} + TBQ_u \\ &= \frac{1{,}320{,}000}{112.3} + 44{,}000 \times 0.355 \times 0.5839 \\ &= 11{,}750 + 9{,}100 \\ &= 20{,}850 \text{ lb per sq in.} \end{aligned}$$

Since this maximum occurs at one spot only, the extreme edge of each flange, and decreases rapidly across the flange, this value may usually be permitted.

The maximum shear in the flanges at the ends of the beam is given by

$$s_s = TN = 44{,}000 \times 0.243 = 10{,}700 \text{ lb per sq in.}$$

The maximum shear in the web at the end of the beam is

$$\begin{aligned} s_s &= \frac{V}{wh} + TZ \\ &= \frac{22{,}000}{0.45 \times 14.19} + 44{,}000 \times 0.161 \\ &= 3{,}440 + 7{,}100 = 10{,}500 \text{ lb per sq in.} \end{aligned}$$

Both of these shear values are satisfactory.

Problem 1–20. Solve Ex. 1–8, substituting a concentrated center load of 22,000 lb for the uniform load.

Ans. Flange fiber stress 19,200 lb per sq in.
Flange shear stress 5,350 " " " "
Web shear stress 5,260 " " " "

Problem 1–21. Compute the total twist for the beams of Ex. 1–8 and Prob. 1–19.

Ans. Ex. 1–8. $\psi = 0.0232$ rad. $= 1° 20'$
Prob. 1–19. $\psi = 0.0299$ rad. $= 1° 43'$

1–9. Compression Flange Buckling. The compression flange of a beam tends to buckle just as a column does. If the flange is restrained, as by encasement in a concrete floor or by other adequate support at frequent intervals, the buckling tendency may be neglected and the beam designed for the allowable tension value; otherwise the buckling tendency must be taken into account in setting an allowable average unit stress for the compression flange, and specifications commonly contain formulas for obtaining allowable stresses under these conditions. As an example, consider the following taken from a specification[1] which permits a unit stress in tension of 16,000 lb per sq in.

"The unsupported length of beams and girders shall not exceed forty times the width of the compression flange. When the unsupported length (L) exceeds ten times the width (b) of the compression flange, the stress per square inch in the compression flange shall not exceed 19,000 − 300 L/b."

This matter will be treated in greater detail in a later chapter when considering design for combined direct stress and bending.

Problem 1–22. On the basis of the paragraph above quoted, what is the maximum simply supported span, without side support for the flanges, on which an 18 I 54.7 may be used? What total uniformly distributed load may the beam carry under these conditions? What would be the allowable load if the beam were used on the same span but with adequate lateral supports at intervals of 5 ft?

Ans. 20 ft; 20.6 kips; 47.1 kips.

[1] "Carnegie Pocket Companion," 1923 Edition, p. 96.

Problem 1-23. The specifications of the American Institute of Steel Construction allow a compressive stress in bending of $\dfrac{20{,}000}{1 + \dfrac{L^2/b^2}{2000}}$ up to 18,000 lb per sq in. with L/b limited to 40. When secondary stress, such as that due to incidental torsion, is computed, it is customary to allow an increase of one-third in the allowable stresses. Is the beam of Prob. 1-19, p. 29, satisfactory on this basis?

Ans. No: the limit of stress is $12{,}300 \times \frac{4}{3} = 16{,}400$ lb per sq in.

1-10. Web Stresses at Concentrated Loads. When a steel beam with unreinforced web carries a concentrated load resting on its top flange or when for support it simply rests on another beam or on wall or shelf angle it is evident that the web is subjected to column action. The student will gain an understanding of the situation if he will draw the free body composed of the end of a horizontal beam resting on a support, the dividing normal plane cutting off the beam end a few inches from the edge of the edge of support. The vertical equilibrium of this free body exists under the action of the upward reaction and the downward shear distributed approximately uniformly over the depth of the section. The unsupported length of this column may evidently be considered to be less than the full depth of the beam (assuming the beam secured against tipping) both on account of this gradual application of load along the length of the column and because of the more or less complete fixation of the ends of the web by the flanges of the beam. (See the review of column theory in the next chapter.) It is customary to take the length of this column as one-half the beam depth. On the basis of unreported tests it is also customary to take the length of beam web participating in column action as equal to the length of the actual bearing plus a distance beyond the bearing edge of one-fourth of the beam depth. On this basis the steel handbooks state that the safe reaction for a beam in end bearing is

$$R = s_b \cdot t \cdot \left(a + \frac{d}{4}\right)$$

and the allowable concentrated load applied at some interior point on a beam equals

$$P = s_b \cdot t \cdot \left(a + \frac{d}{2}\right)$$

where a is the length of bearing, d the beam depth, t the web thickness, and s_b the allowable average load on the column given by the column formula of the specification used. The formula of the A.I.S.C. takes this

form when applied for this purpose

$$s_b = \frac{18{,}000}{1 + \frac{1}{6000}\left(\frac{d}{t}\right)^2}$$

with the maximum not to exceed 15,000 lb per sq in.

Furthermore, designs at points of concentrated loads should be made in the light of tests[1] which have shown that

it is unwise to regard the ultimate compressive fiber stress in the web adjacent to a bearing block as higher than the yield-point strength of the material at the root of the flange. Moreover, the fact should be borne in mind that the material at the root of the flange of an I-beam usually has a yield-point strength somewhat lower than that of the material in the flange or in the web. In the absence of special tests the yield-point strength of the structural steel at the root of the flange of an I-beam may be taken as about 30,000 lb per sq in.

If any proposed loading gives stresses which do not provide a satisfactory factor of safety, stiffener angles should be used. These may be proportioned in the manner illustrated in Ex. 4–9, p. 107.

1–11. Web Buckling. The student has already learned of the phenomenon of diagonal tensile and compressive stresses in the webs of beams from his study of mechanics of materials. This matter will be reviewed in a later article; for the present it is sufficient to recall the existence of these inclined stresses and the tendency of the diagonal compression to cause the relatively thin web of a steel beam to buckle. The results of tests[2] made in the Fritz Engineering Laboratory of Lehigh University on special welded beams with relatively thin webs indicate that there is no danger of this buckling as the cause of failure of a steel beam so long as the ratio of web depth to thickness is less than 70. For these proportions the strength of the web is limited by shear. Since each rolled shape (WF, I, ⊔) has a web thickness which exceeds one-seventieth of its depth, web buckling from shear need not be feared with them.

1–12. Maximum Web Stresses: Principal Stresses. At any section of a beam with shear and bending, the ordinary beam formula will give the maximum stress for only the extreme points; at all other points the maximum stress will be on some other than the normal plane. Occasionally in beams of I-section the maximum web tension or compression next to the flange may exceed that figured for the extreme fiber. The analysis of this stress involves the theory of principal stresses, which

[1] *Bulletin* 86, Engineering Experiment Station, University of Illinois, "Strength of Webs of I-Beams and Girders," p. 33.

[2] *Proceedings*, American Society of Civil Engineers, February, 1934, p. 185.

most students of this book will have encountered in their courses in strength of materials. A demonstration of the theory is here given for the benefit of those who have not studied it before.

In the analysis of stress in a body subjected to coplanar forces, it often happens that the intensities of shear and direct stress are easily found at any point on two mutually perpendicular planes, and the question arises whether these intensities are the greatest which actually happen at that point. For example, if the wedge-shaped figure shown in Fig. 1–7 is a tiny piece of a horizontal beam, the intensities of shear

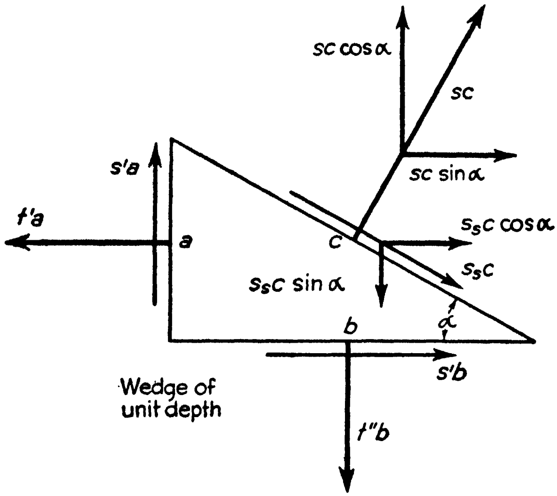


FIG. 1–7

and direct stress on the faces a and b are computed by the formulas we have reviewed in this chapter, t'' being zero, and the shear intensity s' being the same for both faces. This prism is taken so small that the stress intensity may be considered as uniform over any one face; the dimension perpendicular to the plane of the paper is taken as unity. The stress intensities on the sloping face may be greater or less than those on the right faces, which are the ones given by our usual calculations. It is possible, therefore, for an unsatisfactory stress condition to exist on the sloping face at the same time that there are allowable intensities on the right faces, a situation often met with in the design of dams. The intensities on the sloping face are functions of those on the right faces and of the slope of the inclined face which may be taken at any value. Considering this wedge, Fig. 1–7, as a free body in equilibrium under the action of the forces shown, we may write these two equations:

$$\Sigma H = 0 \qquad -t'a + s'b + sc \sin \alpha + s_s c \cos \alpha = 0$$
$$\Sigma V = 0 \qquad s'a - t''b + sc \cos \alpha - s_s c \sin \alpha = 0$$

Dividing both equations by the length c gives:

$$-t' \sin \alpha + s' \cos \alpha + s \sin \alpha + s_s \cos \alpha = 0 \tag{A}$$
$$s' \sin \alpha - t'' \cos \alpha + s \cos \alpha - s_s \sin \alpha = 0 \tag{B}$$

Solving these two simultaneous equations for s and s_s

$$s = t' \sin^2 \alpha - 2\, s' \cos \alpha \sin \alpha + t'' \cos^2 \alpha \qquad (C)$$

$$s_s = t' \sin \alpha \cos \alpha - s' \cos^2 \alpha - t'' \sin \alpha \cos \alpha + s' \sin^2 \alpha \qquad (D)$$

It is desired first to determine under what conditions the intensity of direct stress, s, on the sloping face is a maximum or minimum. Differentiating Eq. C with respect to α and placing equal to zero results in this equation which gives the slope for maximum or minimum direct stress intensity

$$\tan 2\,\alpha = \frac{s'}{\left(\dfrac{t' - t''}{2}\right)} \qquad 1\text{–}13$$

There are two values of $2\,\alpha$ differing by 180° which satisfy this equation, that is, two values of α differing by 90°, one value for the maximum and the other for the minimum value of s. When α has the value given by Eq. 1–13, the intensity of shear, s_s, on the sloping face is zero, as appears when Eq. 1–13 is combined with Eq. D. The maximum and minimum values of s are found by combining Eq. C and Eq. 1–13, resulting in

$$s = \frac{t'' + t'}{2} \pm \sqrt{s'^2 + \left(\frac{t' - t''}{2}\right)^2} \qquad 1\text{–}14$$

So long as there is shear on the right planes the maximum intensity of s given by this equation will be greater than either of the direct stress intensities on the right planes and the minimum value will be less than either of them. If there is no shear on the right planes the maximum value of s is equal to one right plane intensity and the minimum s equals the other.

The stress intensities given by Eq. 1–14 are known as the **principal stresses** at the point represented by wedge of Fig. 1–7, and the sloping planes on which these intensities occur are known as planes of principal stress. From Eq. 1–13 it is clear that these principal planes are at right angles to each other. It has already been pointed out that these are planes of zero shear. When the shear on the right planes of Fig. 1–7 is zero those are the principal planes.

To determine the conditions for maximum shear intensity we proceed as before, placing equal to zero the first differential of the general expression for shear on the sloping plane (Eq. D above) with respect to α, which gives:

$$\tan 2\,\alpha = -\frac{\left(\dfrac{t' - t''}{2}\right)}{s'} \qquad 1\text{–}15$$

When α, the slope of the inclined plane in Fig. 1–7, has the value given by this equation the intensity of the shear on the sloping plane is the greatest existing at that point on any plane. As before, there are two values of α differing by 90° which satisfy this equation. The values of $2\,\alpha$ given by Eq. 1–15 differ by 90° from those given by Eq. 1–13, and the values of α accordingly differ by 45°. We conclude, then, that the planes of maximum shear intensity lie at an angle of 45° with those of maximum and minimum direct stress intensity.

To obtain the value of the maximum shear intensity insert in Eq. D the proper values of the functions of $2\,\alpha$ as just found, giving:

$$s_s = \sqrt{s'^2 + \left(\frac{t' - t''}{2}\right)^2} \qquad 1\text{–}16$$

But this expression is that which results from taking one-half of the algebraic difference of the maximum and minimum direct stress intensities at the point, that is, one-half of the difference of the principal stresses. The equation may then be written as:

$$s_s = \tfrac{1}{2}\,(s_{\text{max.}} - s_{\text{min.}}) \qquad 1\text{–}17$$

Comparison of Eq. 1–14 and 1–16 yields the following useful results:

$$s_{\text{max.}} = \frac{t' + t''}{2} + s_s\,\text{max.}$$

$$s_{\text{min.}} = \frac{t' + t''}{2} - s_s\,\text{max.}$$

These several equations have been obtained with definite assumed directions of the stresses on the right planes, Fig. 1–7. If in any problem any of these directions are different, due changes in the signs must be made in applying the formulas.

The common use for the above formulas in structural analysis arises in connection with stress investigations in beam webs. In this case t'' equals zero and the following simple relations hold. The negative sign in Eq. 1–15 indicates that α is on the opposite side of the horizontal axis from that of Fig. 1–7. Therefore, if a right triangle is drawn, with base, *ab*, equal to s' (to the right from the origin, *a*, for positive shear), and an altitude *bc* equal to $t'/2$ (up from horizontal for tension), the angle made by the hypotenuse, *ac*, with the horizontal will be $2\,\alpha$ (α is the angle which the plane of maximum shear makes with the horizontal) and the hypotenuse will equal this maximum shear. The planes of principal stresses will differ from α by 45° and the values of the principal stresses will be ($t'/2 \pm$ maximum shear).

It is convenient to be able to visualize the position of the plane of maximum direct stress. Assume a square-faced element with negative shear and direct compression on the vertical faces. Owing to shear alone there will be tension across the diagonal extending upward to the left and compression across the other. The direct stress on the second diagonal is of the same character as the direct stress on the left vertical face. Investigate the equilibrium of the triangle lying between these two lines. It will be clear that for equilibrium the resultant of the forces acting on the diagonal must have a slope flatter than 45°, and that the plane of maximum compression must lie somewhere between the vertical and the 45° plane.

The student should remember definitely these facts concerning the maximum stresses at any point in a body subjected to planar stress:

1. The maximum and minimum intensities of direct stress occur on two mutually perpendicular planes, the planes of principal stress, on which planes the shear intensity is zero.

2. The maximum shear intensity at the point occurs on planes midway between the principal planes.

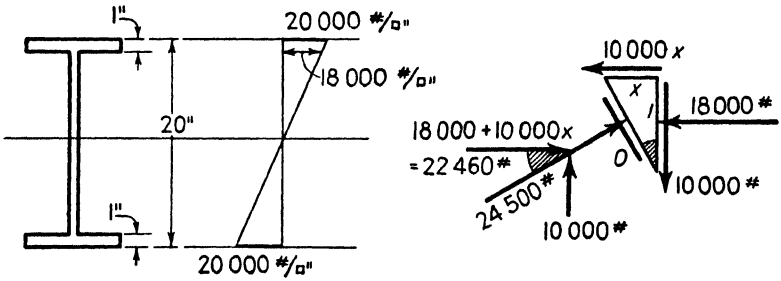
Any involved study of stress intensities in a body will necessitate the use of the equations which have been developed. Simple problems are often much more easily solved by applying the general principles just outlined without bothering to hunt up the appropriate equation and make the necessary review of its exact meaning and the significance of its signs. This is illustrated in Ex. 1–9 below.

Problem 1–24. Complete the derivations of Eq. 1–13, 1–14, 1–15, and 1–16.

Discussion. Eq. 1–14 was obtained from the intermediate Eq. *B* for simplicity. Perhaps it would have been more logical to work from the more general expression, Eq. *C*. The student should make both derivations. When inserting the functions of 2α into an equation remember that the numerator and denominator of the fraction which equals the tangent may be taken as the sides of a right-angle triangle, the square root of the sum of their squares the hypotenuse of that triangle, and that the values of $\sin 2\alpha$ and $\cos 2\alpha$ may be read directly from the triangle.

The derivation has been given in one quadrant only. The student should follow the significance of the equations through all four quadrants.

Example 1–9. The maximum flexural stress on the 20-in. I-section shown is 20,000 lb per sq in., and the shear intensity at junction of web and flange is 10,000 lb per sq in. What is the maximum intensity of compression at that point and the slope of the plane on which it acts?



Ex. 1–9

Solution. A free body is taken consisting of an infinitesimal wedge with one face vertical and one horizontal, considered enlarged to unit depth and width on the vertical face, located at the upper junction of web and flange, in a region of positive shear. This wedge is so chosen that the stress on the inclined face is compression without shear, this being the condition for principal stress. Since the slope of the inclined face is not known, the length of the top horizontal face is taken as x in., its area x sq in. The force system needs no explanation beyond pointing out that the components of the inclined force are plainly fixed by equilibrium. Solving for x [$x : 1 = 10{,}000 : (18{,}000 + 10{,}000\,x)$] gives its value as 0.445; the resultant inclined force then is 24,600 lb acting on an area of 1.10 sq in., an intensity of 22,400 lb per sq in., which is greater than that on the extreme fiber. The plane on which this compression acts makes an angle $\tan^{-1} 0.445 = 24°$, closely, with the vertical.

Why does the horizontal component of the total normal stress on the inclined plane equal the intensity of that normal stress?

Is it common for maximum moment and maximum shear to occur simultaneously at the same section?

In this case the inclined stress at root of web is of secondary effect, permitting a higher limit than for the primary stress.

Problem 1–25. Draw the four possible wedge-shaped free bodies each with a vertical and horizontal face, for determining principal stress at the point investigated in the above example. Does it make any difference whether the unit area, on enlargement, is taken as the vertical or horizontal face? Compute the principal tensile stress and inclination of the plane on which it acts.

Problem 1–26. Compute the principal tensile and compressive stresses and the inclinations of the planes on which they act for the junction of web and lower flange of the beam of Ex. 1–9; the same for points 5 in. above and below the neutral axis. Check your results by use of Eq. 1–13 and 1–14.

To many students the principal stresses at a point, as, for example, in Ex. 1–9, seem quite mysterious compared to the normal fiber stress found at that point by the beam formula. Of course, they are exactly the same sort of forces exerted in quite the same way, with corresponding deformation effects. Remember that the stress on an extreme fiber given by the beam formula for a normal section is the principal stress for that point, the shear there being zero.

Several interesting graphical means have been devised for determining the intensities of direct stress and shear at a point, on any plane through the point, when there are given either the stress intensities on any two planes at right angles through the point or the principal planes and stresses. The relation known as the *ellipse of stress* is referred to so frequently in structural literature that it will be described here, although it has no practical value, except as a graphic representation of the planes of principal stress in a body.

Consider the elementary prism *abc* in Fig. 1–8, representing a point in a body with three planes passed through it. Planes *a* and *b* are planes of principal stress with no shear. It is desired to find the stress intensity on the inclined plane *c*.

An ellipse is constructed as shown, with major and minor semi-diameters equal respectively to the maximum and minimum principal stress intensities; also two circles are drawn with center *O*, radii equal to the principal stress intensities. From *O* the line *Omn* is drawn, perpendicular to plane *c*, cutting the circles at *m* and *n*. A vertical is dropped from *n* and a horizontal drawn from *m*, intersecting at *p*. The line *Op* equals the intensity of stress on plane *c* and has the same slope as that stress; it may be resolved readily into direct and shear components. Point *p* lies on the ellipse. The proof of this is as follows.

The prism shown is in equilibrium under the action of three forces, $s_a a$, $s_b b$, and $s_c c$, *a*, *b*, and *c* being the areas of the respective faces so denoted. On construction of the triangle of force for these three forces we would have

$$(s_c c)^2 = (s_a a)^2 + (s_b b)^2$$

and

$$s_c^2 = s_a^2 \sin^2 \alpha + s_b^2 \cos^2 \alpha$$

The line of *Op*, Fig. 1–8, satisfies this equation for s_c, since it is the hypotenuse of a right triangle, the side *Or* of which equals *On* sin α, equals $s_a \sin \alpha$, by construction, and side *pr* of which equals *Om* cos α, equals $s_b \cos \alpha$. The proof for direction follows directly.

From the well-known trigonometrical relation we may write

$$\frac{(s_a \sin \alpha)^2}{s_a^2} + \frac{(s_b \cos \alpha)^2}{s_b^2} = 1$$

or

$$\frac{(Or)^2}{s_a^2} + \frac{(pr)^2}{s_b^2} = 1$$

But this is the equation of an ellipse with major semi-diameter equal to s_a and minor semi-diameter equal to s_b. Point p accordingly lies on the ellipse as drawn.

The *circle of stress* is a very practical graphical method and will be found described by S. TIMOSHENKO in "Theory of Elasticity," p. 16.

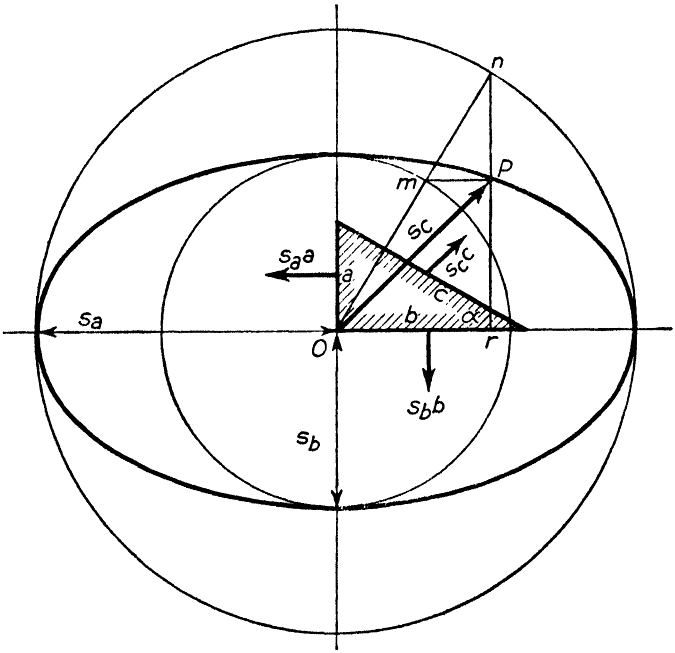


FIG. 1-8

1-13. Composite Beams. Occasionally different materials, or different sizes of the same material, are bolted together to act as a beam. The various parts of these beams must, at any section, deflect the same amount. On the basis of this fact it is possible to compute fiber stresses. Suppose that Fig. 1-9 represents a cross section of a beam composed of two planks of different kinds of wood and a steel plate. Reference to a text or table of deflections will show that all deflection formulas are

in the form:

$$y = K\frac{WL^3}{EI}$$

where K is a constant depending on the kind of loading. Since all three parts of the beam of Fig. 1-9 deflect the same amount

$$y = K\frac{W_1L^3}{E_1I_1} = K\frac{W_2L^3}{E_2I_2} = K\frac{W_3L^3}{E_3I_3}$$

where W_1, W_2, and W_3 are parts of the total load carried by each beam part, or

$$\frac{W_1}{E_1I_1} = \frac{W_2}{E_2I_2} = \frac{W_3}{E_3I_3} = \frac{W_1 + W_2 + W_3}{E_1I_1 + E_2I_2 + E_3I_3} = \frac{W}{\Sigma EI}$$

(by the law of addition in proportion), where $W_1 + W_2 + W_3 = W$. Therefore:

$$W_1 = W\frac{E_1I_1}{\Sigma EI}; \quad W_2 = W\frac{E_2I_2}{\Sigma EI}; \quad W_3 = W\frac{E_3I_3}{\Sigma EI}$$

Obviously a total bending moment will divide among the various parts in the same manner as a total load.

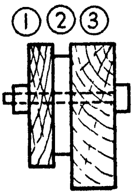


FIG. 1-9

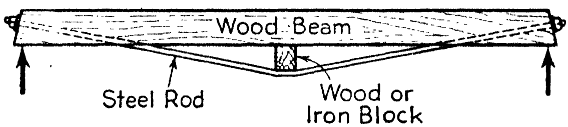


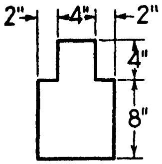
FIG. 1-10

A common form of composite beam in timber construction is a wood beam reinforced by a tension rod as shown in Fig. 1-10. This "trussed beam," as it is called, is usually considered as continuous over the three supports provided by the two end supports and the intermediate blocking supported by the truss rod. The reaction developed at the intermediate block being known, the stress in the rod is easily calculated. A more exact analysis may be made by the methods developed in "Structural Theory."

Problem 1-27. A beam is formed by bolting together two pine timbers 4 × 12 in. and one steel plate 12 × ½ in. What percentage of the load is carried by the steel? E, steel = 30,000,000; E, pine = 1,500,000 lb per sq in.

Ans. 55.5%.

Problem 1–28. (*a*) A piece of timber of the size shown is cut from a solid 8×12 in. and used as a beam on a span of 20 ft to carry a uniform load of 300 lb per ft. What is the maximum unit fiber stress?



PROB. 1–28

(*b*) Another beam of the same form is made by bolting together two 2×8 in. and one 4×12 in. planks in such a manner that they will deflect the same amount but will be free to slide one along the other. What will be the maximum fiber stress if this beam is used for the span and load above?

Ans. (*a*) $s_c = 1486$ lb per sq in.
$s_t = 1136$ lb per sq in.
(*b*) $s_{4 \times 12} = 1447$ lb per sq in.
$s_{4 \times 8} = 965$ lb per sq in.

Problem 1–29. Three 8×16 in. wood stringers are used on a 14-ft span to support one rail of a line designed for Cooper E–60 loading. (E–60 loading on a 14-ft span produces a moment, per rail, of 165 ft-kips.)

(*a*) Neglect impact and dead load and determine the maximum bending fiber stress in the stringers when new.

(*b*) Assume that in the course of time an inspection shows that decay has set in and the three are now equivalent to one 8×16, one 8×14, and one 8×13 in. What is the stress in each under these conditions?

Ans. (*a*) 1933 lb per sq in.
(*b*) $s_{8 \times 16} = 2625$ lb per sq in.
$s_{8 \times 14} = 2300$ lb per sq in.
$s_{8 \times 13} = 2140$ lb per sq in.

CHAPTER II

COLUMNS

2-1. A column (post, pillar, or strut) is any straight member which resists the compression caused by two equal and opposite forces applied at its ends. Two columns of the same cross section and same material but of greatly unequal length will be wholly unlike in the amount of load which they can carry. In a **short column** — one in which the ratio of length, L, to least radius of gyration, r, is less than, approximately, 30 — failure will follow the application of a relatively large load and will be by crushing, similar to the failure in a block of the material. In a **long column** — one with L/r greater than, approximately, 150 — the strength will depend on the modulus of elasticity, that is, on the stiffness of the material, rather than on its crushing strength, and failure will be due to buckling under a relatively small load. In a column of **medium** length — one between the above limits — failure will come from a load of intermediate magnitude and will be due to a combination of crushing and buckling. Heavy chord members in bridges, and columns in the lower stories of high buildings, generally fall in the short classification. Long columns are used infrequently in engineering construction. Many columns, perhaps a majority of those built, come in the medium class. Ideally there is no limit to the length for which the failure would be by crushing, but actually, as the length increases, the piece bends laterally and the internal stresses are no longer those of uniform compression over all sections. This bending is inevitable since no piece can be made exactly straight, no load can be applied absolutely axially, and no material is entirely uniform, homogeneous, free from internal stress along the whole length.

It is important to note that a column is a much more critical member in a structure than a beam or a tension piece. The deviations of the material and of the piece from the ideal conditions assumed in deriving the theoretical stress relations have a far greater effect upon the strength of a column than upon that of a beam or a tie. In a column these variations give eccentricity to the load and consequently cause bending, which in its turn tends to cause lateral deflection and more bending. In a beam this does not happen, and in a tension member eccentricity of loading causes a bending which decreases the eccentricity.

Bibliography. The student should review the theory of columns in his textbook on strength of materials and also consult these important references.

"Columns," E. H. SALMON, London, 1921.

Reports of the Special Committee on Steel Column Research, *Transactions,* American Society of Civil Engineers, Vols. 89, 1926; 95, 1931; 98, 1933.

Since columns involve bending with axial stress it is desirable to review the general case of combined bending and direct stress in a short block before proceeding with column theory.

2-2. Bending and Direct Stress. The short block shown in Fig. 2-1 is acted upon by the force N, with line of action parallel to the axis of the block, applied at point b. The block is of uniform cross section and of homogeneous material. Its length is so small that no appreciable deflection is caused by the bending. It will be understood that the force N applied at a point outside the body represents the resultant of the direct stress and the couple actually applied to the section by the action of some portion of the block above the part shown. In graphical analysis the line of resistance of an arch or dam is found (see "Theory," p. 52), and the analysis of stress at a section presents the case shown here in the figure, except that the force acting at the intersection, b, of the line of resistance and the plane of the section is not, in general, normal to the section. This force may be resolved into a normal component, as N in the figure, and a shearing component whose effect would be determined separately.

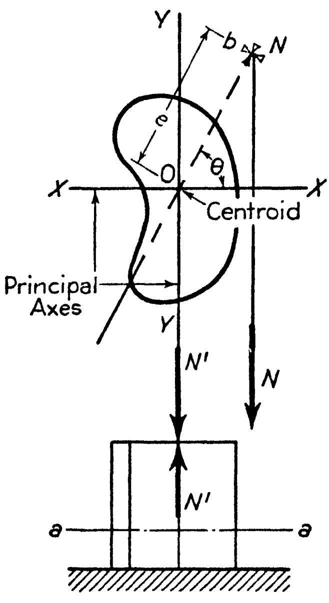


FIG. 2-1

The common use of the graphical method for determining the line of resistance is perhaps the chief reason for starting this analysis with a force N, applied as shown, since the first step in our solution of the stress relationships is to replace the force N with an axial force and a couple.

The deformation of the block is not in any way affected by applying at the centroid the two equal and opposite axial forces, N', each equal and parallel to the original force N. The stress effect of N, then, is equal

to that of the three forces in combination, and these effects may be found by considering any combination of the forces. We may say, accordingly, that the stress at any point in a normal section through the block is that due to an axial force N' and a couple Ne. The stress at any point due to the axial force is N/A, N and N' being equal; that due to a couple in a plane making an angle θ with the axis X, a principal centroidal axis, we have already determined (see p. 10) as

$$s = \frac{(M \sin \theta)y}{I_X} + \frac{(M \cos \theta)x}{I_Y} \qquad 1\text{–}4$$

The expression for unit stress at any point is the sum of the two expressions, M being equal to Ne,

$$s = \frac{N}{A} + \frac{(Ne \sin \theta)y}{I_X} + \frac{(Ne \cos \theta)x}{I_Y} \qquad 2\text{–}1$$

That is, s equals the algebraic sum of the fiber stresses due to the axial load and to each of the components of moment along the principal axes.

It is obvious that the neutral axis does not pass through the centroid. The intercept of the neutral axis and one of the principal axes, for example the X axis, may be found by solving Eq. 2–1A (below) for x when both s and y are zero.

Problem 2–1. Derive the equation of the neutral axis in combined flexure and direct stress.

From the general form of expression for fiber stress in unsymmetrical bending, Eq. 1–4A, p. 13, where I_X and I_Y are for any set of centroidal axes, we may write the corresponding equation for combined direct stress and bending.

$$\begin{aligned} s &= \frac{N}{A} + \frac{M \sin \theta\,(I_Y y - K \cdot x) + M \cos \theta\,(I_X x - K \cdot y)}{I_X I_Y - K^2} \\ &= \frac{N}{A} + \frac{M}{S} \end{aligned} \qquad 2\text{–}1A$$

The advantages of Professor Cross's treatment of the problem of unsymmetrical bending have already been pointed out, and obviously the same advantages exist when direct stress is combined with bending. From Eq. 1–4B, p. 14, we obtain Professor Cross's equation

$$s = \frac{N}{A} + \frac{M'_X y}{I'_X} + \frac{M'_Y x}{I'_Y}$$

Beams with Axial Loads. Occasionally a piece whose major loading is transverse carries, in addition, an axial load. This is by no means as common as transverse bending applied to a member with major axial

loading. An exact determination of stress in these members is a complex matter, and only the usual approximate solutions are given here.

A beam deflects under the action of its transverse load, and, in consequence, if an axial force is added, there is a bending moment throughout the beam due to it, increasing the transverse bending moment if the axial force is compression, decreasing it in the case of tension. This bending due to deflection is usually small, permitting analysis of stress by Eq. 2–1*A* above. If this effect is not negligible it is a simple matter to compute the beam deflection, D, at the center, which gives the center moment due to the axial force, P, as PD. The stress due to this moment, $(PD)c/I$, is to be added algebraically to that due to transverse bending and that due to direct axial stress. This is, of course, an approximate analysis since the action of the axial force is to increase or decrease the deflection due to beam loads, but the approximation is not serious.

Very commonly beams are restrained from lateral buckling by the floor system through which the load is applied so that column action, the bending inevitable in relatively long unsupported pieces with axial loads, is not present. The analysis in this article is valid only under this condition. The problem of columns with transverse bending will be considered later, and beams which are free to buckle come in this category.

If the longitudinal load is not axial but is applied with an eccentricity e, the stress caused by this moment, Pe, is to be added to that due to the other causes enumerated.

Problem 2–2. A common formula to give the stress in a member with both longitudinal and transverse loads is

$$s = \frac{P}{A} \pm \frac{Mc}{I} = \frac{P}{A} \pm \frac{M_T c}{I \mp \dfrac{PL^2}{10\,E}}$$

The upper signs are for a compressive and the lower for a tensile longitudinal force. Here

M_T = bending moment due to transverse loads and to eccentricity, if any, of the longitudinal force,

M = $M_T + PD$, P being the longitudinal force and D the center deflection due to transverse loads,

= total bending moment at the center, approximately.

This formula is based on the approximation that the center deflection of a loaded beam is

$$y = \frac{1}{10}\frac{M_T L^2}{EI} \quad \text{closely (for a center concentrated load, } y = PL^3/48\,EI = (PL/4)\,L^2/12\,EI = M_T L^2/12\,EI)$$

$$= \frac{1}{10}\frac{ML^2}{EI} \quad \text{approximately}$$

assuming the increment of moment due to the longitudinal force to have the same relative effect on deflection as the transverse bending moment. Complete the derivation.

Problem 2–3. Demonstrate that the change in deflection of a beam under the action of a longitudinal force, P, is approximately given by the expression

$$y' = \frac{5\,PL^2y}{48\,EI}$$

where L is the span, and y is the deflection under transverse load, the deflection curve being assumed a parabola.

Suggestion. See the approximate derivation for Euler's formula, p. 49.

Problem 2–4. A 24 WF 160 carries a total uniformly distributed load of 186 kips on a 20-ft span. It is also subjected to a longitudinal force of 235 kips centrally applied. What is the maximum intensity of compressive stress using Eq. 2–1A? What intensity results if the beam deflection is regarded? How much increase of deflection is caused by the longitudinal force? If the longitudinal force is applied 1 in. below the center of the beam, what change is made in the unit compression?

Ans. 18,500 lb per sq in.
18,630 lb per sq in.
about 0.002 in.
a decrease of 570 lb per sq in.

2–3. The Core. Near the center of gravity of each cross section there is a small area — the *core* (sometimes *kern*, or *kernel*) — which is the portion of the surface on which a load may be applied to cause stress of one character — all tension or all compression — over the entire cross section.

For an eccentrically loaded column the point of application of the load and the center of gravity of the section determine the plane of loading (bO, Fig. 2–1). On the side of the center of gravity where the load is applied the bending stress is compression, on the other side tension. If the eccentricity is small the resultant stress at all points will be compression; if large, the stress at some points will be tension.

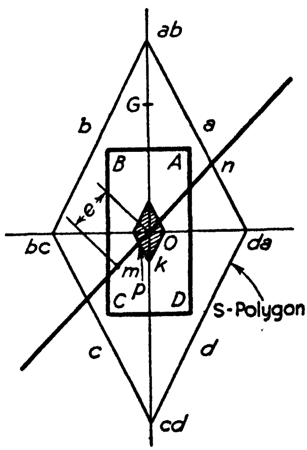


FIG. 2–2

Fig. 2–2 illustrates this. If a load is applied at m the stress at point A is

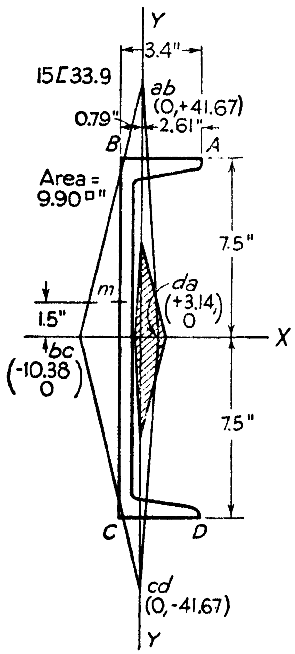
$$s_a = -\frac{N}{A} + \frac{Ne}{S} = -\frac{N}{A} + \frac{Ne}{\overline{On}}$$

For some value of e the two terms will be equal and s_a will be zero. For this case $N/A = Ne/S$, that is, $e = S/A = \overline{On}/A$. This value may be plotted as $\overline{Op}$. Clearly, for each point on the S-polygon there will be a

corresponding point on the opposite side of the origin, the points thus located circumscribing the shaded area shown — the core — within which a load must be applied to have resultant compression (or resultant tension) over the entire cross section.

The core may be most readily plotted by determining for each apex of the S-polygon a corresponding apex of the core by dividing the S-polygon apex coordinates by A, the area. Since the S-polygon values are inches cubed, *the core values are inches and are to be plotted to the same scale as the figure itself.*

In plotting an S-polygon or core only those corners of the figure need be considered which are most distant from the neutral axis for any conceivable position of that axis. If the plotting is for a rectangle, an I-beam, or a channel, four corners are involved in the computation. The S-polygon and core for the rectangle and I-beam are symmetrical about two axes — as are the rectangle and I-beam; those for the channel are symmetrical about one axis only. This is illustrated in Prob. 2-5.



Prob. 2-5

Where stress is due to the eccentric application of a load, the expression for fiber stress may be transposed as follows:

$$s = \frac{N}{A} \pm \frac{M}{S} = \frac{N}{A} \pm \frac{Nc}{S}$$
$$= \frac{N}{S}\left(\frac{S}{A} \pm e\right)$$

But, as has been seen, S/A is a distance, for example, Fig. 2-2, the distance $\overline{Op}$ when A is the point whose stress is being considered, and mn is the plane of loading. This fact is useful where the load is applied along a principal axis, as is usual in dealing with thrust in an arch rib or with bending in the columns and girders of a rigid frame. If, for illustration, a load, N, is applied at point G, Fig. 2-2, the stress along edge AB will have the value

$$s = \frac{N \times \overline{Gk}}{S} = \frac{N \times \overline{Gk}}{I/c}$$

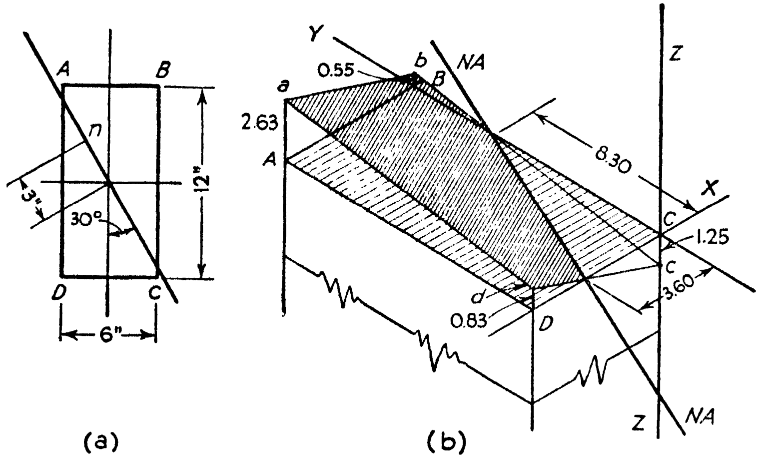
Problem 2-5. Check the plotting of the S-polygon and core for the 15 ⊔ 33.9 shown.

Problem 2-6. The channel of Prob. 2-5 acts as a column with a load of 15 kips applied at m on the back of the web, $1\frac{1}{2}$ in. from the X axis. What is the stress at D?

Ans. $S = 5.90$ in.^{3}
$s_d = -1515 + 4325$
$= 2810$ lb per sq in. tension.

The following example emphasizes the concept of planar stress distribution.

Example 2-1. A short pier, 6×12 in. in cross section, is loaded by a normal force of 50 kips acting 3 in. from the centroid on a line making an angle of 30° with the long axis. Compute the unit stress at each corner, and demonstrate that when these four values are plotted graphically they lie in a common plane.



Ex. 2-1

Note. This is Ex. 1-1 essentially, p. 12, with the addition of an axial stress to the moment previously given.

Solution. Since the bending moment and the plane of loading are the same as in Ex. 1-1 the corner bending stresses are as before and there remains now simply to combine algebraically with each stress that due to an axial load of 50 kips, i.e., 50,000/72 = 690 lb per sq in. compression. The corner stresses are as follows, indicating compression by plus and tension by minus: *A* 2630; *B* 550, *C* −1250, *D* 830 lb per sq in. These values are plotted and the plane of the four points indicated with its intersection with the plane of the cross section, this being the neutral axis. Taking the coordinate axes as shown, the equation of the plane of stress is $z = -1250 - 347\,x + 150\,y$ (347 is the slope of *cd*, 2080/6) as determined by points *b*, *c*, and *d*. Substituting in this equation the coordinates of point *a*, $x = -6$, $y = +12$, gives $z = -1250 + 2080 + 1800 = 2630$, which is the value already given for the stress at *a*. Accordingly *a* lies in the plane.

Problem 2-7. (*a*) Solve Ex. 2-1 using a rectangular coordinate system with origin at the centroid of the cross section. Compare the resulting equation for the stress plane with

$$s = \frac{N}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y} = 690 \pm 150\,y \pm 347\,x$$

(*b*) Locate the neutral axis in the figure for Ex. 2-1 by dimensions from C. Compare the location of this line with the neutral axis of Ex. 1-1, p. 12.

(*c*) Compute the average intensity of normal stress, and multiply it by the cross-section area. What should the result equal?

Suggestion: In studying this problem note that the load is applied in the second quadrant; that, so far as bending about the horizontal axis goes, the stress at A and B is compression, that at C and D tension; that the bending about the vertical axis causes compression at A and D, tension at B and C. This visualization of the character of stress simplifies the matter of signs.

2-4. Column Strength: Euler's Formula. A strut fails when the intensity of internal stress equals the elastic limit of the material. If the

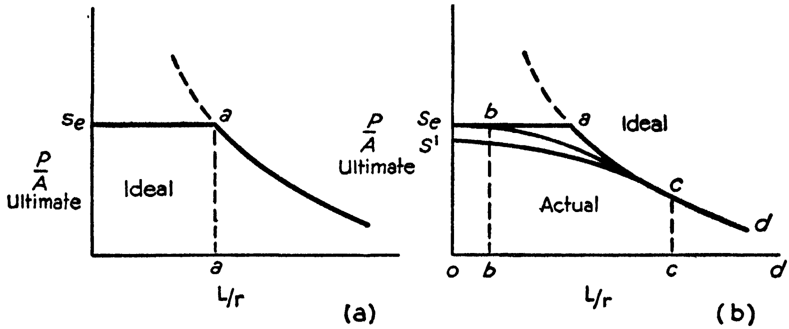


FIG. 2-3

length of an imaginary ideal (without the imperfections of the actual) strut or column were increased there would come a point (a, Fig. 2-3a) where the slenderness would be such that, if in some way the piece were bent, failure would ensue by buckling at an average unit stress less than the elastic limit. Slenderness is measured by the ratio, L/r, of the length, L, of the column to the least radius of gyration, r, of the cross section. The strength of an ideal column is plotted in Fig. 2-3a with the average load intensity, P/A, as ordinates and slenderness ratio, L/r, as abcissas; here s_e is the elastic limit of the material in compression. The curved portion of the strength line may be determined as follows.

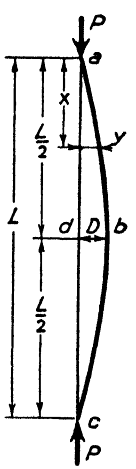


FIG. 2-4

The Euler Formula. (Leonard EULER [Oi-ler], 1707–1783.) In Fig. 2-4 is shown an unrestrained, ideal (perfectly straight, homogeneous, axially loaded, etc.) column maintained in a bent position by an axial load, P, whose magnitude cannot be decreased without the column straightening up. It is evident that whatever agency first causes the

bending must not give sufficient curvature to make the stress on any fiber (due to combined direct stress and bending) greater than the elastic limit. It is to be noted that there is a very considerable range of value of the deflection, D, consistent with stability under load P. Experiment shows that a small increase in magnitude of load P will cause a progressive increase of bending, and failure. This load, then, may be considered the maximum load the column can carry without breaking.

The deflection curve of the column shown in Fig. 2-4 is also the bending-moment curve to some scale, since the bending moment at any point distant x from the top is equal to the lateral deflection, y, at that point, multiplied by the load, P. Assuming that the moment area theorems (see "Theory," p. 168) hold for this deflection curve as for a beam, the maximum deflection, D, at the center of the column, equals the moment about the top, a, of the area abd, multiplied by P/EI. Assuming that the curve of deflection here is a parabola, this results in

$$D = \left(\frac{2}{3} \times \frac{L}{2} \times D\right)\left(\frac{5\,L}{16}\right)\left(\frac{P}{EI}\right) = \frac{5\,P\,DL^2}{48\,EI}$$

and $$P = \frac{9.6\,EI}{L^2}$$

or $$\frac{P}{A} = \frac{9.6\,E}{(L/r)^2} \qquad (A)$$

If we take the actual shape of the deflection curve we obtain

$$\frac{P}{A} = \frac{\pi^2 E}{(L/r)^2} = \frac{9.87\,E}{(L/r)^2} \qquad 2\text{–}2$$

which is Euler's formula for long columns.

Euler's formula shows that the magnitude of the maximum load on a long column depends on the stiffness of the material and on the slenderness ratio and that it is not affected by the strength of the material.

The curve given by Eq. 2–2 is plotted on Fig. 2–3. Its left end in the figure comes when P/A given by the formula equals the elastic limit of the material.

Problem 2–8. For what maximum slenderness ratio will the average load intensity at failure for a long column equal the elastic limit of the material for structural steel?

Ans. Taking the elastic limit as 35,000 lb per sq in. and $E = 29{,}000{,}000$ lb per sq in. results in $L/r = 90.3$.

End Condition. The lack of restraint assumed in the derivation signifies that the ends of the column are free to rotate, a condition which results if the ends are round and without friction on the bearing sur-

faces, or, considering one plane of possible deflection only, if the supports are frictionless pins. Actual pins are not frictionless and induce considerable restraint. Fixed ends result in points of inflection at the quarter points, and in applying the Euler formula to this case the length L is the distance between inflection points. A fixed-ended long column, accordingly, is four times as strong as an identical column with round ends. A long column fixed at one end and free to deflect laterally at the other is one-fourth as strong on the same column, round-ended, L in the Euler formula evidently being twice the actual length.

Actual columns differ markedly in action from the straight line and curve of Fig. 2-3*a*. Test results for very low and for very large values of L/r will give ultimate loads substantially on the curve and straight line there shown, but at intermediate values the column fails at loads below those for the ideal column. This is as would be expected since the actual column differs from the ideal by the presence of defects, which result in eccentricity of load and consequent bending. This is shown in Fig. 2-3*b*, where, at some point b on the straight line representing s_e and tangent to that line, a curve starts which becomes tangent to the Euler curve at some point c. Columns in the range o-b would be considered short; b-c, medium; c-d, long.

Query: How does fiber stress, s, vary with P/A along curve bcd?

There are many records of tests of short columns which developed strengths far above the elastic limit of the material, even approaching the crushing strength of short blocks. Structural columns in practice cannot be expected to approach the perfection of precisely made test specimens and in fact should not be counted on to develop an average stress equal to the elastic limit. The curve $s'cd$, Fig. 2-3*b*, indicates what may be depended on for actual columns.

The Euler formula is considered further in the subsequent articles.

2-5. The Straight-Line Formula. The process of devising practical column formulas consists in finding equations whose plotted values fit test results for ultimate strength and choosing the proper factor of safety to adjust them to design purposes. The equations may be entirely empirical, or may be devised on a rational basis and adjusted to fit test results.

In bridge design many column formulas are of the straight-line type, as illustrated by a former (1920) American Railway Engineering Association specification for steel railway bridges,

$$\frac{P}{A} = 15{,}000 - 50\frac{L}{r} \qquad \text{2-3}$$

but not to exceed 12,500 lb per sq in. This is equivalent to saying that

columns with L/r equal to 50 or less shall be designed for a unit stress of 12,500 lb per sq in. and columns with greater values of L/r shall be designed by the formula, Eq. 2–3. This type of formula with lower limit is accordingly sometimes referred to as the broken-line formula. This setting of a lower limit comes from the fact already pointed out that

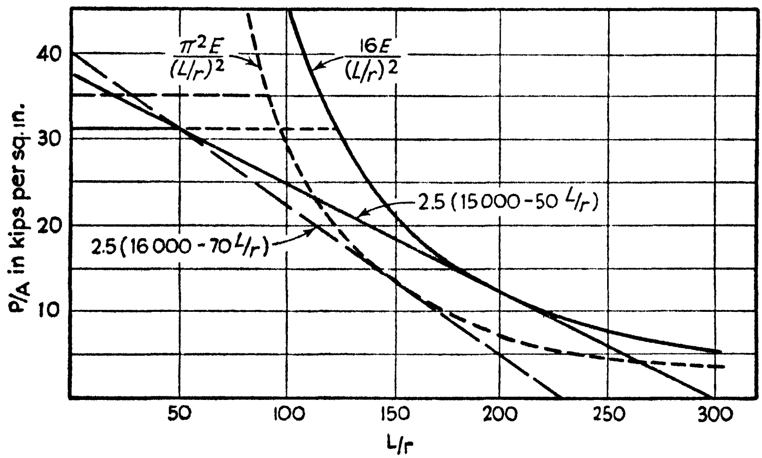


FIG. 2–5

short struts give tested strengths in agreement with Fig. 2–3*a* approaching the elastic limit of the material. As the slenderness ratio increases, bending intervenes and the column fails at a lower average unit stress, long before L/r has reached the value a, the intersection with the Euler curve in the figure.

Recalling the derivation of the Euler formula it will be remembered that the area moment theorem was applied as though the column were an end-supported beam, that is, the ends of the column were assumed to slope under load without restraint. Tests of pin-ended columns were reported by T. H. JOHNSON in 1886 in the paper in which he proposed the straight-line formula (*Transactions*, A.S.C.E., 1886) which indicated that Euler's formula gives too low results for this common type of column. Mr. JOHNSON proposed that the 9.87 of the Euler formula be replaced by 16 to make the curve fit the test results. It is evident that the friction of the pins introduces sufficient restraint at the ends of a column to stiffen it and so strengthen it materially, as indicated by this increased factor. Both the Euler curve and Johnson's modification are plotted in Fig. 2–5 for $E = 30{,}000{,}000$ lb per sq in. On the same diagram are plotted the straight-line formula of the A.R.E.A. specification already quoted, multiplied by a factor of safety of 2.5 to make it

applicable to ultimate loads, and also, similarly multiplied by 2.5, the equation

$$\frac{P}{A} = 16{,}000 - 70\frac{L}{r} \qquad \text{2–3}A$$

which was the formula used instead of Eq. 2–3 in earlier editions of the A.R.E.A. specifications, the lower limit being set at 14,000 lb per sq in.

The A.R.E.A. specifications for railway bridges limits the value of L/r to 100 for main compression members, and to 120 for wind and sway bracing, both values well below the intersection of the plot of Eq. 2–3, adjusted for ultimate loads, with the modified Euler curve of T. H. JOHNSON. The corresponding values for structural steel for buildings are 120 and 200 as set by the A.I.S.C. Should there be occasion to estimate the strength of a column more slender than this it is evident that the modified Euler formula should be used and not the straight-line formula.

It should be noted that Eq. 2–3 is homogeneous, being in the form $y = a - bx$, where b is the rise of the line per unit measured along the X axis, and a is the intercept on the Y axis.

The original proposal of T. H. JOHNSON was for a straight line tangent to Euler's curve, without the limit for small values of L/r. Professor Mansfield MERRIMAN of Lehigh University proposed a straight-line formula in 1882, using the least side (which is the practice today in many formulas for timber columns) instead of the least radius of gyration.

Problem 2–9. The small-scale diagram of Fig. 2–5 apparently shows a line $P/A = 2.5(15{,}000 - 50\ L/r)$ tangent to the curve $P/A = 16\ E/(L/r)^2$ at about $L/r = 200$ and $P/A = 12.5$ kips per sq in. Assuming these values to be correct, determine whether the slope of the line and its intercept on the vertical axis are consistent with tangency. For convenience replace P/A with y and L/r with x in applying the calculus.

Discussion. What is the value of P/A where the curve crosses the $L/r = 200$ line? Were the point $x = 200$ and $y = 12.5$ on the curve the line would be tangent, since the slope of the curve at any point equals $-2\ y/x$ and the intercept equals three times the ordinate at the point of tangency.

Problem 2–10. Determine the values of L/r for the intersections of the straight line and curve of the above problem.

Ans. About 176 and 222.

2–6. The Gordon-Rankine Formula. In this country, until recently, the greatest rival of the empirical straight-line formula was the Gordon-Rankine formula whose basis is largely rational. Its derivation proceeds from the logical assumption that the imperfections of the actual column are equivalent in effect to an initial bending of an ideal column. The

bent column of Fig. 2–4, p. 48, may be considered such an equivalent ideal column, and the fiber stress at an extreme fiber at the section of maximum deflection may be expressed in familiar fashion

$$s = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{P\,Dc}{I}$$

which may be written

$$s = \frac{P}{A}\left(1 + \frac{Dc}{r^2}\right)$$

In a beam it can be shown that the deflection is proportional to sL^2/c. Assuming that the same relation holds for the column we may substitute for D its equivalent aL^2/c, where a is a constant which can be evaluated by experiment. Making this substitution and solving for P/A there results

$$\frac{P}{A} = \frac{s}{1 + a(L/r)^2} \qquad 2\text{-}4$$

which is the Gordon-Rankine formula. A relation in this form, except that r^2 was replaced by the square of the least side, was proposed by the English engineer Lewis GORDON. William J. M. RANKINE (1820–1872), the great Scotch engineer and teacher, gave the equation its present form, and it is often known by his name alone.

For the calculation of the ultimate load on a column, s in Eq. 2–4 is equal to the elastic limit of the material. Many values of the constant a have been proposed for different conditions. Early investigators made much of the way in which the ends of a column are supported, distinguishing several basic cases: fixed ends, flat ends, round ends, pin or hinged ends. In the derivation of the Euler formula, we have already noted that it is for ends freely revolving, either round ends or ends on frictionless pins. Modern practice largely disregards the conditions of the ends. In the first place the friction of a pin connection tends to restrain the end from revolving freely; also absolutely fixed ends are difficult to secure. More important, however, is the fact that in most structures a column is rigidly connected to the rest of the frame and end rotations may be determined entirely by factors external to the column. These end rotations may weaken or strengthen the column and render any quibbling over fixed or pin ends quite futile. When the effects of restraint are definitely known this may be taken into account in design.

The constants recommended for Eq. 2–4 by the A.I.S.C. (1929) are $s = 18{,}000$ lb per sq in. and $a = 1/18{,}000$, working values for use in design, with a maximum value of P/A of 15,000 lb per sq in., which holds for values of L/r up to 60. The A.I.S.C. column formula is, therefore,

a combination of a straight (horizontal) line and a Rankine curve. If the value of s in the Rankine formula is made equal to the elastic limit of the steel, with the curve tangent to the Euler curve at a high value of L/r, the middle portion of the curve lies below the results of tests on the plot.

Problem 2–11. Lay off on cross-section paper Eq. 2–3 multiplied by the safety factor 2.5 and the modified Euler curve, as in Fig. 2–5. On the same diagram plot the Gordon-Rankine formula with $s = 31{,}250$ (i.e., $12{,}500 \times 2.5$) and $a = 1/18{,}000$.

Problem 2–12. Lay off on cross-section paper the broken line given by Eq. 2–3 with its limiting value for small values of L/r and also Eq. 2–4 with $s = 18{,}000$ lb per sq in. and $a = 1/18{,}000$. Add also the curve of $\dfrac{P}{A} = \dfrac{1}{2.5}\left(\dfrac{16\,E}{(L/r)^2}\right) = \dfrac{6.4\,E}{(L/r)^2}$.

Problem 2–13. The center deflection of a symmetrically loaded beam can be expressed by the relation $D = KWL^3/EI$, where K is a constant which varies with the nature of the load W. The moment at the center is $M = K'WL$, where K' is also a constant dependent on the loading; the maximum fiber stress at the center section equals $s = Mc/I$. Show that the deflection is proportional to sL^2/c.

Ritter's Rational Constant. A rational value of the constant a in the Gordon-Rankine formula, Eq. 2–4, may be obtained by noting that at large values of L/r this formula should give the same values of P/A as the Euler formula, Eq. 2–2, or its modified form based on tests proposed by T. H. JOHNSON, noted on p. 51. Equating the two equations

$$\frac{P}{A} = \frac{\pi^2\,E}{(L/r)^2} = \frac{s_e}{1 + a(L/r)^2}$$

where s_e is the elastic limit of the material. Neglecting the value of unity as it is small relative to $a(L/r)^2$ when L/r is *very* large, we obtain

$$a = \frac{s_e}{\pi^2\,E} \quad \text{or} \quad \frac{s_e}{16\,E}, \text{ using Johnson's modification}$$

This relation was found by Professor RITTER and is called Ritter's rational constant. For $s_e = 30{,}000$ and $E = 30{,}000{,}000$ this constant equals 1/9870, or 1/16,000 from Johnson's equation. From the diagrams which the student has constructed in Prob. 2–11 he will see at once that, with this value of the constant a, the curve of the Gordon-Rankine formula will lie considerably below the straight line of Eq. 2–3 for intermediate values of L/r if the two are made to agree for small values.

2–7. The Parabolic Formula. Professor J. B. JOHNSON proposed the parabolic formula which fits test results better than the straight-line and Rankine formulas already discussed. The equation is in the form

$$\frac{P}{A} = s - b\left(\frac{L}{r}\right)^2 \qquad \text{2–5}$$

where s equals the value of P/A for low L/r values in the working formula, equals the elastic limit of the material for ultimate loads, and b is a constant so chosen that the parabola is tangent to the Euler curve or the modified Euler curve as desired. In "Modern Framed Structures," Part III, the authors, the late Professor J. B. JOHNSON, Dean F. E. TURNEAURE, and the late C. W. BRYAN, recommend this form

$$\frac{P}{A} = 13{,}000 - 0.25\left(\frac{L}{r}\right)^2 \qquad 2\text{-}5A$$

which is based on $s_e = 36{,}000$ for ultimate loads and a safety factor of 2.75.

The Special Committee on Steel Column Research, A.S.C.E. previously referred to, in its 1933 report, points out that it is a good substitute for the theoretically correct secant formula within the usual range of L/r, and recommends its use as a working formula. Their equation is the same as Eq. 2-5A except that 15,000 replaces 13,000. For ultimate loads the Committee recommends

$$\frac{P}{A} = \frac{\text{Yield point}}{1 + ec/r^2} - K\left(\frac{L}{r}\right)^2 \qquad 2\text{-}5B$$

K being determined by the point chosen for the intersection with the secant curve which is being approximated, suggested as $\frac{3}{4}$ $L/r = 100$. With a yield point of 32,000 lb per sq in. the equation becomes

$$\frac{P}{A} = 25{,}600 - 0.425\left(\frac{L}{r}\right)^2 \qquad 2\text{-}5C$$

for ultimate loads. The value of E used in calculating the corresponding secant curve was 29,400,000 lb per sq in.

The parabolic formula has now become the standard for American practice with its appearance in the specifications of the A.R.E.A. (1935) and the A.I.S.C. (1936), in these forms:

A.R.E.A. $P/A = 15{,}000 - L^2/4\,r^2$ for riveted ends 2-5D

$P/A = 15{,}000 - L^2/3\,r^2$ for pin ends with L/r limited to 140 in both cases 2-5E

A.I.S.C. $P/A = 17{,}000 - 0.485\,(L/r)^2$ with L/r limited to 120 2-5F

The A.R.E.A. specification is for steel bridges; that of the A.I.S.C., for steel buildings. For values of L/r greater than 140 and for columns with known eccentricity of loading the A.R.E.A. specification (1935) recommends the secant formula; see Appendix A of the specification.

Within recent years the steel manufacturers have made available certain high-strength alloy steels, and these are being used in increasing quantities in situations where weight reduction is desirable, even at an increase in first cost. However, as will be shown in the following paragraphs, no advantage is gained by the use of these more costly steels if the member length is such that buckling is the governing factor (increased resistance to corrosion is another matter).

It has been pointed out that the ultimate strength of long columns may be learned approximately by use of a modified Euler formula, $P/A = KE/(L/r)^2$. Also, it may be shown that a parabolic formula, $P/A = s - b(L/r)^2$, will be tangent to the modified Euler curve if $b = s^2/4\,KE$, and that the coordinates of the point of tangency are $L/r = \sqrt{2\,KE/s}$, and $P/A = s/2$. (See Prob. 2-14.)

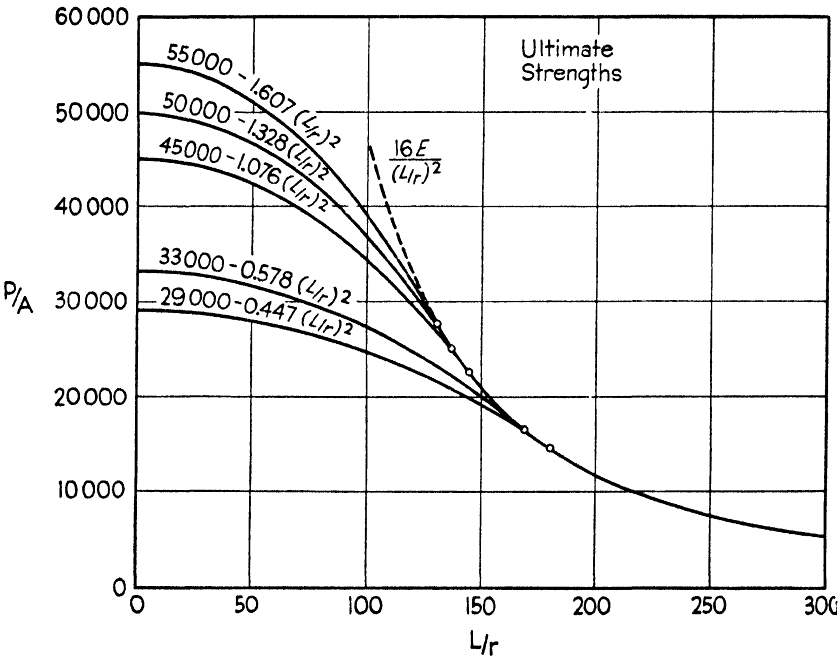


Fig. 2-6

Using Johnson's value for K, (16), E = 29,400,000, and s = yield point, there result the formulas for ultimate strengths given in the fourth column of the adjacent table. These and the modified Euler formula are plotted in Fig. 2-6 which serves to emphasize the fact that for values of L/r exceeding 180 the softer A.A.R. steel is just as strong

in column action as any of the harder steels. With a proper factor of safety — 2.2, for example — Fig. 2–6 becomes available for use in design.

Kind of Steel	Ultimate, lb per sq in., min.	Yield Point, lb per sq in., min.	$\frac{P}{A} = \text{Y.P.} - b\left(\frac{L}{r}\right)^2$	Point of Tangency		Modified Euler Formula
				$\frac{L}{r}$	$\frac{P}{A}$	
Man-Ten	85,000	55,000	$= 55{,}000 - 1.607\left(\frac{L}{r}\right)^2$	130.8	27,500	
Cor-Ten	65,000	50,000	$= 50{,}000 - 1.328\left(\frac{L}{r}\right)^2$	137.2	25,000	
Sil-Ten	80,000 to 95,000	45,000	$= 45{,}000 - 1.076\left(\frac{L}{r}\right)^2$	144.6	22,500	$\frac{16\,E}{(L/r)^2}$
A.S.T.M. A 7–34 A 9–34	60,000 to 72,000	33,000	$= 33{,}000 - 0.578\left(\frac{L}{r}\right)^2$	168.9	16,500	
A.A.R. M 116–34	50,000 to 65,000	29,000	$= 29{,}000 - 0.447\left(\frac{L}{r}\right)^2$	180.2	14,500	

Problem 2–14. Demonstrate that the curves represented by $y = KE/x^2$ and $y = s - bx^2$ are tangent if $b = s^2/4\,KE$. Also, show that at the point of tangency $x = \sqrt{2\,KE/s}$ and $y = s/2$.

Problem 2–15. (*a*) Plot Eq. 2–5 above with $s = 36{,}000$ lb per sq in. and tangent to the modified Euler curve $P/A = 16\,E/(L/r)^2$.

(*b*) Show on a plot: (1) the curve of the 1935 A.R.E.A.[1] formula for columns with pin ends [**301**], extended to $P/A = 0$; (2) the curve of the 1935 A.R.E.A. formula for columns with riveted ends [**301**] extended to $L/r = 220$; (3) the curve of the formulas

$$P/A = (33{,}000 - b(L/r)^2)/2.2$$

and

$$P/A = (16\,E/(L/r)^2)/2.2$$

Determine the value of b to make the curves tangent. Extend the curve to $L/r = 220$; (4) the curve of the 1929 A.I.S.C. formula, carried to $L/r = 200$; (5) the curve of the 1936 A.I.S.C. formula for values of L/r up to 120.

2–8. The Secant Formula. The best solution of the column problem is the secant formula, but it is little used because it is difficult to solve,

[1] Bold face numerals in brackets in this chapter refer to the corresponding articles in the 1935 A.R.E.A. specification, Appendix D.

since it gives the values of P/A implicitly instead of explicitly. Its derivation comes from the assumption that the actual column with its irregularities of shape, material, and loading is equivalent to an ideally perfect column with an eccentric load. The load on such a column which causes a maximum fiber stress, s, may be found as follows.

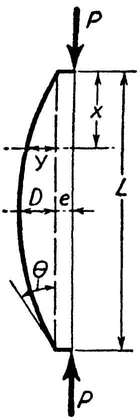


FIG. 2-7

Fig. 2–7 represents an ideal column eccentrically loaded, the magnitude of the load being P and the eccentricity being e. Under action of this load the column takes a bent position as shown and the maximum compressive stress equals

$$s = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{P(D + e)c}{I} \tag{A}$$

where D is the maximum deflection at the center. The value of D may be found from the equation of the elastic curve, which is

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} = -\frac{P(y + e)}{EI}$$

The general integral of this differential equation[1] is

$$y = C_1 \sin \alpha x + C_2 \cos \alpha x - e$$

where
$$\alpha = \sqrt{\frac{P}{EI}}$$

Since
$$y = 0 \quad \text{when} \quad x = 0$$
$$C_2 = e$$

Also, since $y = 0$ when $x = L$

$$0 = C_1 \sin \alpha L + e \cos \alpha L - e$$

$$C_1 = \frac{e\,(1 - \cos \alpha L)}{\sin \alpha L}$$

$$= e \tan \frac{\alpha L}{2}$$

The general integral then takes this form

$$y = e\left(\tan \frac{\alpha L}{2} \sin \alpha x + \cos \alpha x - 1\right) \tag{B}$$

[1] It is not necessary to resort to the methods of differential equations for a solution. It is equally satisfactory to employ the calculus simply as in SEELY's "Resistance of Materials," p. 241, and other textbooks.

Since $D = y$ when $x = \frac{L}{2}$ we may write for D

$$D = e\left(\tan\frac{\alpha L}{2}\sin\frac{\alpha L}{2} + \cos\frac{\alpha L}{2} - 1\right)$$

$$= e\left(\frac{\sin^2\frac{\alpha L}{2} + \cos^2\frac{\alpha L}{2}}{\cos\frac{\alpha L}{2}} - 1\right) = e\left(\sec\frac{\alpha L}{2} - 1\right)$$

Inserting this value for D in Eq. A above gives

$$s = \frac{P}{A}\left(1 + \frac{\left[\left(e\sec\frac{\alpha L}{2} - e\right) + e\right]c}{r^2}\right)$$

$$= \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\frac{\alpha L}{2}\right)$$

Solving for P/A and inserting the value of α

$$\frac{P}{A} = \frac{s}{1 + \frac{ec}{r^2}\sec\frac{L}{2}\sqrt{\frac{P}{EI}}} = \frac{s}{1 + \frac{ec}{r^2}\sec\frac{L}{2r}\sqrt{\frac{P}{EA}}} \qquad 2\text{–}6$$

By tests on a given column where A, c, r, L, and EI are known, the value of the eccentricity e can be determined for any value of the load P, thus relating the actual and the ideal column. The value of e which makes the ideal column equivalent to the actual column is called the "equivalent eccentricity."

Various values have been proposed for the eccentric ratio ec/r^2 to be used in Eq. 2–6 in order to make it generally applicable. JOHNSON, BRYAN, and TURNEAURE in "Modern Framed Structures," Part III, state that a value of 0.001 (L/r) for ec/r^2 makes Eq. 2–6 a fairly accurate formula for pivoted columns of wrought iron. Professor BASQUIN[1] proposed the value of $0.1 + 0.001(L/r)$ to make the formula comparable with those already given. Mr. MONCRIEFF[2] gives the range of the values of ec/r^2 as from 0.6 to 0.15 for columns of cast iron, wrought iron, steel, and several kinds of timber.

The Special Committee on Steel Column Research of the A.S.C.E. in their final report[3] give this form for columns in riveted structures,

[1] *Journal*, Western Society of Engineers, Vol. 18, 1913.

[2] "The Practical Column under Central or Eccentric Loads," by J. M. MONCRIEFF, *Transactions*, A.S.C.E., Vol. XLV, p. 334, 1901.

[3] *Transactions*, Vol. 98, 1933, p. 1449. Dean F. E. TURNEAURE was chairman of this committee.

based on extensive tests

$$\frac{P_u}{A} = \frac{\text{Yield point}}{1 + 0.25 \sec \sqrt{\frac{P_u}{EA}} \left(\frac{0.75\,L}{2\,r}\right)} \qquad \text{2-6}A$$

This assumes the restraint at the ends to give a free column length of three-fourths of the total length, and takes ec/r^2 as 0.25, to cover both crookedness of the member and end eccentricity due to secondary stress.

For working loads, P, which produce a stress equal to the ultimate divided by the factor of safety, $s = s_e \div m$, we have

$$\frac{P}{A} = \frac{\frac{s_e}{m}}{1 + \frac{ec}{r^2} \sec \sqrt{\frac{mP}{EA}} \left(\frac{0.75\,L}{2\,r}\right)} \qquad \text{2-6}B$$

It is evident that the moment area method is just as applicable to the derivation of the secant formula as to the Euler formula, and that the same difficulty will be met with in both cases, the determination of the area and centroid of the moment area. Mr. MONCRIEFF used this method for his derivation, making the approximation used in this text for the Euler formula that the moment curve is a parabola, obtaining this approximation to the secant formula for maximum compressive unit stress

$$s = \frac{P}{A}\left[1 + \frac{ec}{r^2}\left(\frac{48\,E + \left(\frac{P}{A}\right)\left(\frac{L}{r}\right)^2}{48\,E - 5\left(\frac{P}{A}\right)\left(\frac{L}{r}\right)^2}\right)\right] \qquad \text{2-6}C$$

Problem 2–16. Complete Probs. 2–11, p. 54, and 2–15, p. 57, by plotting Eq. 2–6A with s_e = 32,000 lb per sq in., E = 30,000,000 lb per sq in.

Problem 2–17. Complete Prob. 2–12, p. 54, by plotting Eq. 2–5A with 13,000, replaced by 15,000, and Eq. 2–6B with s_e = 32,000, m = 2.1333, and ec/r^2 = 0.25.

Problem 2–18. Derive Eq. 2–2, Euler's formula, using the differential equation of the elastic curve.

Note. The general integral is the same as that given in the derivation of the secant formula except that the last term, $-e$, does not appear. Constant C_2 is easily found to be zero. When $x = L$, $y = 0 = C_1 \sin \alpha L$. Here C_1 cannot be zero, for the general integral would then reduce to zero, that is, it would state that there is no curvature, which is not possible. The term $\sin \alpha L$ is zero when $\alpha L = 0$, $\alpha L = \pi$, $\alpha L = 2\,\pi$, etc., that is when $\alpha = n\pi/L$. It is not necessary to go on to evaluate C_1 in the general integral since the above relation reduces to the Euler formula taking n as unity. In order to complete the mathematical sequence it is advisable that the student demonstrate that $C_1 = D$, the maximum deflection, the value of y when $x = L/2$.

The general integral, $y = D \sin \alpha x$, is the equation of a sinusoidal curve. For single curvature (Fig. 2–3, p. 48) αx varies from 0 to π and $n = 1$.

Problem 2–19. Derive Eq. 2–6C.

Inspection of the parabolic and secant formulas for ultimate loads demonstrates what is shown graphically by the result of the work of Probs. 2–15 and 2–16, that for the low values of the slenderness ratio the breaking loads on actual structural columns in frames are estimated as giving less than an average stress equal to the yield point of the material. This is due to the inevitable imperfections of column and loading already discussed. Short columns made and tested with precision often develop stresses far above the yield point.

Euler Formula. In a long slender column to which the Euler formula would be applied, the greatest stress is the bending stress; that is, the direct stress is small (almost negligible), and *ec* is also small. Therefore, in the equation

$$s = \frac{P}{A} + \frac{P}{A}\left(\frac{ec}{r^2} \sec \frac{L}{2\,r}\sqrt{\frac{P}{EA}}\right)$$

for s to become large enough to cause failure, $\sec \sqrt{\dfrac{PL^2}{4\,EI}}$ must become infinite, which it will for an angle of 90°. That is,

$$\sqrt{\frac{PL^2}{4\,EI}} = \frac{\pi}{2}$$

$$P = \frac{\pi^2 EI}{L^2}$$

This is the Euler formula, which may also be written

$$\frac{P}{A} = \frac{\pi^2 E}{\left(\dfrac{L}{r}\right)^2} \qquad \text{2–2}$$

Note that the angle $\dfrac{L}{2\,r}\sqrt{\dfrac{P}{EA}}$ is simply an expression arising in the analysis and is without physical significance.

2–9. Columns With Bending. The design of members subjected to both transverse bending and axial loads but *not* free to buckle was considered in Art. 2–2 under the heading "Beams with Axial Loads." Members which support these loads and which *are* free to buckle are frequently met in practice. Common cases are mill-building columns with eccentric loads (perhaps applied through brackets), and girders and columns of rigid frames (see "Theory," Art. 6–6, and Chapters

VIII and IX). Two methods, based on two different ways of specifying allowable working stresses, are in use for the design of such members. In either method, provision may be made for the added stress due to the bending, $P \times D$, which arises from the deflection due to transverse loads. As is shown in Prob. 2–2, this may be disregarded, usually, as an unnecessary refinement.

First Method. The only difference between the case of direct stress and bending previously considered and that presented by the column with transverse bending is that the effect of the axial compression on the column is to induce bending in addition to direct stress through the inevitable eccentricity of its application and irregularity of the member. The maximum compressive stress in a column with both axial and lateral loads is

$$s = \left(\frac{P}{A} + \frac{Pec}{Ar^2}\right) + \frac{Mc}{Ar^2} \tag{A}$$

where the bending stress due to column action is represented by the second term in the parenthesis, e being the equivalent eccentricity (which would not have the same value for this simple relation as for the more elaborate analysis of the secant formula; Art. 2–8). The third term gives the effect of transverse bending which, as has been shown, will arise from the following causes: transverse loads; secondary bending; eccentric application of the longitudinal load; the lateral deflection of the column under transverse load and consequent bending action of the longitudinal force. This latter deflection is in addition to the fortuitous deflection represented by the equivalent eccentricity.

Solving Eq. A for the required column area

$$A = \frac{P}{\dfrac{s}{(1 + ec/r^2)}} + \frac{Mc}{sr^2} \tag{B}$$

$$= \frac{P}{p} + \frac{Mc}{sr^2}$$

the denominator of the first fraction, $s/(1 + ec/r^2) \doteq p$, being the value of P/A given by the column formula. When applying this in design, the limiting stress, s, used in the second term is often the same as the s in the fraction expressing the results of the column formula, being given the value specified for beams. The case where $s = p$ is discussed below under the heading "Second Method."

A trial selection is made of a section to carry the loads and the allowable unit stresses both as a column and as a beam are computed. Then

the area required to carry the column load is added to the area required to carry the beam load and comparison is made with the area of the trial section. An identical[1] method is given in the rule: A satisfactory design is one in which $\dfrac{P_{\text{actual}}}{P_{\text{allowable}}} + \dfrac{M_{\text{actual}}}{M_{\text{allowable}}} < 1$

Example 2-2. A 15 I 55 $\times$ 10′-0″ carries a direct axial load of 90 kips and a transverse load which produces a moment of 450 kip-in. about the centroidal axis parallel to the web. The specification of the A.I.S.C., Sec. 10(*a*), states: "Members subject to both direct and bending stresses shall be so proportioned that the greatest combined stresses shall not exceed the allowed limits." Is the design in accordance with the specifications?

Solution. As a column only the allowable load is

$$P = 16.06\left(\frac{18}{1 + \dfrac{L^2}{18{,}000\, r^2}}\right) = 165 \text{ kips}$$

As a beam only the allowable moment is

$$M = 67.8\left(\frac{20}{1 + \dfrac{L^2}{2000\, b^2}}\right) = 1100 \text{ kip-in.}$$

In the beam under consideration

$$\frac{90}{165} + \frac{450}{1100} = 0.55 + 0.41 = 0.96 < 1$$

Therefore, the design is satisfactory.

Second Method. Other specifications — the A.R.E.A., for instance — are more severe in dealing with combined stresses. In [**216**] we read: "Members subject to both axial and bending stresses shall be so proportioned that the combined fiber stresses will not exceed the allowed axial stress." That is, s, in the equation $s = P/A + M/S$, must not exceed the allowable value as determined by the column formula. The following transformation will aid in the use of this method.

$$s = \frac{P}{A} + \frac{M}{S} = \frac{P}{A} + \frac{MA}{SA}$$

$$sA = P + M\frac{A}{S}$$

Values of sA are given in column tables,[2] and it is seen that, if the

[1] This rule is identical with the requirement of Sec. 6(a), A.I.S.C. Spec., 1936.

[2] A.I.S.C., "Steel Construction," Second Edition, pp. 181–205, Third Edition, pp. 208–235; Carnegie, "Pocket Companion," Twenty-fourth Edition, pp. 286–310. Values of A/S are given in "Steel Construction," under the heading "Bending Factors, B_x, B_y"; and in "Pocket Companion," under "A/S_{1-1}, A/S_{2-2}." Elsewhere the bending factor is sometimes given as c/r^2. Show that $c/r^2 = A/S$.

moment be multiplied by A/S and added to the direct central load, P, the combined value may be used as though it were the load on a centrally loaded column, that is, sA in the tables.

2-10. Column Shapes. The principal types of columns made of combinations of structural shapes are shown in the figures for Prob. 2-21, p. 67. It is essential that the several pieces forming any column be securely fastened together so that the whole will act as a unit and each part will carry safely and efficiently its proportional share of the load. In order to assure uniform distribution of load the rivets at the ends of a column are required to be closely spaced [**63**[2]]. By itself a plate is not an efficient compression piece, lacking sufficient stability to develop its strength unless definitely restricted in proportions. Experiment and analysis have led to the usual rules[1] limiting the width-thickness ratio of thin elements of columns [**65**[2], **405**].

The experiments conducted by the special committee of the A.S.C.E. previously referred to included as principal types the rolled H-section and the built-up forms lettered d, f, g, and h in Prob. 2-21. Of these, the column formed of two channels connected by lacing (f) showed itself decidedly (about 10 per cent) weaker than the theoretical ultimate when subjected to bending in the plane of the lattice bars.[3] (For a view of lacing see Fig. 8-2, p. 192.) The design of lacing involves the shear developed in the column by bending.

It is sometimes ruled that, where a column is composed of separated parts connected by lacing, each segment shall have a slenderness ratio not greater than that of the member as a whole. Consider a column of square section, made up of four angles, one at each corner, connected by lacing. Usually in such a column the points of lattice connection on one leg of an angle would be intermediate between those on the other leg. For simplicity assume that these connections come together. This rule would demand that the length, L, between connections must be such that the slenderness ratio, L divided by least r, will equal that of the column as a whole. If this value is in the intermediate range, however, this length of angle will not be able to develop the yield point of the material, which it must do if it is on the concave side and the column as a whole is to develop its computed strength. It is necessary therefore that each isolated individual component of a column have a slenderness ratio definitely in the short-column range, regardless of the ratio for the column itself.

[1] *Transactions*, A.S.C.E., Vol. 95, 1931, pp. 1237ff.
[2] A.R.E.A., 1931.
[3] *Transactions*, A.S.C.E., Vol. 98, 1933, pp. 1393–1394.

2–11. Shear in Columns.[1] Two cases are to be considered in estimating the shear developed in a column: (1) that in a column bent in single curvature, and (2) that in a column bent into a reversed curve by the action of the secondary bending at the ends. The first case is represented by Fig. 2–7 (p. 58). Here the transverse shear is

$$V = P \sin \theta = P \tan \theta \tag{A}$$

since the angle is very small. Assuming that at failure the curve taken by the bent column is a parabola, we may write approximately

$$\tan \theta = \frac{2(D + e)}{L/2}$$

The maximum fiber stress caused by the curvature of the column is

$$s_b = \frac{P(D + e)c}{I}$$

whence

$$(D + e) = \frac{s_b r^2}{(P/A)c}$$

Rewriting Eq. A above

$$\begin{aligned} \frac{V}{P} &= \frac{s_b}{P/A} \frac{4\, r^2}{Lc} \\ &= \frac{(s_e - P/A)}{P/A} \frac{4\, r^2}{Lc} \end{aligned} \tag{B}$$

since at failure the fiber stress due to bending equals the difference between the elastic limit and the average stress.

In the second case the curvature may be neglected and the primary and secondary stress effects may be represented by the eccentric application of the primary compression, Fig. 2–8. Again we have the relation

$$V = P \sin \theta = P \tan \theta \tag{A}$$

where $\tan \theta = 2\, e/L$. Also

$$s_b = \frac{(Pe)c}{I}$$

$$\frac{s_b}{P/A} = \frac{ec}{r^2}, \text{ the secondary-primary stress ratio}$$

FIG. 2–8

[1] The material in this article is given on pp. 1218–1219, *Transactions*, A.S.C.E., Vol. 95, 1931.

We may rewrite Eq. A

$$V = P\frac{2\,r^2(ec/r^2)}{Lc} \tag{C}$$

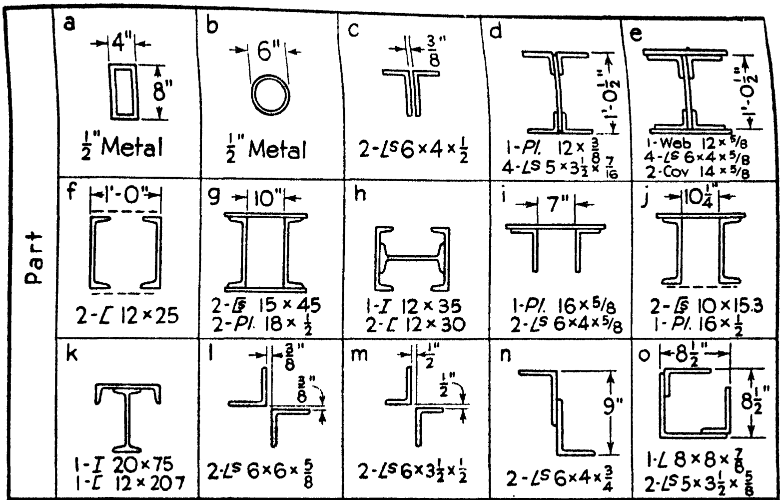
The following considerations are to be observed in attempting to estimate the significance of Eq. B and C. The ratio r/c appears in both and may be estimated at about 0.8 for average conditions. Evidently both expressions give increasing values with decreasing slenderness ratios. The secondary-primary stress ratio, ec/r^2, may be taken at an upper value of 0.5 for bending in the plane of the truss and at 0.2 for bending perpendicular to the truss. The latter is usually the plane of lacing and so gives the value chosen. Estimating P/A and s_b by the aid of the secant formula, Eq. 2–6A, p. 60 (see plot made for Prob. 2–16), $s_e = 32{,}000$, eccentric ratio of 0.25, effective length of three-fourths total length, for $L/r = 10$ we have $V/P = 0.07$ from Eq. B, and 0.03 from Eq. C. The 1935 A.R.E.A. specification gives a formula [**421**] to be followed in design.

Problem 2–20. Carry through the computation outlined in the paragraph above for $L/r = 5$ and $L/r = 100$.

Ans. $(L/r = 5)$ 0.16 0.06
$(L/r = 100)$ 0.015 0.003

Problem 2–21. Determine the allowable total load on each of the columns shown. Make full use of handbook data.

Part	Formula	Length	Ans., kips	Part	Formula	Length	Ans., kips
a	β	9′–6″	125	i	β	15′–0″	218
b	γ	15′–0″	105	j	β	25′–0″	188
c	δ	10′–0″	112	k	γ	18′–0″	345
d	β	12′–3″	214	l	α or δ (?)	29′–0″	108
e	γ	15′–0″	727	m	α or δ (?)	22′–0″	40
f	γ	20′–0″	220	n	α or δ (?)	17′–6″	41
g	β	18′–6″	554	o	α or δ (?)	25′–0″	219
h	δ	16′–3″	342				



Formula	α	β	γ	δ
	$\frac{P}{A}=\left(\frac{16E}{(L/r)^2}\right)\frac{1}{2.75}$	$\frac{P}{A}=15000-50\,L/r$ Max., 12500	$\frac{P}{A}=\frac{18000}{1+\frac{L^2}{18000r^2}}$ Max., 15000	$\frac{P}{A}=[36000-0.675(L/r)^2]\frac{1}{2.75}$

Prob. 2–21

CHAPTER III

BOLTS AND RIVETS

3-1. Structural parts are connected by bolts, rivets, and pins, and by welding. The last method is discussed in Chapter XI. The usual rules for the computation of the strength of bolted and riveted connections and for the details of workmanship are given in the "General Specifications for Steel Railway Bridges," of the American Railway Engineering Association (hereafter referred to as the A.R.E.A.), editions of 1931 and 1935, portions of which are printed as Appendix D of this volume. In the following articles, bold-face numerals in braces, { }, refer to articles in the 1931 specification; in brackets, [ ], to the 1935. Some of the articles to which references are made are not reprinted because their contents are sufficiently explained below.

3-2. Bolts. In structural work, bolts are generally used only in minor structures, temporarily in the erection of more important structures, and occasionally in inaccessible connections where rivets cannot be driven. Ordinary bolted connections have holes $\frac{1}{16}$ in. larger than the bolt, which is made of rolled material. For the few cases where bolts are used in important connections a tight fit is necessary and turned bolts are used in drilled holes [**522**].

3-3. Rivets. Structural rivets are made of a soft grade of steel which does not become brittle as a result of hot working. The A.R.E.A. specification for this material [Part II, Section 8 of that specification] is that of the American Society for Testing Materials (hereafter A.S.T.M.) and printed in the yearbook of the society. Although both larger and smaller rivets are employed in special cases, the general run of structural work takes either $\frac{3}{4}$-in. or $\frac{7}{8}$-in.-diameter rivets, the smaller of these being used for relatively light building and bridge work. General information concerning rivets will be found in structural-steel handbooks.

There are three types of rivet holes: punched, subpunched and reamed, and drilled [**505**]. *Punched holes* are punched full size, which is $\frac{1}{16}$ in. larger than the nominal size of the rivet, in metal of definitely limited thickness, varying with different specifications. Inevitably the metal at the edge of a punched hole is damaged by the operation, the damage increasing with the thickness of the material; hence there is a limit to thickness so far as the metal itself goes. The limit set ranges from a conservative $\frac{5}{8}$ in. for main material, $\frac{7}{8}$ in. for secondary material

[**504**] with $\frac{3}{4}$-in. and $\frac{7}{8}$-in. rivets, to a thickness equal to the rivet diameter plus $\frac{1}{8}$ in. Structural steel thicker than $1\frac{1}{8}$ in. cannot be punched for $\frac{3}{4}$-in. or $\frac{7}{8}$-in. rivets without likelihood of breakage of punches.

A more expensive process is that of punching the holes $\frac{3}{16}$ in. smaller (*subpunching*) and, after assembly of the pieces to form the member, *reaming* to $\frac{1}{16}$ in. larger than the nominal rivet size, thus removing the metal damaged by punching [**506, 507**]. As the reaming is done after the pieces composing the member are assembled, this method results in holes which are accurately lined up (fair holes) and cylindrical [**505**]. Material thicker than the limit set for full-size punching may be either subpunched and reamed, or drilled, according to the specification. The requirement that when there are more than three thicknesses of main material the holes shall be subpunched and reamed [**504**] is explained by the difficulty of lining up holes accurately, a difficulty which increases greatly with the number of thicknesses of material. Poor matching of holes results in deformed rivets of impaired section and strength, and so is not allowed.

Drilled holes are used in thick metal when called for by the specification and also for rivet groups where great accuracy of fit is essential, as in the main field connections of trusses. Metal templates may be used to locate the holes in this work [**512, 513**]. Drilled holes are avoided as far as possible on account of the extra expense involved.

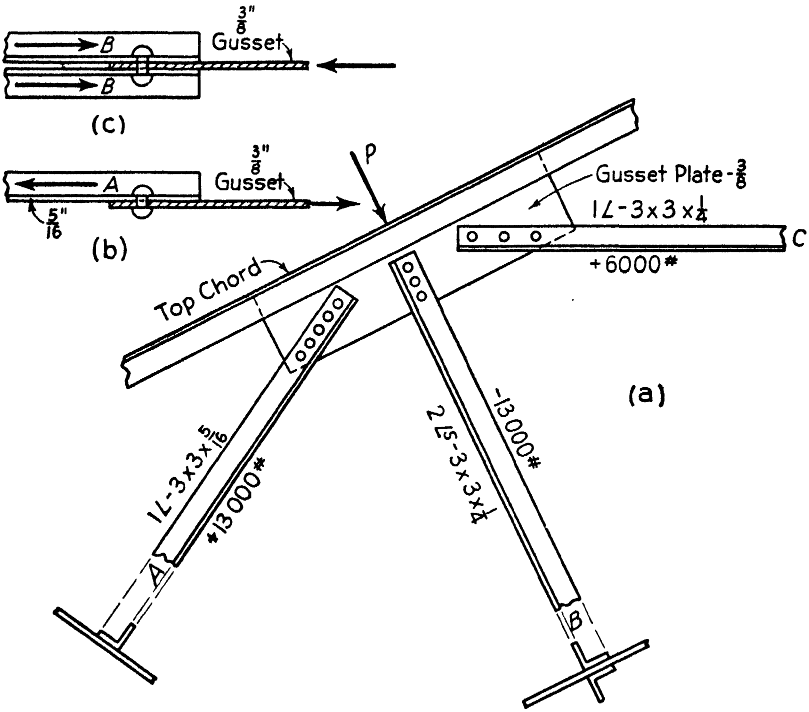
After the parts to be connected by rivets are punched and assembled, the rivet — a cylinder of metal with a head on one end — is heated and inserted in the hole, and a head is formed on the other end [**517, 518, 519**]. The contraction of the hot rivet when cooling causes it to be placed in a state of tension, which draws tightly together the parts connected by the rivet. Tests have shown that in many cases, where the stress on the rivet does not exceed the usual working values, the transfer of stress between the parts connected takes place by friction, but this fact is never taken into account in design.

Specifications list allowable unit values for shear and bearing for use in the design of riveted joints. The strength of a rivet in shear is taken as the allowable shearing stress in pounds per square inch times the cross-sectional area of the nominal rivet, that is, a $\frac{3}{4}$-in. rivet is taken as $\frac{3}{4}$ in. in diameter although the driven rivet is assumed to fill a $\frac{13}{16}$-in.-diameter hole. The allowable load on a rivet in bearing is taken as the product of the allowable bearing stress, the nominal rivet diameter, and the thickness of the material on which the rivet bears {**41, 42**}.

Computations involving the net areas of tension members, on the contrary, are made on the assumption that the diameter of the rivet hole is $\frac{1}{8}$ in. greater than the nominal rivet diameter [**409**]. This not only pro-

vides for the fact that the hole is actually $\frac{1}{16}$ in. larger than the rivet but also allows for some possible injury that the punching may cause in the adjacent material.

3-4. Riveted Connections. The simplest use of rivets for carrying measured stress in structural work is in the connection of tension and compression members in trusses. The principle of proportioning riveted connections of this type is illustrated by the following example.



Ex. 3-1

Example 3-1. Determine the number of $\frac{5}{8}$-in. rivets required to connect members A and B to the top chord gusset plate here shown. Allowable stresses in rivets: shear 10,000 lb per sq in.; bearing 20,000 lb per sq in.

Solution. The load on each rivet connecting A to the gusset is transmitted to it by bearing of $\frac{5}{16}$-in. metal on the rivet shank; is carried across to the plate with shear on a single cross section of the rivet; is given over to the gusset by bearing of the shank on $\frac{3}{8}$-in. material. The rivet is described as in *single shear.* There must be enough rivets so that each load-carrying function just listed is properly carried through. A $\frac{5}{8}$-in. rivet can receive or transmit load by contact with $\frac{5}{16}$-in. metal, with the given unit stresses, in the amount of $\frac{5}{8} \times \frac{5}{16} \times 20{,}000 = 3900$ lb; it can carry in single shear $0.307 \times 10{,}000 = 3070$ lb. (The cross-section area of a $\frac{5}{8}$-in. rivet is

0.307 sq in.) In this case, by inspection, it is seen that the rivets are limited either in single shear or in bearing on $\frac{5}{16}$-in. thickness of metal, and the computation shows that of these two the single shear value is the smaller. The number of rivets required is 13,000 ÷ 3070 = 5 rivets.

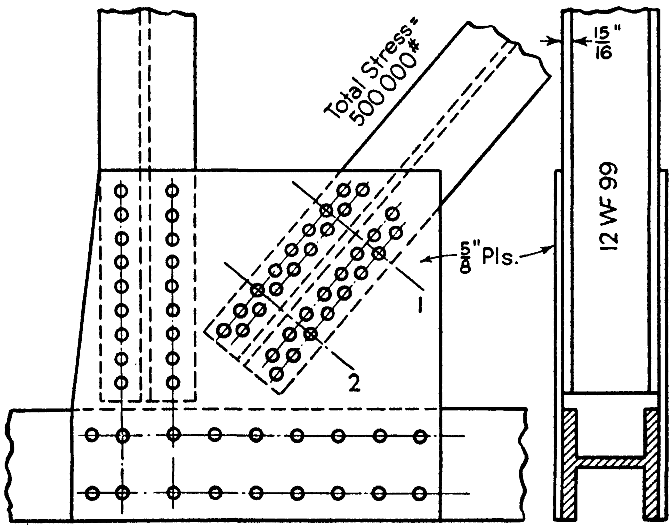
The load from member *B* comes equally from each angle, and accordingly each rivet in the connection receives its load by two bearings on $\frac{1}{4}$-in. metal and gives it over by bearing on $\frac{3}{8}$-in. metal. In the transmission the two sections on each side of the gusset plate are equally stressed in shear, and the rivets are said to be in *double shear*. Each rivet is limited either by bearing on $\frac{3}{8}$-in. metal ($\frac{3}{8}$ being less than $2 \times \frac{1}{4}$) or by double shear. The bearing value is $\frac{5}{8} \times \frac{3}{8} \times 20{,}000 = 4690$ lb; the double shear value is $2 \times 3070 = 6140$ lb. The number of rivets required is that needed in bearing, 13,000 ÷ 4690 = 3 rivets.

Problem 3–1. Compute the number of rivets required to connect member *C* to the gusset in the previous example.

Ans. Two

Problem 3–2. Compute the number of rivets required to connect the diagonal member here shown to the two gusset plates. Use $\frac{7}{8}$-in. rivets. Stresses: shear 12,000 lb per sq in.; bearing 24,000 lb per sq in.

Ans. 35 rivets through each plate



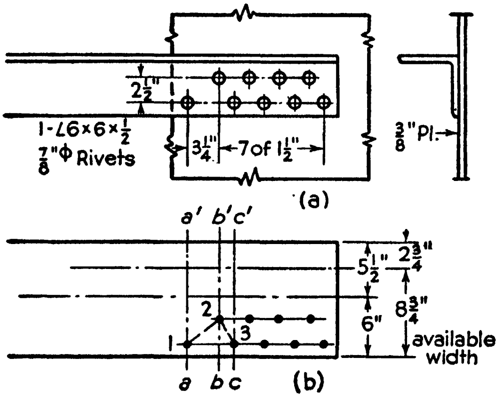
Prob. 3–2

The rivet group shown for Prob. 3–2 is symmetrical both in itself and as regards the load transmitted. Whenever the line of action of the forces passes through the centroid of the rivet group all the rivets are assumed to be equally stressed. That this cannot be actually the case is

evident on considering the deformations at any two sections, as 1 and 2, across the end connection of the diagonal of Problem 3–2. At section 1 the member carries more stress than at section 2, less stress having been transferred to the gusset plates by the rivets, and accordingly the unit deformation of the diagonal must be larger at section 1 than at 2. For equal bearing on each rivet, and therefore equal load, these two unit deformations should be the same.[1] Theoretically the uppermost rivets in this group would carry more load than those at the lower end. Tests have shown, however, that satisfactory results are obtained when equal distribution of load among the rivets is assumed.

When a tension member is connected by more than one line of rivets, there is the possibility that a zigzag section (as for example a–1–2–b', part (*b*) of figure of Ex. 3–2) will be weaker than a right section. A rational method for computing the strength of such a section has not been developed, and recourse must be had to one or other of the several rules-of-thumb available, the best of which are based on tests and are reliable. Procedure in this situation is illustrated by the example which follows.

Example 3–2. Using the 1935 A.R.E.A. Specification for Steel Railway Bridges, compute the allowable tension on the angle shown.



Ex. 3–2

Solution. If the rivets prove to be sufficient and properly spaced, the allowable tension will equal the working tensile stress times the net area,

[1] What effect upon this argument has the unequal deformation of the gusset plate? In studying this question consider two strips of metal of equal area fastened together longitudinally by a row of three rivets. Sketch the deformed connection as tension is transferred across the joint. Repeat for four and for five rivets. It will be necessary to do this to some assumed scale. Evidently the distribution of load is more nearly equal than it would be were one strip riveted to another very much stiffer.

section a–a', having regard for Art. 410 of the specification which permits only one-half of the unconnected leg to be considered.

$$\begin{aligned} T &= 18(5.75 - 0.5 - 0.5 \times 0.5 \times 5.5) \\ &= 18 \times 3.87 = 69.75 \text{ kips} \end{aligned}$$

The full pull in the angle will come also on the zigzag section a–1–2–b', and accordingly the net area here must equal that on the right section a–a', using the rule for computing width given by Art. 409.

$$3.87 = 0.5\left(11.5 - \frac{5.5}{2} - 2 \times 1 + \frac{s^2}{4 \times 2.5}\right)$$

which gives $s = 3.17$ in. Use $3\frac{1}{4}$ in.

The section b'–2–3–c is called upon to carry only $\frac{8}{9}$ of the total pull, assuming each rivet to carry its proportional share of the load, or it may be assumed that the pull at this section equals the total less the bearing or single shear value of the first rivet, taking the smaller of the two. The student should complete the computation for this section and should check the total number of rivets used.

3–5. Rivet Groups in Torsion. In bracket and similar connections rivets must frequently resist a force whose line of action does not pass through the center of the group. In such cases the group is subjected to a twisting moment in addition to direct stress.

This is illustrated in Fig. 3–1, where is shown a plate loaded by the inclined force P, and attached by five rivets to a vertical supporting

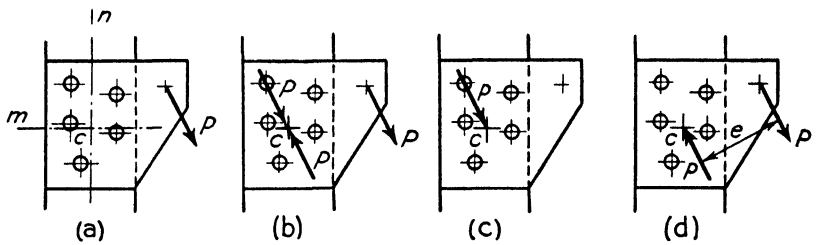


FIG. 3–1

member. If the line of action of the force passed through c, the centroid of the rivet group, there would be no torsion and all rivets would be assumed to be equally loaded. The effect of the load applied as shown is made clear by applying at c, the centroid, two equal and opposite forces, each equal to P, in a line parallel to P, Fig. 3–1b. These two forces do not change the stress situation in the slightest; the deformation conditions in Fig. 3–1b are exactly the same as those for Fig. 3–1a. For the purpose of stress analysis the effect of these three forces may be considered as those of a centric load, P, Fig. 3–1c, and a couple, Pe, Fig. 3–1d.

The centroid of the rivet group is found as follows. Apply to each rivet a unit vertical force and find n, the line of the resultant of these five forces. Similarly, apply five unit horizontal forces and locate m, the line of their resultant. The intersection of lines n and m is called the centroid of the rivet group.[1]

For the rivet group shown in Fig. 3-1 the effect of the centric load P is to place upon each rivet a load equal to $P/5$ acting parallel to the load line.

The effect of the couple Pe is to cause a tendency of the plate to rotate about the centroid. Assume that a slight rotation takes place about this point, c. If the plate be considered rigid and the rivets elastic the deformation at each rivet will vary as the distance of the rivet from c. Consequently, the stress in any rivet due to torsion — and its opposite, the force exerted by the rivet on the plate — will also vary as the distance from c, and this force will act perpendicular to the line joining the rivet with c. The force system acting on the plate due to torsion alone is shown in Fig. 3-2. Calling the stress on the extreme rivet of the group r_1 and the distance of this rivet from the centroid d_1, the force acting on any other rivet, distant d from c, is $r_1 d/d_1$; the moment of this force about c is $r_1 d^2/d_1$. The plate is in equilibrium under the action of the balanced force system shown in Fig. 3-2, and accordingly we may write

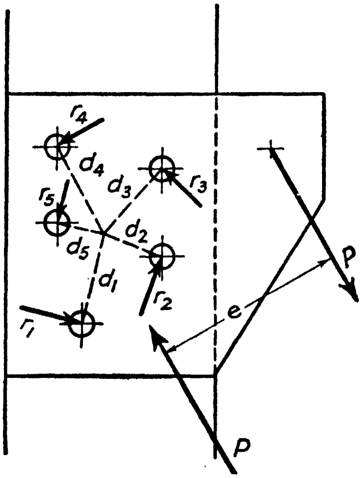


FIG. 3-2

$$\frac{r_1}{d_1}\Sigma d^2 = Pe = M$$

and
$$r_1 = \frac{Md_1}{\Sigma d^2}$$

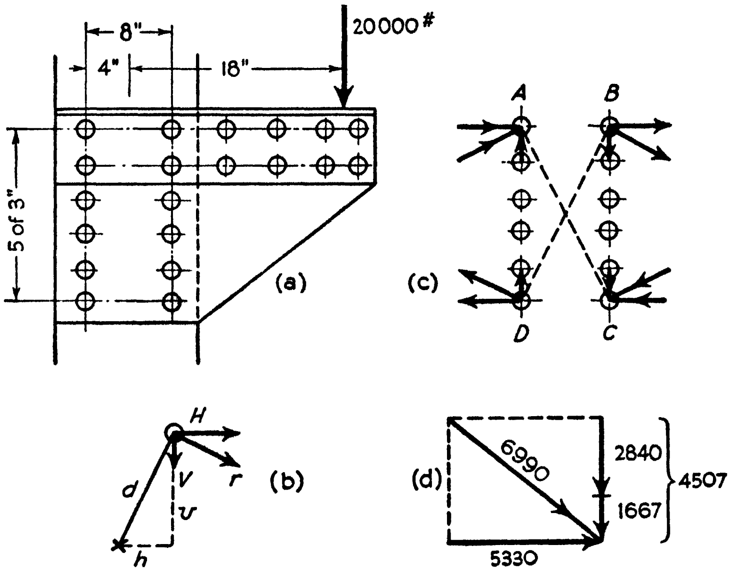
Similarly the stress due to torsion on any rivet is

$$r = \frac{Md}{\Sigma d^2}$$

The *total* force on any rivet will be the resultant of the torsional force and a direct force $P/5$ acting parallel to P. This resultant may be found graphically, but the example which follows indicates a simple slide-rule method for its determination.

[1] It may be shown by calculus that the point so located is the point about which Σd^2 (where d is the distance from this point to any rivet) is a minimum. As will be shown later, the stress on any rivet in a group in torsion varies as d^2, where d is the distance from the center of rotation. Therefore, the resistance of the group to torsion is *least* about this point.

Example 3–3. A plate and angle bracket which supports a load of 20,000 lb is attached to the flange of a column by 12 rivets as shown. Which is the most stressed rivet, and what is the stress in that rivet?



Ex. 3–3

Solution. Each rivet carries a direct vertical stress of 20,000/12 = 1667 lb. In addition each rivet carries a torsion stress of $\frac{M}{\Sigma d^2} \times d$, where $\frac{M}{\Sigma d^2}$ is a constant for the rivet group. This stress, r, acts normal to the line joining the rivet with the centroid of the group. If this stress be resolved into V and H components, it is clear from (b) in the figure, by similar triangles, that

$$V = r\frac{h}{d} = \frac{Md}{\Sigma d^2} \times \frac{h}{d} = \frac{M}{\Sigma d^2}h$$

and

$$H = r\frac{v}{d} = \frac{Md}{\Sigma d^2} \times \frac{v}{d} = \frac{M}{\Sigma d^2}v$$

also, $\Sigma d^2 = 4 \times 1\tfrac{1}{2}^2 + 4 \times 4\tfrac{1}{2}^2 + 4 \times 7\tfrac{1}{2}^2 + 12 \times 4^2 = 507$

(In practical work arrange the computation of Σd^2 vertically.) Therefore, each top and bottom rivet of the group carries a horizontal component due to torsion of

$$\frac{20{,}000 \times 18}{507} \times 7\tfrac{1}{2} = 5330 \text{ lb}$$

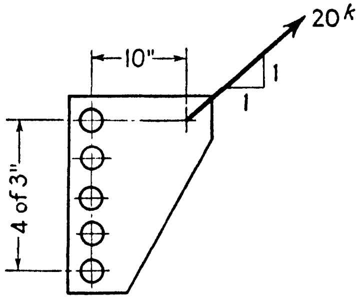
Furthermore, each rivet of the group carries a vertical component due to torsion of

$$\frac{20{,}000 \times 18}{507} \times 4 = 2840 \text{ lb}$$

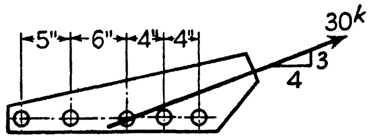
In the case of A and D this component opposes the component due to direct stress. In B and C the components add, giving the total components shown (d). Therefore, rivets B and C are most stressed and each carries a stress of 6990 lb.

Problems 3–3, 3–4, 3–5. Which of the rivets is most stressed and what is the stress in that rivet?

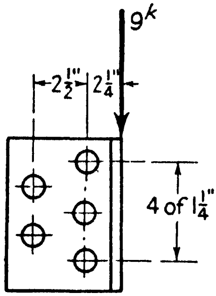
Ans.	3–3	7180 lb
	3–4	6450 lb
	3–5	4400 lb



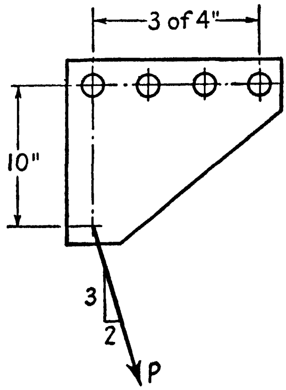
PROB. 3–3



PROB. 3–4



PROB. 3–5



PROB. 3–6

Problem 3–6. Due to P, the stress in the most stressed rivet in the group is 5400 lb. Find P.

Ans. $P = 5360$ lb

3–6. Rivets in Tension.[1] Many modern specifications permit a tension value in rivets equal to 50 per cent of the shear value. This is

[1] "Tensile Working Stress for Rivets," by C. R. YOUNG, A.I.S.C., 1927.
"Tensile Tests of Rivets," by Wilbur M. WILSON and William A. OLIVER, *Bulletin* 210, Engineering Experiment Station, University of Illinois.

amply justified by tests and by the further fact that in past years many details have given satisfactory service which were designed with rivets carrying stress in tension equal to 100 per cent of the allowable shear value. The recommended value of 50 per cent appears very conservative when it is remembered that a rivet in tension does not take additional stress until the load exceeds the initial tension caused by cooling in the rivet. The tests mentioned above show that this initial tension is close to the elastic limit of the material.

In arranging rivets in connections where they are subjected to tension the following precautions should be observed:[1]

1. A tension detail should have at least four rivets symmetrically placed with respect to the pull.
2. In addition to the rivets needed for tension there should be enough to take the shear.
3. The connection should be designed with thick enough metal to take the bending stress at the heel of the angles.

3–7. Bracket Connections. Loads are frequently supported on the faces of columns by brackets of the type shown in Fig. 3–3. Downward movement of the bracket is resisted by shear in the rivets which connect

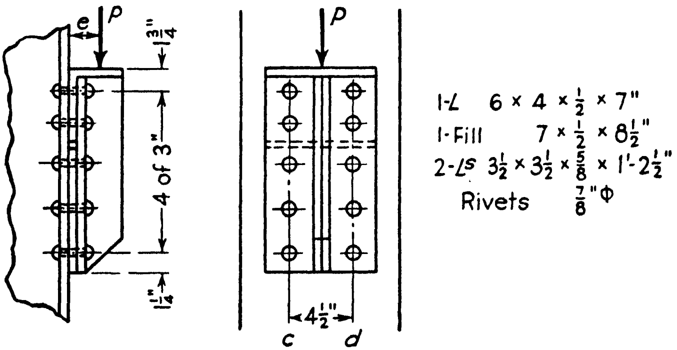


FIG. 3–3

the bracket to the column. The downward load and the upward shears constitute a couple, of moment Pe, which must be resisted by an equal couple consisting of tension in some of the rivets in lines c and d and compression of the lower part of the stiffener angles through the filler and against the column.

Analysis of such a connection for flexural stress is simplified by assuming that the rivet material in tension may be replaced by a vertical fin of equivalent area. The thickness, a, of this fin will be the area of two

[1] Edward GODFREY, *Engineering News-Record,* March 8, 1928, p. 416.

rivets divided by the vertical spacing of the rivets

$$\left(a = \frac{2 \times 0.60}{3} = 0.4 \text{ in.}\right)$$

Some assumption must be made as to the width, b, Fig. 3-4, which acts in compression. It seems evident that in each angle this width will be something greater than the thickness of the metal in the outstanding leg, but will be considerably less than the full width of the angle leg. For the present discussion let it be assumed that the bearing area under each angle is two times the thickness of the angle material

$$(b = 2 \times 2 \times \tfrac{5}{8} = 2.5 \text{ in.})$$

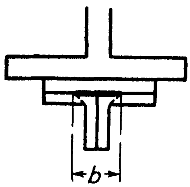


Fig. 3-4

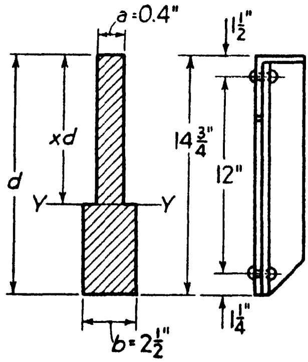


Fig. 3-5

Therefore the bending stress will be carried by the area shown shaded in Fig. 3-5. If it be further assumed that this area may be treated in the same manner as any other homogeneous area it will be clear that axis Y-Y is the neutral axis which, for simple bending, passes through the centroid of the cross section. That is

$$\frac{a(xd)^2}{2} = \frac{bd^2(1 - x)^2}{2}$$

$$x\sqrt{a/b} = 1 - x$$

$$x = \frac{1}{1 + \sqrt{a/b}}$$

where x is the decimal part of the depth to the neutral axis.

Furthermore

$$T = \frac{axds}{2}$$

where s is the unit tensile stress in the most stressed fibers due to bending and

$$M = T \cdot \frac{2}{3} d = \frac{axd^2s}{3}$$

For purposes of design this may be written

$$d = \sqrt{\frac{3\,M}{asx}} = \sqrt{\frac{3\,M(1 + \sqrt{a/b})}{as}}$$

On the rivets which carry tension there is also a shearing stress. The resultant maximum tensile stress due to such a combination (see p. 34) is

$$s_{\text{max.}} = \frac{s_{\text{tension}}}{2} + \sqrt{\left(\frac{s_{\text{tension}}}{2}\right)^2 + (s_{\text{shear}})^2} \qquad 1\text{–}14$$

Example 3–4. Determine the load P which may be resisted by the bracket of Fig. 3–3 if $e = 2\frac{1}{4}$ in. and $s_{\text{max.}}$ may not exceed 12,000 lb per sq in.
Solution.

$$a = \frac{2 \times 0.60}{3} = 0.4 \text{ in.}$$

$$b = 2 \times 2 \times \tfrac{5}{8} = 2.5 \text{ in.}$$

$$x = \frac{1}{1 + \sqrt{a/b}} = \frac{1}{1 + \sqrt{0.4/2.5}} = \frac{1}{1 + 0.4} = 0.715 \quad xd = 10.5 \text{ in.}$$

$$M = Pe = \frac{axd^2s}{3}$$

$$P \cdot 2\tfrac{1}{4} = \frac{0.4 \times 0.715(14.75)^2 \cdot s_{\text{tension}}}{3}$$

$$s_{\text{tension}} = 0.108\,P \text{ lb per sq in.}$$

$$s_{\text{shear}} = \frac{P}{10 \cdot 0.6} = 0.167\,P \text{ lb per sq in.}$$

$$s_{\text{max.}} = P\left(\frac{0.108}{2} + \sqrt{\left(\frac{0.108}{2}\right)^2 + (0.167)^2}\right)$$

$$= P(0.054 + 0.175) = 0.229\,P$$

$$\therefore 12{,}000 = 0.229\,P$$

$$P = 52{,}500 \text{ lb}$$

It is clear that the upper part of the legs of the vertical angles which are in contact with the shelf angle and filler plate must carry a bending stress. Commonly these legs are assumed to bend as shown in Fig. 3–6 with a point of contraflexure midway between the rivet and the face of the outstanding leg. The direct stress on the upper rivet is (approximately, since the stress is uniformly decreasing)

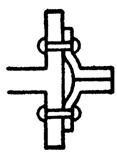


FIG. 3–6

$$(52{,}500 \times 0.108)\left(\frac{9}{10.5}\right)(0.4 \times 3)\tfrac{1}{2} = 2910 \text{ lb}$$

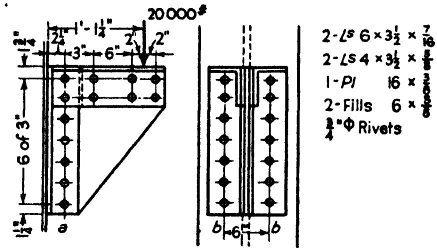
The bending moment is

$$2910 \times \frac{2.25 - 0.625}{2} = 2370 \text{ in-lb}$$

The corresponding unit stress in the angle leg is

$$\frac{2370}{\frac{1}{6} \times 3 \times (\frac{5}{8})^2} = 12{,}200 \text{ lb per sq in.}$$

Problem 3-7. Determine the magnitude of the maximum tension in Ex. 3-4, $s_{\max.} = 0.229\,P$, without using Eq. 1-14. *Suggestion.* See Ex. 1-9, p. 36.



PROB. 3-8

Problem 3-8. Determine: (*a*) the maximum shearing and bearing stresses in the rivets of line *a*; (*b*) the maximum stress in the rivets of lines *b*; (*c*) the bending stress in the legs of the vertical angles. Assume that the two $6 \times 3\frac{1}{2} \times \frac{7}{16}$ angles transfer the load to the vertical plate but may otherwise be neglected.

CHAPTER IV

THE PLATE GIRDER

4–1. A plate girder is a beam of I section built up usually of plates and angles with the several parts joined by rivets. When the connection is by welding, plates alone may be used in forming the girder. The manner of assembling the parts is shown for a few typical cases in Fig. 4–1, where also are shown cross sections of a rolled wide-flanged beam, and a standard I-beam. Plate girders are common objects, and the student is urged to familiarize himself with neighboring examples of their use.

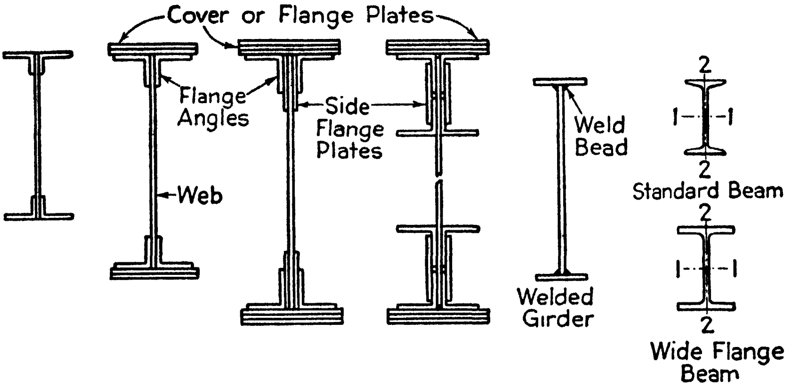


FIG. 4–1

4–2. Web. It was shown in Art. 1–3 that the unit shearing stress in the web of an I-beam is distributed in a manner that varies comparatively little in the depth of the web. In the design of a plate girder it is common to assume a uniform distribution of unit shearing stress. Specifications follow either of two methods in listing web stress: the allowable stress is based on (*a*) the gross area of the web [**301**][1] or (*b*) the net area of the web. In the latter case it is common to assume that the net area is equal to three-fourths of the gross area. This assumes that at the section of maximum shear there is a possibility that there may be a row of rivet holes spaced on the average 4 in. on center, the usual $\frac{7}{8}$-in.

[1] Bold-face figures in brackets ([ ]) refer to the 1935 A.R.E.A. Specifications for Steel Railway Bridges; figures in braces ({ }) to the 1920 A.R.E.A. Specifications, Fourth Edition, 1931. See Appendix D.

rivet hole being considered as 1 in. in diameter when account is taken of its effect in reducing any section. This is a generous allowance in ordinary cases: for some unusually heavy girders a smaller net area may exist.

Problem 4–1. What is the ratio of the actual net area to gross for the web of the plate girder shown in Plate II, taking a section through the inner end stiffener?

4–3. Approximate Flexure Formula. Bending stresses in plate girders may be figured by the use of Eq. 1–1, $s = Mc/I$ (p. 2), but this formula is not satisfactory for use in design as it requires a trial section to have been determined by some other method before it may be applied. Approximate formulas which do not have this disadvantage are therefore used, both for design and for the investigation of existing designs, and the most common of these will now be developed. The significance of the word "approximate" when applied to plate girder flexure formulas will be considered in Art. 4–5, p. 88.

Some writers refer to the approximate formula as the *truss-chord* formula, a name which suggests the method of approach. In a truss, the total stress in a chord at any section is taken as the moment at the section divided by the depth of the truss (the distance between the centroids of the chords), and the unit stress is assumed to be uniform over the chord section. If the same method be followed in the girder, the total stress in the angles and cover plates of one flange will be $F = M/h$, and the unit stress in the flange will be $s = \dfrac{F}{A} = \dfrac{M}{Ah}$, and the resisting moment

$$M = Ash \qquad 4\text{–}1$$

where M = the bending moment, equals the resisting moment at the section (in-lb),
s = unit stress in flange (lb per sq in.),
A = area of the angles and cover plates of the flange (in.^{2}),
h = distance between centroids of flanges (in.).

Sometimes in the design of buildings the formula is used as above but recognition is generally given to the fact that the web provides help in resisting bending. By Eq. 4–1 the moment resisted by the flange is

$$M_{\text{flange}} = Ash \qquad (a)$$

If the web were not weakened by rivet holes (as at points where stiffeners are connected) its resisting moment would be

$$M_{\text{web}} = \frac{s_1 t h_w^2}{6}$$

where s_1 = bending fiber stress in web due to the moment M_{web},
t = web thickness,
h_w = web height.

It is obvious, however, that there is a weakening effect from rivet holes, and the resistance of the web to bending is commonly taken as three-fourths of the above (Prob. 4–2) {**116**}, or

$$M_{\text{web}} = s_1 \frac{3}{4}\frac{th_w^2}{6} = \frac{1}{8} s_1 t h_w^2 \qquad (b)$$

The sum of (a) and (b) must be the total resisting moment of the girder and must also equal the bending moment at the section. That is,

$$M = Ash + \frac{1}{8} s_1 t h_w^2 \qquad (c)$$

In a well-designed girder, unless it is required by circumstances to be very shallow, the distances h and h_w will be very nearly equal. Plates as received from the rolling mill may vary slightly from the nominal width. The girder flange angles are consequently spaced $\frac{1}{2}$ in. farther apart, back to back, than the web depth to avoid the necessity of chipping the overrun. This increases h by $\frac{1}{2}$ in. and quite commonly makes it more nearly equal to h_w. It is therefore permissible in an approximate formula to assume that $h_w^2 = h \times h_w$, and $s_1 = s$. Making these substitutions,

$$M = Fh = Ash + \frac{1}{8} th_w sh = s\left(A + \frac{th_w}{8}\right)h \qquad 4\text{–}2$$

Of all approximate formulas for plate girder design, Eq. 4–2 is most generally known and used. It will be seen that Eq. 4–1 may be transformed into Eq. 4–2 by including in *each* flange an area equal to one-eighth of the area of the web, called the *web equivalent,* that is, web area equivalent to added flange area.

Common practice in the calculation of h is to locate the center of gravity of the angles and cover plates of the compression flange (i.e., the "web equivalent" is neglected in this calculation) and to assume the center of gravity of the tension flange to be similarly located. This practice will be followed in this chapter.

Problem 4–2. Compute and compare the moments of resistance of a 60 by $\frac{1}{2}$ in. web plate, with a maximum allowable stress of 15,000 lb per sq in., for the following conditions:

(a) For gross cross section.

(b) With a line of rivet holes for $\frac{7}{8}$-in.-diameter rivets spaced as follows: a rivet $2\frac{1}{2}$ in. from each edge and an additional group of 16 rivets, 3 in. on centers, symmetrically located about the center line of the plate.

(*c*) Same as for (*b*) above, except that the inside group consists of 12 rivets at 4-in. spacing.

Ans. (*a*) 4,500,000 lb-in. (1.00)
(*b*) 3,357,000 " (0.75)
(*c*) 3,550,000 " (0.79)

As stated in Art. 1–9, the allowable unit stress in the compression flange is less than that in the tension flange. In using Eq. 4–2 $\left(\text{in the form, } A + \frac{th_w}{8} = \frac{M}{sh}\right)$, the area obtained will be: (*a*) the required gross area of the compression flange when *s* is taken as the allowable compressive stress, and (*b*) the required net tension area when the corresponding tensile stress is used.

4–4. Net Section of the Tension Flange. The number of rivet holes to be deducted from the gross area of the tension flange of a girder is a matter that can be known with certainty only after the shop drawing

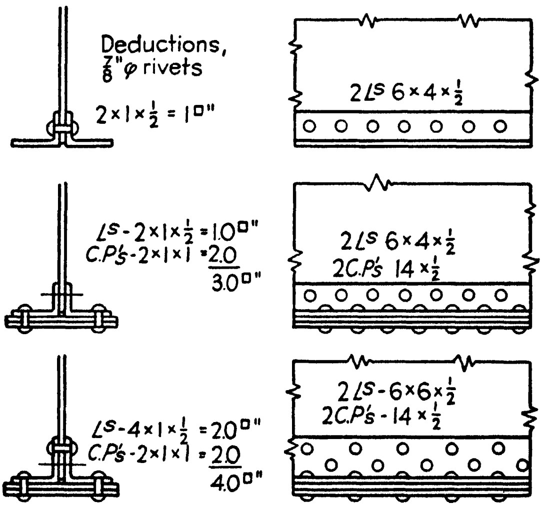


FIG. 4–2

has been prepared. The designer must proceed without this definite information and needs a ready rule for practical use. The following is proposed (see Fig. 4–2):

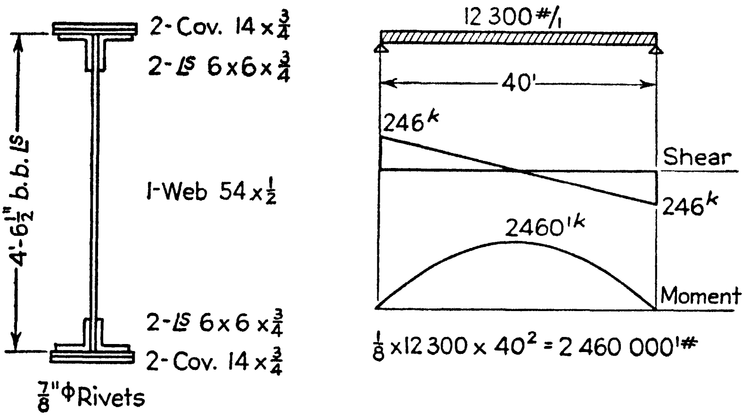
(*a*) For plate girders without cover plates, one hole shall be deducted from the area of each tension angle.

(*b*) For plate girders with cover plates, where the *vertical* legs of the flange angles are 4 in. or less, two holes shall be deducted from the area of each tension cover plate and one hole from each tension angle.

(*c*) For plate girders with cover plates, where the *vertical* legs of the flange angles are more than 4 in., two holes shall be deducted from the area of each tension cover plate and two holes from each tension angle.

The examples which follow show the application of the foregoing articles.

Example 4–1. A girder with the cross section shown carries a uniform load of 12,300 lb per ft of length, including its own weight. The span length is 40 ft. Find the maximum unit stresses in the web and in the tension and compression flanges, assuming uniform distribution of shear in web and employing the truss-chord method.



Ex. 4–1

Solution. The curves of shear and bending moment are as shown.

Shear. Unit shearing stress on gross web $= \dfrac{246{,}000}{54 \times \frac{1}{2}}$

$= 9120$ lb per sq in.

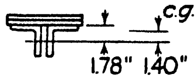
Flange Stress. $s = M \div \left(A + \dfrac{th_w}{8}\right)h$

	Areas	
	Compression	Tension
$\frac{1}{8} \times 54 \times \frac{1}{2}$	3.37 sq in.	3.37 sq in.
2 — ∠ $6 \times 6 \times \frac{3}{4}$	16.88	$-(4 \times 1 \times \frac{3}{4}) = 13.88$
2 — Pl $14 \times \frac{3}{4}$	21.00	$-(2 \times 1 \times \frac{3}{2}) = 18.00$
	41.25	35.25

h: (taking moments about centroid of angles)

$$h = 54.5 - 2\left(1.78 - \frac{21.00(1.78 + 0.75)}{21.00 + 16.88}\right)$$

$$= 54.5 - 2\ (1.78 - 1.40) = 53.74 \text{ in.}$$



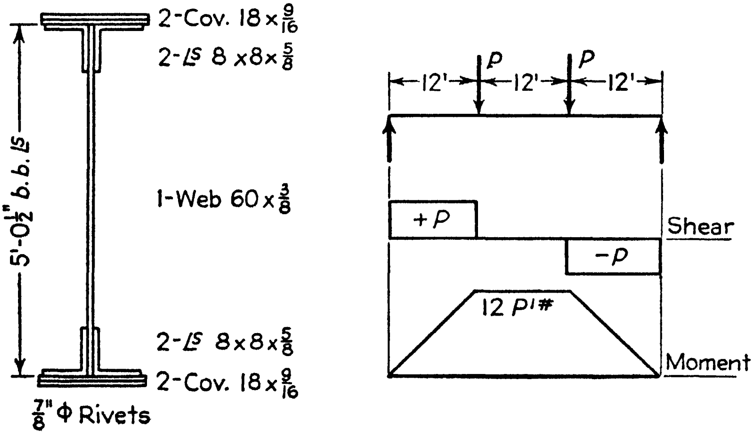
Total flange stress: $F = \dfrac{M}{h} = \dfrac{2{,}460{,}000 \times 12}{53.74} = 549{,}000$ lb

Unit flange stress:

$$\text{Compression,}\ s_c = \frac{549{,}000}{41.25} = 13{,}300 \text{ lb per sq in.}$$

$$\text{Tension,}\ s_t = \frac{549{,}000}{35.25} = 15{,}600 \text{ lb per sq in.}$$

Example 4-2. A girder built up as shown is used to support two equal concentrated loads at the third points of the 36-ft span. Neglecting the weight of the girder, how large may the loads be without exceeding the given fiber stresses? Allowable unit stresses: compression 14,000; tension 16,000; shear on gross web 10,000 lb per sq in.



Ex. 4-2

Solution. The curves of shear and moment in terms of the unknown P are as shown.

Shear: The maximum allowable end shear, which equals the unknown load P, is

$$V = P = 60 \times \tfrac{3}{8} \times 10{,}000 = 225{,}000 \text{ lb}$$

$$\text{Moment:}\quad M = 12\,P = s\left(A + \frac{th_w}{8}\right)h$$

	Areas	
	Compression	Tension
$\frac{1}{8} \times 60 \times \frac{3}{8}$	2.81 sq in.	2.81 sq in.
2—∠ $8 \times 8 \times \frac{5}{8}$	19.22	$-(4 \times 1 \times \frac{5}{8}) = 16.72$
2—Pl $18 \times \frac{9}{16}$	20.25	$-(2 \times 1 \times \frac{9}{8}) = 18.00$
	42.28	37.53

h:
$$h = 60.5 - 2\left(2.23 - \frac{20.25(2.23 + 0.56)}{20.25 + 19.22}\right)$$
$$= 58.90 \text{ in.}$$

Moment of resistance:

Compression: $M = 14{,}000 \times 42.28 \times 58.90$
$= 34{,}850{,}000$ in-lb $=$ limit

Tension: $M = 16{,}000 \times 37.53 \times 58.90$
$= 35{,}350{,}000$ in-lb $>$ above

Allowable load: $12\,P = \dfrac{34{,}850{,}000}{12}$

$P = 242{,}000 > 225{,}000$ above

Shear value governs; maximum $P = 225{,}000$ lb

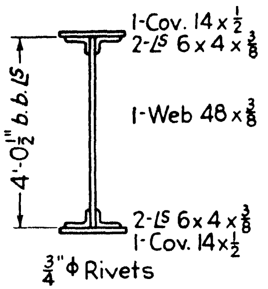
Problem 4–3. Derive Eq. 4–2, $M = s(A + \frac{1}{8}th_w)h$, directly from Eq. 1–1, $M = sI/c$, writing $I = \frac{1}{12}th_w^3 +$ twice the I of one flange about its own center of gravity + twice the correction factor for transfer to neutral axis of section. Use notation already employed, plus h_o for total depth of girder; neglect rivet holes. In order to arrive at desired equation it will be necessary to neglect the moment of inertia of the flanges about their own centroid, to take moment of inertia of the web as $\frac{1}{16}th_w^3$, and to assume $h_0 = h_w = h$. Are these approximations on the safe or unsafe side?

Problem 4–4. Same as Prob. 4–3 except that the net section is to be used, taking the moment of inertia of the net web section as $\frac{1}{16}th_w^3$, making allowance for holes in both tension and compression portions of the section. Is there any difference in the derivation beyond having A stand for the net flange area instead of the gross?

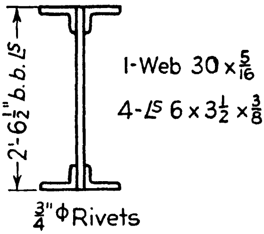
Problem 4–5. Inspection of the elevation of the girder in Plate II shows that toward the center of the girder a vertical cross section can be taken cutting: (*a*) the gross section and no rivet holes; (*b*) a net section with holes in flange only as in the section drawn for Fig. 4–2; and (*c*) a net section including, in addition to holes through flange, intermediate holes through the web at a stiffener or at the web splice. Assuming that design is by the moment of inertia formula and that holes in the compression parts are completely filled by the rivets, which of these sections would you use in locating the neutral plane? Compare next article.

Problem 4–6. This girder supports a load which causes a moment of 970,000 ft-lb. Determine the maximum fiber stresses in the tension and compression flanges.

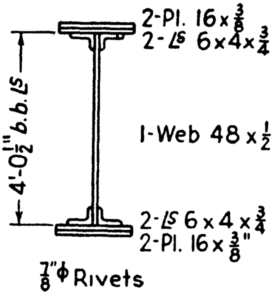
Ans. s_t = 16,300 lb per sq in.
s_c = 14,800 lb per sq in.



Prob. 4–6



Prob. 4–7



Prob. 4–8

Problem 4–7. This girder, without intermediate top flange support, is used on a span of 20 ft (L) to carry a uniform load. How heavy may this load be without exceeding:

$s_{\text{shear on gross web}}$	10,000 lb per sq in.
s_{tension}	18,000 lb per sq in
$s_{\text{compression}}$	$21{,}000 - 300\dfrac{L}{b}$

b = flange width = 12.31 in.

Ans. w = 5850 lb per ft

Problem 4–8. A girder with the cross section shown is used on a span of 40 ft to carry a uniform load of 8000 lb per ft. Determine the maximum unit fiber stresses in the tension and compression flanges.

Ans. s_t = 15,550 lb per sq in.
s_c = 13,940 lb per sq in.

4–5. Approximate Compared with Exact Formula. It would seem that the only step necessary to take to determine the degree of approximation involved in the use of the so-called approximate formula would be to compute stresses by that formula and compare with the stresses obtained by means of Eq. 1–1, $s = Mc/I$. As a matter of fact, this comparison has been made in a number of textbooks, but, unfortunately, an examination of them does not furnish the answer desired because of the variety of ways in which the supposed exact formula has been used. For example, it has been applied as follows:

(*a*) With the moment of inertia computed for the gross area of both flanges.

(*b*) With the moment of inertia computed for the net area of both flanges [**426**].

(c) With the moment of inertia computed for the gross area of the compression flange and for the net area of the tension flange, with the position of the center of gravity and the neutral axis located accordingly.[1]

(d) With the moment of inertia figured as in (a) above, but with the tension stress taken as the compression stress multiplied by the ratio of the gross flange area to the net flange area.

It is clear that not all the above methods can be correct and that tests are needed to give an answer to the question under discussion. Fortunately, experience teaches that the approximate formula may be used with confidence because the many girders designed by its use are giving satisfactory service. Probably the percentage of error in stresses obtained by its use is less than the percentage of variation in the determination of live and impact loading. It is believed that the approximate formula will give satisfactory results wherever the girder is proportioned as directed in the next article.

By way of illustration, the following example is worked by method (d) above. Emphasis is directed to the fact that the per cent of variation will change with girders of different proportions.[2]

Example 4–3. Determine, by method (d) above, the flange stress in the girder of Ex. 4–1.

Solution.

Moment of inertia[3]	Web	6 561 in.^{4}
	Angles	22 013
	Covers	32 936
		61 510

$$s_{\text{compression}} = \frac{Mc}{I} = \frac{2{,}460{,}000 \times 12(27\frac{1}{4} + 1\frac{1}{2})}{61{,}510} = 13{,}800 \text{ lb per sq in.}$$

$$s_{\text{tension}} = s_{\text{compression}} \times \frac{\text{Gross flange area}}{\text{Net flange area}}$$

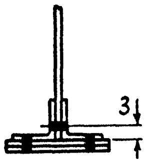
$$= 13{,}800 \frac{41.25}{35.25} = 16{,}100 \text{ lb per sq in.}$$

[1] This is done in spite of the obvious fact that the net tension area occurs at relatively infrequent intervals along the girder.

[2] Mention was made in an earlier article of the fact that the approximate formula is sometimes used with the web equivalent neglected. Perhaps the extreme in this direction is the practice of certain English engineers who consider the flange to consist of cover plates and horizontal legs of flange angles only.

[3] KETCHUM'S "Structural Engineer's Handbook," the A.I.S.C. handbook, "Steel Construction," the Carnegie "Pocket Companion," and the "Bethlehem Manual of Steel Construction," contain tables which greatly shorten the labor of computing the moments of inertia of girders.

Problem 4–9. Taking P equal to 242,000 lb for the girder of Ex. 4–2, determine the fiber stress in the tension and compression flanges by the approximate method and by methods (*a*), (*b*), and (*d*) of this article.



PROB. 4–9

		s_c	s_t
Ans.	(*a*)	14,400	14,400 lb per sq in.
	(*b*)	16,380	16,380 " " " "
	(*d*)	14,400	16,220 " " " "
	(Approx.)	14,000	15,770 " " " "

Problem 4–10. It is desired to ascertain the degree of approximation of Eq. 4–2 for varying depths of girder. Let the web of the girder of Ex. 4–1, p. 85, vary from 18 in. to 90 in. in depth, with 12-in. intervals, flange angles and cover plates remaining unchanged. The cross section chosen will cut four rivet holes in the angles of each flange and a series of holes in the web spaced 3 in. apart. The clear depth of web between legs of flange angles will be 6½ in., 18½ in., 30½ in., etc., and the intermediate holes in the web are so disposed that the extreme holes are always 1¾ in. from the edge of the angle; the horizontal holes through the flange and angles are 2½ in. from the edges of the web. Compute the moments of resistance for each web depth: by Eq. 1–1, using gross moment of inertia; by Eq. 1–1, using net moment of inertia, allowing for holes in both tension and compression sides; by Eq. 4–2, using allowable unit stress in compression and gross flange area; by Eq. 4–2, using allowable tensile stress and area of tension flange. Note that the same web equivalent is generally used in both the last two cases. Allowable unit stresses: compression 14,000 lb per sq in.; tension 16,000 lb per sq in.

Plot results with web depth as abscissas and resisting moment and percentage results as ordinates.

4–6. Flange Proportions. Many specifications contain a provision which states that the gross area of the compression flange shall not be less than the gross area of the tension flange [**427**]. Common practice is to make the two flanges alike, determining the gross area from that one which requires the more.

As regards the division of area between plates and angles, a satisfactory rule is as follows:

After deducting the web equivalent, the remaining area shall lie between the following limits:

Angles ½ to ⅖ of total flange area
Plates ½ to ⅗ of total flange area

If this rule is followed, the distance between centers of gravity of flanges will always be less than the back to back of angle distance and there will be no occasion to apply the rule sometimes found in specifications which states:

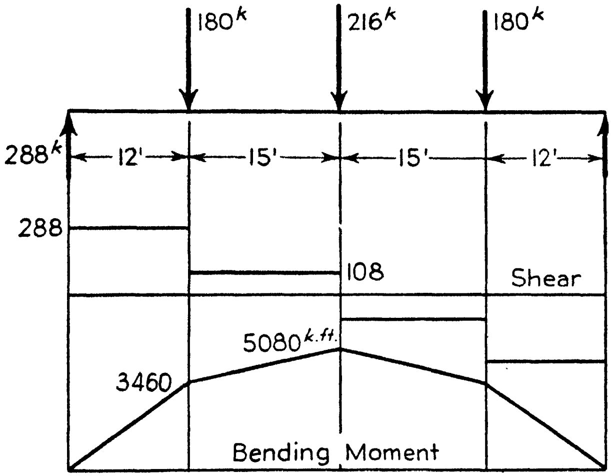
" Where the distance between centers of gravity exceeds the back to back of angle distance, the latter shall be used in strength computations."

Queries. What objection is there to a flange made of 2 × 2 × ⅜-in. angles and an 18 × 1-in. cover plate? How does stress develop in the cover plates?

Return to these rules after studying Art. 4–9 on flange rivets.

Cover plates are commonly selected which project slightly beyond the outstanding legs of the flange angles. For example, 14-in. (sometimes 16-in.) plates are used with 6-in. outstanding angle legs; 18-in. (or 20-in.) plates with 8-in. legs.

Example 4-4. Design a plate girder, $54\frac{1}{2}$ in. back to back of flange angles, to carry the loads shown. Allowable stresses in lb per sq in.: compression flange 15,000; tension 18,000; shear on gross web section 12,000. Use $\frac{7}{8}$-in.-diameter rivets. Neglect the dead weight of the girder.



Ex. 4-4

Solution. The first step was to draw the shear and moment curves.

Web: $t = \dfrac{288}{54 \times 12} = 0.445$ in. Use $54 \times \frac{1}{2}$ in. web plate

Flanges: Assume $h = 54.5 - 1 = 53.5$ in.

Total flange stress $C = T = \dfrac{5080 \times 12}{53.5} = 114.0$ kips

		sq in. gross		sq in. net
Area required:	$A = \dfrac{114}{15} =$	76.0	$\dfrac{114}{18} =$	63.4
Web equivalent:	$\frac{1}{8} \times 54 \times \frac{1}{2}$	3.4		3.4
Required for angles and cover plates:		72.6		60.0
($\frac{2}{5}$ × area) =			(29.0) (24.0)	
2 − 8 × 8 × 1 angles		30.0	less 4 × 1 × 1 =	26.0
Required for cover plates		42.6		34.0
2 − 20 × $\frac{3}{4}$	1 − 20 × $\frac{5}{8}$	42.5	less 2 × 1 × 2.13 =	38.2
	Excess	− 0.1	+	4.2

Check of h:

$$h = 54.5 - 2\left(2.37 - \frac{42.5(2.37 + 1.06)}{42.5 + 30.0}\right) = 53.8 \text{ in.}$$

$$\text{Actual required gross area} = 76.0 \times \frac{53.5}{53.8} = 75.6 \text{ sq in.}$$

The section chosen is satisfactory.

4–7. Cover Plate Length. An advantage of the plate girder is that the section may be varied by using cover plates of different lengths. It is customary to keep the tension and compression flanges the same at all sections, which involves using cover plates in pairs, that is, corresponding tension and compression plates have the same length and thickness. An exception to this is in bridge work where one top flange plate always extends the whole length to provide cover against water penetrating between the parts {**119**}. Some specifications require a full-length cover plate on each flange [**427**].

There are two ways of finding the points where cover plates may be stopped. (1) The moment of resistance of the whole section with all cover plates being known, compute the moments of resistance of the sections formed by dropping successive pairs of cover plates, which involves finding the effective depth (h) of each section. Find the points where the bending moments are of the same values as these moments of resistance, by writing a general expression for moment or by scaling from a carefully constructed bending-moment diagram.

(2) If the variation in h is neglected it is evident from Eq. 4–2 that the flange area required, including web equivalent, varies directly with bending moment, and that *the bending-moment curve is, to some scale, the curve of required flange areas.* This, the more common method, consists in scaling the necessary cover plate lengths from a curve of required flange areas.

The bending-moment curve and the corresponding curve of required flange area for a girder uniformly loaded are parabolas, and for this common case it is a simple matter, following the second method described above, to derive a formula for cover plate length which will save the labor of graphical construction. Such a parabolic flange-area-required curve is shown in Fig. 4–3, whose center ordinate, A, represents to scale the flange area required at the section of maximum moment. On the line of this center ordinate, the areas of the several elements composing the flange (the web equivalent, the angles, and each cover plate individually, a_1, a_2, a_3) are laid off to scale. In this figure the area furnished by these elements just equals that required. Usually there is excess area, but *it is common to include this excess and assume that the*

area required equals the area furnished. This necessitates drawing a slightly larger area-required diagram than the actual one and so results in errors on the safe side. The next step is to draw horizontal lines through the several points on the center ordinate plotted to represent flange element areas. It is evident from the diagram that cover plate

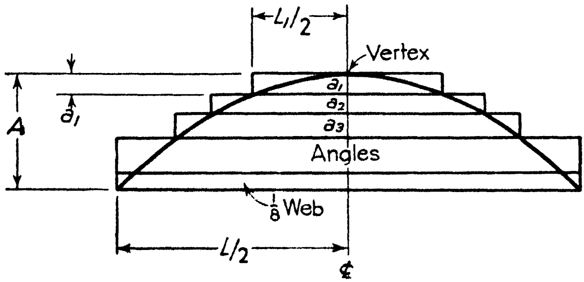


FIG. 4–3

a_1 is not required to extend farther toward the ends of the girder than the intersections with the parabola of the horizontal through the point on the center line whose distance from the base line represents the area of the flange, less one cover plate. From the properties of the parabola (offsets from the tangent through the vertex vary as the squares of their distances from the axis) we may write

$$\frac{a_1}{A} = \frac{(L_1/2)^2}{(L/2)^2}$$

whence
$$L_1 = L\sqrt{\frac{a_1}{A}}$$

or, more generally, the length, L_c, of any cover plate for uniform loading on a simple end-supported beam is given by

$$L_c = L\sqrt{\frac{A_c}{A}} \qquad 4\text{–}3$$

where A_c equals the area of the cover plate whose length is in question plus the areas of all the plates outside it.[1]

Fig. 4–4 shows a girder with two equal concentrated loads, giving a moment curve of the form shown. The cover plate lengths may be measured from a scale sketch, or a slide-rule solution may be made by

[1] A common error of the "formula-hound," who disdains both to observe and to think, is to apply Eq. 4–3 in other than uniform load conditions.

means of similar triangles. That is, for the outer plate

$$L_1 = 2\left(\frac{10.50}{38.22}\right)10 + 12 = 17.50 \text{ ft}$$

and for the inner plate

$$L_2 = 2\left(\frac{21.00}{38.22}\right)10 + 12 = 23.00 \text{ ft}$$

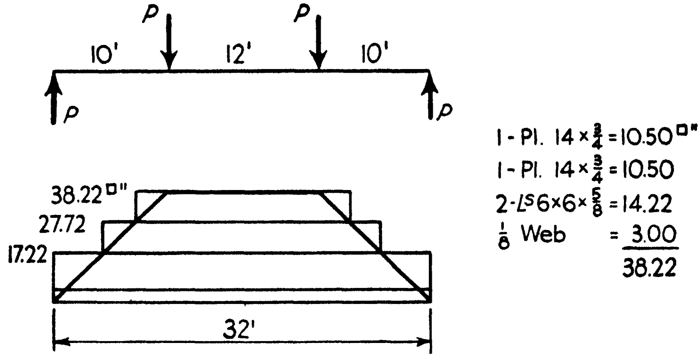


FIG. 4–4

The lengths as here computed are *theoretical* lengths, and in practice the plate is extended a short distance beyond the theoretical point at each end, 1 ft being the usual allowance (compare {**119**} and [**427**]).

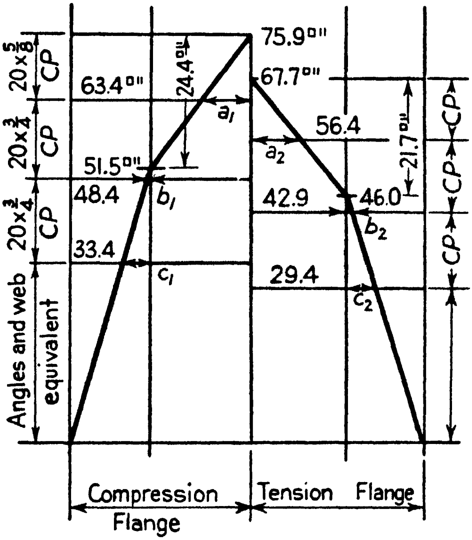
Example 4–5. Determine the cover plate lengths for the girder designed in Ex. 4–4.

Solution. The following table of areas facilitates the construction of the area-furnished diagram.

	Compression	*Flange Areas in sq in.*	*Tension*
Angles	30.0		26.0
Web equivalent	3.4		3.4
	33.4		29.4
1 plate 20 × $\frac{3}{4}$	15.0	$-2 \times 1 \times \frac{3}{4} =$	13.5
	48.4		42.9
1 plate 20 × $\frac{3}{4}$	15.0	$-2 \times 1 \times \frac{3}{4} =$	13.5
	63.4		56.4
1 plate 20 × $\frac{5}{8}$	12.5	$-2 \times 1 \times \frac{5}{8} =$	11.3
Total	75.9		67.7

The assumption is made that the maximum area of flange furnished equals that required, although there is a slight excess. Consulting the bending-moment curve, Ex. 4–4, it is seen that at each 180-kip load there is required

3460/5080 = 0.68 as much area as at the point of maximum moment. This enables the curve of modified area requirement to be drawn, the load ordinates being 0.68 × 75.9 = 51.5 sq in. in compression and similarly 46.0 in tension. The computation of certain elements of plate lengths are shown in the figure; the fact that the dimensioning is incomplete should not cause



$$\left.\begin{aligned} a_1 &= 15 \times \frac{12.5}{24.4} = 7.70 \\ a_2 &= 15 \times \frac{11.3}{21.7} = 7.82 \end{aligned}\right\}$$

$$\left.\begin{aligned} b_1 &= 12 \times \frac{3.1}{51.5} = 0.73 \\ b_2 &= 12 \times \frac{3.1}{46.0} = 0.81 \end{aligned}\right\}$$

$$\left.\begin{aligned} c_1 &= 12 \times \frac{18.1}{51.5} = 4.22 \\ c_2 &= 12 \times \frac{16.6}{46.0} = 4.34 \end{aligned}\right\}$$

Ex. 4–5

trouble as the subtractions are easily made mentally. The length of the outside plate is $2 \times a_2 = 15.64$ ft, the longer plate being required by the tension flange; the two flanges will be made alike. The length of the next plate is $30 + 2 \times 0.81 = 31.62$ ft, that of the third 38.68 ft.

The make-up of the girder finally is

Web plate	54 × ½ in., 54½ in. back to back of angles
Flange angles	4 angles 8 × 8 × 1 in.
Cover plates	2 plates 20 × ¾ in. × 38.68 ft (theoretical length)
	2 plates 20 × ¾ in. × 31.62 ft (theoretical length)
	2 plates 20 × ⅝ in. × 15.64 ft (theoretical length)

4–8. Balanced Design. Another way to regard the problem of cover plate length is as follows. At the point of maximum moment the girder flange is stressed to the allowable unit stress. In going away from this point of maximum moment, the flange unit stress will decrease until a point is reached where the flange *with the outer plate removed* will resist, at the allowable unit stress, the total stress from the moment. This is the theoretical point of cut-off for the cover plate.

Usually it is impossible to furnish in the flanges of a girder the exact amount of material called for in the design computations, and most girder flanges have a slight excess of area. This means that at the point of maximum moment the unit stress in the flange is less than the allowable.

In determining cover plate lengths this reduced stress may be used as though it were the allowable stress in order that at the point of maximum moment and at points where cover plates end the unit stress will be the same. A design so made is called a *balanced design;* it will permit a certain amount of overload without one critical point being stressed beyond another. The method previously illustrated, of using the area furnished as though it were the required area, accomplishes the same purpose.

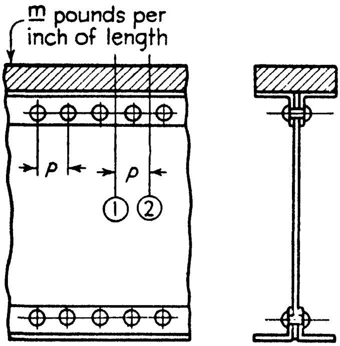


FIG. 4–5

4–9. Rivet Pitch. If a load of m pounds per inch of length is applied to the top flange of a girder, Fig. 4–5, and if the rivets joining the vertical legs of the flange angles to the web are spaced p inches apart, it is evident that each rivet will carry a vertical load of pm pounds. In addition, each rivet will carry a horizontal load equal to the change in flange stress in a distance p, that is between sections 1 and 2. At section 1 the flange stress is M_1/h, and at section 2, M_2/h. The change in stress[1] between the sections is

$$\frac{M_1 - M_2}{h} = \frac{Vp}{h}$$

If the effect of the web equivalent be neglected, all this change is horizontal load on the rivet and the rivet stress is

$$r = \sqrt{\left(\frac{Vp}{h}\right)^2 + (mp)^2} = p\sqrt{\left(\frac{V}{h}\right)^2 + m^2}$$

or

$$p = \frac{r}{\sqrt{(V/h)^2 + m^2}} \qquad \text{4–4A}$$

where r is the load on a rivet.

[1] The change in moment between any two sections is equal to the area of the shear curve between those sections.

In many girders, the principal load is applied through stiffeners or other web connections. In such cases m is zero, and

$$p = \frac{r}{V/h} \tag{4-4B}$$

where V/h is the rate of increase of flange stress.

This is the common formula for rivet pitch. Usually rivets are spaced on the detail drawing by a draftsman who does not have at hand the design computations. In order to save time and avoid the determination of h, there is substituted instead the distance between the gage lines in the flanges.

The foregoing method of stress or pitch computation is in error in that it neglects the web equivalent and assumes that the entire increase in flange stress goes into flange angles and cover plates. Actually the increase per inch in the stress in angles and plates is VQ/I, where Q is the statical moment of angles and covers (Art. 1–3). Therefore, an exact value of p is

$$p = \frac{r}{\sqrt{(VQ/I)^2 + m^2}} \tag{4-4C}$$

$$= \frac{r}{VQ/I}, \text{ if } m \text{ is zero} \tag{4-4D}$$

Comparable results are found and a simpler formula is obtained by assuming that the increment of flange stress is uniformly distributed over the entire flange area, including the web equivalent. By this assumption, the increase per inch in the stress in angles and plates is

$$\frac{V}{h} \times \frac{\text{Area angles} + \text{Area plates}}{\text{Area angles} + \text{Area plates} + \frac{1}{8}\text{ Area web}}$$

and the corresponding value for required pitch is

$$p = \frac{r}{\sqrt{\left(\dfrac{V}{h} \times \dfrac{A_{\angle s} + A_{\text{Pls}}}{A_{\angle s} + A_{\text{Pls}} + \frac{1}{8} A_w}\right)^2 + m^2}} \tag{4-4E}$$

It should be evident from the foregoing discussion that, for the rivets connecting cover plates and horizontal legs of flange angles (that is, rivets in lines a, Fig. 4–6), the expression for pitch is (since the rivets are spaced in pairs)

$$p = \frac{2\,r}{VQ/I} \tag{4-5A}$$

where Q is the statical moment of the cover plates of one flange about the center line of the web, or

$$p = \frac{2r}{\dfrac{V}{h} \times \dfrac{A_{\text{Pls}}}{A_{\measuredangle s} + A_{\text{Pls}} + \frac{1}{8} A_w}} \qquad \text{4-5}B$$

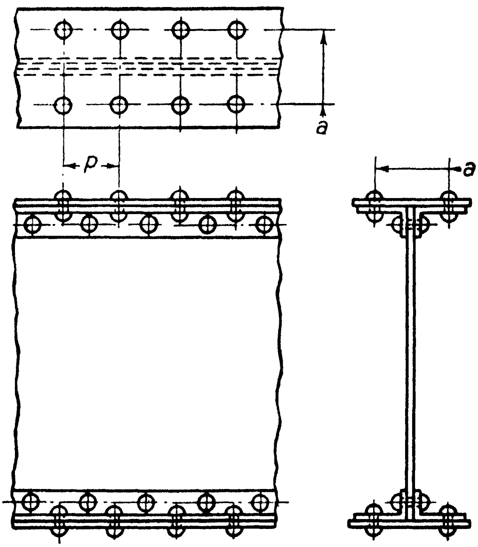


FIG. 4–6

Generally Eq. 4–5A and 4–5B will give results greater than 6 in., the maximum allowable value commonly set by specifications because of the belief that a larger spacing might permit local buckling of the plates {59}.

Problem 4–11. Draw the free body made up of the portion of a girder compression flange between two sections taken on each side of a single flange rivet through the web, with the rivet removed, in a region of positive shear, and show the forces acting. Draw the free body consisting of the single rivet above mentioned with the forces acting on it.

Draw a horizontal end-supported beam made up of a heavy timber with a plank of the same length and width as the timber lying on top of it and another plank placed underneath the timber. Sketch the shape of this deflected combination under load, with no connection between the parts. Consider the result of spiking the planks to the timber. Compare this case with the plate girder.

Example 4–6. Determine by means of Eq. 4–4A the allowable pitch of the rivets joining the flange angles and the web at the end of the girder of Ex. 4–1, p. 85. Assume that the load is applied to the top flange. Allow-

able rivet stresses are $s_{shear} = 12{,}000$ lb per sq in., $s_{bearing} = 24{,}000$ lb per sq in.

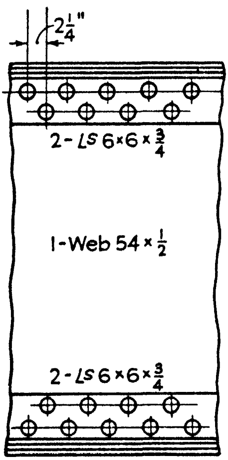
Solution. The rivets may fail by (*a*) shearing each rivet on *two* planes (i.e., *double shear*), or (*b*) crushing the metal where the rivet is in contact with the $\frac{1}{2}$-in. web plate (i.e., *bearing*). (Why not where the rivet is in contact with the flange angles?) Allowable rivet values are:

Double shear	$2 \times 0.601 \times 12{,}000 = 14{,}420$ lb
Bearing	$\frac{7}{8} \times \frac{1}{2} \times 24{,}000 = 10{,}500$ lb

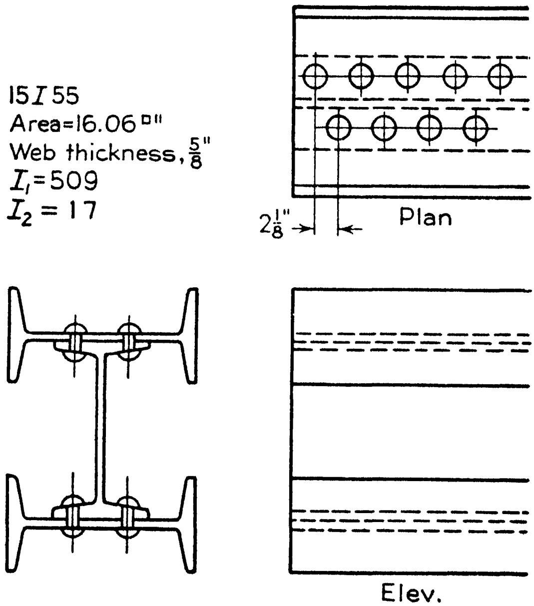
Therefore, bearing governs.

The end shear in the girder is $20 \times 12{,}300 = 246{,}000$ lb. Therefore

$$p = \frac{10{,}500}{\sqrt{\left(\dfrac{246{,}000}{53.74}\right)^2 + \left(\dfrac{12{,}300}{12}\right)^2}} = \frac{10{,}500}{\sqrt{4580^2 + 1025^2}} = 2.24 \text{ in.}$$



Ex. 4-6



Ex. 4-7

Rivets are commonly spaced in multiples of quarter inches. The computation shows that a rivet is needed each $2\frac{1}{4}$ in. along the girder. Since rivets may not be spaced closer than three diameters center to center {**59**}, it will be necessary to use two gage lines in the vertical legs of the flange angles. The rivets will be spaced as shown in the sketch.

Example 4-7. A girder is built up by riveting together three 15 I 55. The maximum end shear is 50,000 lb. A $\frac{3}{4}$-in. rivet is good for 5300 lb. What is the required pitch of the rivets joining the beams?

Solution. The shear per linear inch on the plane between beams is VQ/I.

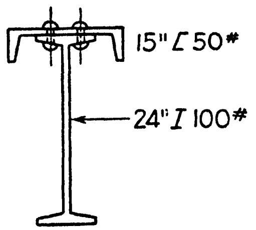
$$Q = 16.06 \times 7.81 = 125.3 \text{ in.}^3$$
$$I = 509 + 2(17 + 16.06 \times 7.81^2)$$
$$= 2503 \text{ in.}^4$$
$$\text{Shear per linear inch} = \frac{50{,}000 \times 125.3}{2503} = 2500 \text{ lb.}$$
$$\text{Rivet pitch} = \frac{5300}{2500} = 2.12 \text{ in.}$$

Problem 4–12. On the basis of column loads of 225,000 lb each, determine the required pitch of the rivets joining the flange angles and web in the end 12-ft sections of the girder of Ex. 4–2, p. 86. Allowable rivet values are s_{shear}, 11,000 lb per sq in., s_{bearing}, 22,000 lb per sq in.

Ans. $p = 1.89$ in.

Problem 4–13. Determine by means of Eq. 4–5*B* the required pitch of the rivets joining the flange angles and the cover plates at a point 10 ft. from the left end of the girder of Ex. 4–2, p. 86 ($P = 225{,}000$ lb). Use the rivet values of Prob. 4–12.

Ans. $p = 7.23$ in. (Use 6 in.)



PROB. 4–14

Problem 4–14. A girder built up as here shown is used to support a single moving concentrated load of 70,000 lb. Neglecting dead load and impact, determine the required pitch of the rivets at the end of the girder. One rivet may be assumed as good for a safe load of 8000 lb; rivets are staggered.

Ans. $p = 3.71$ in.

4–10. Intermediate Web Stiffener Angles. A thin plate, like a girder web, when stressed in the plane of the plate by shear, or by bending, or by a combination of shear and bending, buckles sidewise at comparatively small stress, and its resistance to load depends, not on the strength of the plate material, but upon a variety of circumstances, such as the stiffness (measured by E) of the material, the proportions of the plate, and the manner of edge support. The complicated problem of plate buckling has long been studied both theoretically and experimentally, but only recently have engineers attempted to proportion plate girder webs by any but empirical rules or the most elementary of approximate rationalizing. The usual rules and formulas give webs which are adequate and conservative in proportions. The usual basis for design may be understood by consideration of the nature of the buckling action to be expected.

In the regions of the girder where shear is small and bending moment large, the web approximates the condition of pure bending which, in a horizontal girder, would result in a horizontal compression in the

upper part of the web plate and a tendency to form vertical wrinkles. This could not occur without yielding of the flange and is never considered in design except in some extremely large girders where horizontal stiffener angles are sometimes riveted to the web.

Toward the ends of a girder where shear is large and moment small the action of the shear alone induces a diagonal tension and compression of equal intensity with the shear. In a region of positive shear in a horizontal girder, this compression acts along approximately a 45° slope upward to the right from a point on the lower flange. This tends to cause wrinkles at right angles.

To prevent this buckling, stiffener angles, arranged in symmetrical pairs, as shown at section A, Fig. 4–7, are used. Common practice is to place these stiffeners on fillers (shown shaded in the section). Sometimes, however, the stiffener angles are heated and crimped (bent), see Fig. 4–8, in order to avoid the use of fillers.

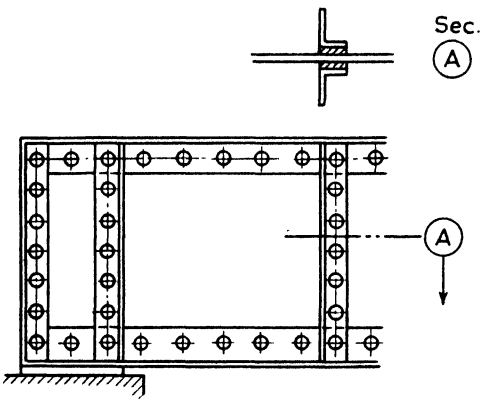


FIG. 4–7

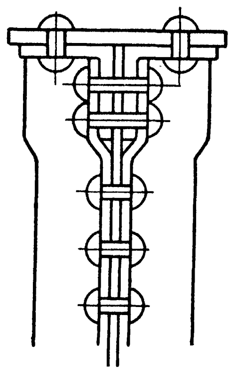


FIG. 4–8

Many formulas have been proposed for finding the required spacing of the intermediate stiffeners. In some of them, a strip of the web is considered to act as a column and a column formula is applied. Normal to the column, and in the plane of the web, are other strips in tension. This fact makes uncertain the constants for use in the column formula, and as a consequence stiffener spacing formulas give a wide range of results, depending on the judgment of their makers. Some formulas have been proposed which require a spacing much closer than experience shows to be necessary. In fact, the most liberal of present-day formulas requires a closer spacing than was used in proportioning many old girders which are still rendering satisfactory service.

Common practice is represented by the (1931) A.R.E.A. rules in which the formula for stiffener spacing {**125c**} is $d = \frac{t}{40}(12{,}000 - S)$ where t is the web thickness and S is the shear intensity. This formula may be rewritten *$S = 12{,}000 - 40\ d/t$, which, it will be noted, is in the form of* the column formula in the same specification, $P/A = 15{,}000 - 50\ L/r$ {**38**}; and this gives the clue to the origin of the expression. In Fig. 4–9 two stiffeners are shown a distance, d, apart, in this case equal to the

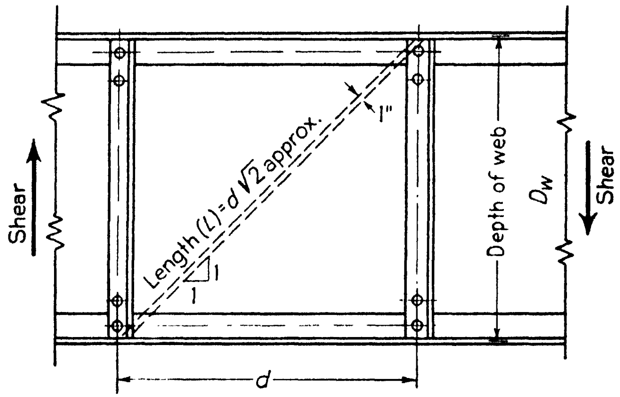


Fig. 4–9

web depth, D_w. It may be assumed that shear here is very large with very small moment and that an imaginary column, width unity, along the diagonal as shown, is stressed in axial compression throughout, at the same time that a diagonal tension acts at right angles to the column axis along its length. Assuming that the unit load on the column section, 1 in. by t, is equal to the shear intensity, which is strictly true at the neutral axis, its limiting value is here taken to be given by a straight-line column formula dependent on ratio of stiffener spacing to web thickness, where the term $40\ d/t$ proves to be equal to about $8.2\ L/r$, where $L = d\sqrt{2}$ and

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1 \times t^3}{12(1 \times t)}} = \frac{t}{3.46}$$

Since the shear working stress determines the web thickness, except as rules for minimum thickness of metal prevail, the effect of this formula is to limit the column length by bringing the stiffeners closer together as shear intensity, and therefore column stress, increases.

The 1935 A.R.E.A. specifications give the following formula for the

spacing of intermediate web stiffeners [**433**]:

$$d = \frac{255{,}000\, t}{S} \sqrt[3]{\frac{St}{a}}$$

with the notation as before except that d = clear distance between stiffeners and a = clear depth of web between flanges or side plates, in inches. This formula is derived from one expressing the elastic stability of flat plates, derived by Professor George H. Bryan.[1] The transition from the Bryan formula is given by Mr. Otis E. Hovey in "Elastic Stability of Plates Subjected to Compression and Shear," *Bulletin*, A.R.E.A. February, 1935. This article contains a brief bibliography to which the student is referred. A chart for the solution of this formula is printed here as Fig. 4–10 by courtesy of the Bethlehem Steel Co.

Problem 4–15. Prove that, when the intensity of the vertical and horizontal shear at any point in a beam equals any value, s, the intensity of the tension and compression on 45° planes at the same point has the same value. Compare Prob. 1–2*b*, p. 4.

Problem 4–16. Demonstrate that the formula $d = \frac{t}{40}(12{,}000 - S)$ is the same as $S = 12{,}000 - 8.2\, L/r$, referring to the imaginary column in Fig. 4–9.

Plot the above special column formula and also the regular one given by the A.R.E.A. specification above referred to, $P/A = 15{,}000 - 50\, L/r$, between the values zero and 200 for L/r.

Why is the special formula the more radical?

Problem 4–17. In the region of a girder exposed to both shear and bending there is a tendency for wrinkles to form at angles between 45° and the vertical. Explain this direction.

The discussion of the requirements of {**125c**} above serves at once to explain the other provisions of that article; they ensure that the longest web column shall not exceed $\sqrt{2}$ times the web depth, or times 6 ft. By taking the distance between gage lines of stiffener angles as the depth of the web it is ensured that clear distance between stiffeners approaches the clear distance between flange angles for the web. In this matter it is plain that precision is of academic interest only.

On the basis of tests Professors Moore and Wilson have suggested that probably stiffeners are not needed where the unsupported distance between flange angles is not more than 60 times the web thickness.[2] The A.R.E.A. specification formerly set the more conservative ratio of 50 but has now adopted the value 60 [**433**].

[1] *Proceedings*, London Mathematical Society, 1891.

[2] *Bulletin* 86, University of Illinois, 1916.

PLATE GIRDER STIFFENER SPACING

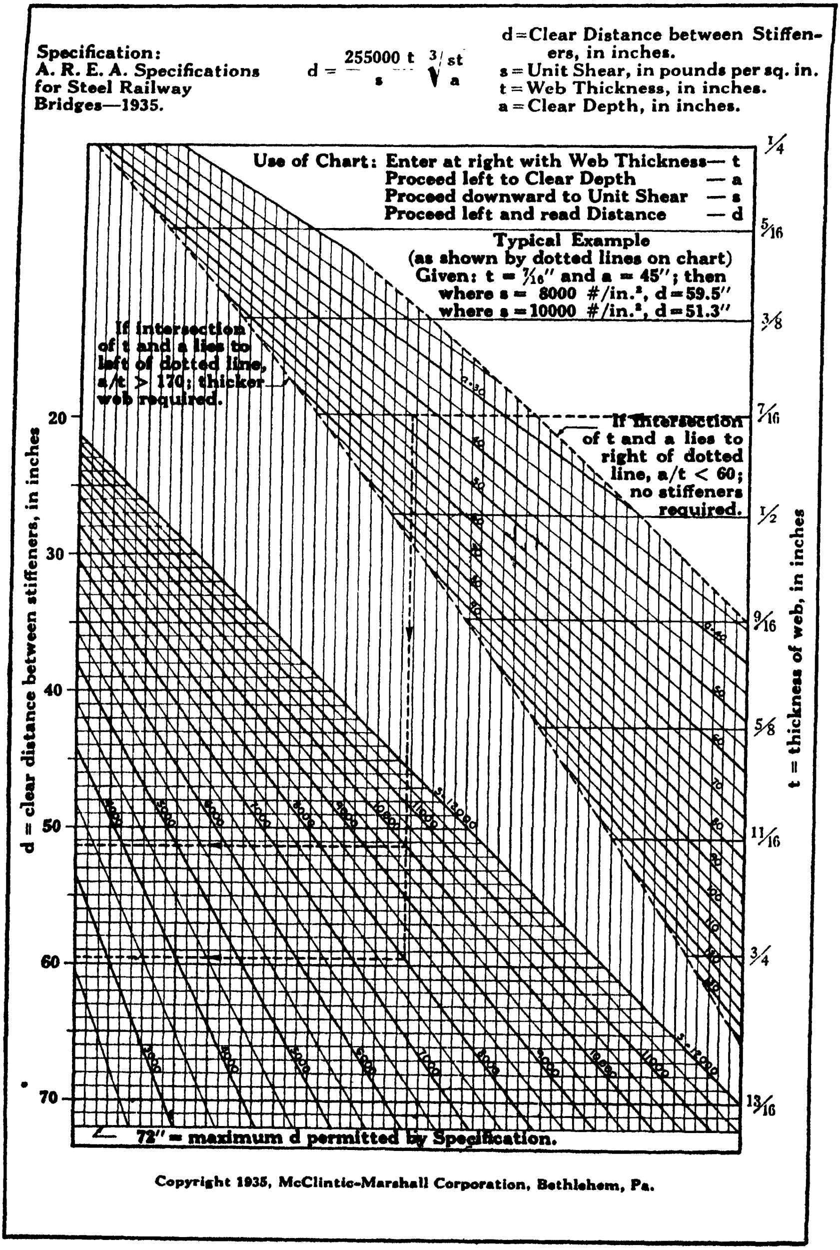


FIG. 4–10

Professor TIMOSHENKO points out that since total load on a railroad bridge is somewhat proportional to span, and also since depth is quite commonly a fixed fraction of the span, the proportioning of the web to resist shear at a given allowable stress results in nearly constant web thickness for all spans and depths of railroad bridge girders. Since this would give too thin webs for deep girders, the A.R.E.A. sets the lower limit web thickness at $\frac{1}{170}$th of the clear distance between flanges [**431**].

The usual method of proportioning intermediate stiffener angles is to make the outstanding leg as wide as possible without having it project beyond the outstanding leg of the flange angle, to make the other leg the smallest that will provide space for riveting, and to make the thickness the minimum permitted by the specification (usually $\frac{5}{16}$ in. for building work, $\frac{3}{8}$ in. for bridge work) [**433**]. There is no way to figure the necessary spacing of rivets in intermediate stiffener angles. A few specifications require a $4\frac{1}{2}$-in. spacing, but most permit 6 in.

Example 4–8. Determine by the 1931 and the 1935 A.R.E.A. rules the required spacing and size of the intermediate stiffener angles near the end of the girder of Ex. 4–1, p. 85.

Solution. The allowable spacings are (1931) {**125**}:

(*a*) $d = 6$ ft 0 in.

(*b*) $d = 4$ ft 6 in.

$$(c)\ d = \frac{t}{40}(12{,}000 - S)$$

$$= \frac{1}{80}(12{,}000 - 9120) = \frac{2880}{80} = 36 \text{ in.}$$

$$= 3 \text{ ft } 0 \text{ in. governs}$$

Outstanding legs not less than $\frac{84}{30} + 2 = 4$ in. However, since the flange angles have 6-in. outstanding legs, common practice will be followed and the outstanding legs of the stiffener angles will be made 5 in. Therefore, use for each pair 2 ∠s $5 \times 3 \times \frac{3}{8}$, 3 ft 0 in. c.c.

The 1935 rules give [**433**]:

(*a*) 72 in.

(*b*) 68 in. (equation solved by chart).

Discussion: This wider spacing of stiffeners by the 1935 specification returns practice to that of earlier days. Is the presence of diagonal tension, tending to prevent buckling under action of diagonal compression, an argument for more radical spacing?

Problem 4–18. Determine by the rules (A.R.E.A.) of this article the required spacing and size of the intermediate stiffener angles in the end 12-ft sections of the girder of Ex. 4–2, p. 86.

Ans. 2 ∠s $6 \times 3\frac{1}{2} \times \frac{3}{8}$, 1 ft $6\frac{3}{4}$ in. c.c. (1931)
3 ft 6 in. clear (1935)

4–11. Stiffeners at Concentrated Loads. It is not uncommon in building construction to have the column spacing vary between the upper and lower parts of a structure because of architectural considerations. This means that somewhere the loads from the upper columns must be distributed by girders or trusses to other columns. At points where girders support columns, and also at the ends of girders which rest on walls or abutments, heavy concentrated loads come to the members.

Fig. 4–11 shows a column supported by a girder. It will be evident that: (*a*) the rivets connecting the flange angles and the web directly under and near the column are too few to take the column load into the web; and (*b*) the outstanding legs of the flange angles, together with the cover plates, are not strong enough to support the column load in bending. It is necessary, therefore, to place stiffener angles on the girder. These load stiffeners are never crimped {**124**}.

Angles are rolled with a fillet at the junction of the legs. To make the stiffeners fit tightly to the flange angles at this point would require an expensive grinding operation. Instead, the stiffeners are *chamfered*, as shown in Fig. 4–12, both for intermediate stiffeners and for stiffeners at concentrated loads.

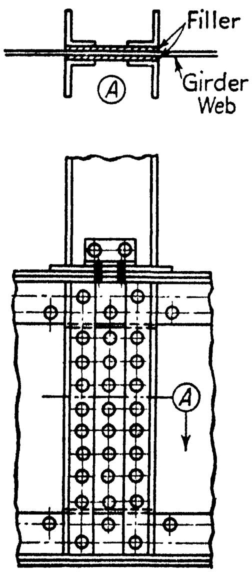


Fig. 4–11

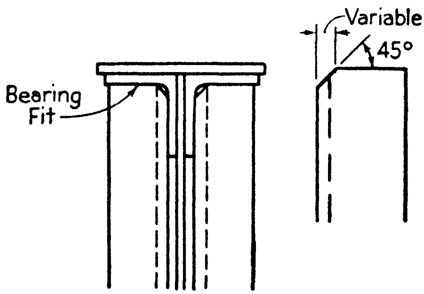


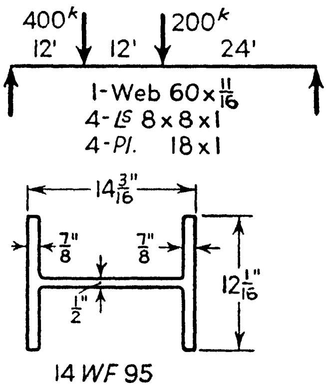
Fig. 4–12

The usual method for proportioning stiffeners like those in Fig. 4–11 is to provide enough bearing area at the top of the stiffener angles to receive the column load at a safe bearing value, and then furnish sufficient rivets to transfer the stiffener load to the girder web. (Note that the bearing area on one stiffener angle equals [length of outstanding leg minus chamfer] × [angle thickness].)

Some designers go one step further. The angles (four in Fig. 4–11,

more for heavier loads) are considered as a column and investigated by means of a column formula. This column differs from a typical column in a number of ways, one being that the load varies from a maximum at one end to zero at the other. This fact is commonly taken into account by assuming that the column length is one-half the depth of the girder. It is the experience of the present writers that this computation is unnecessary since stiffeners proportioned for bearing will always be good as a column. However, in several of the examples which follow the column computation is added by way of illustration.

Example 4–9. A girder, with the section noted, carries two loads as shown. The column which carries the 400,000-lb load is a 14 WF 95. The column is placed with its web parallel to and directly over the girder web. Design the stiffeners which must be placed under the column and compute the number of rivets which must be used to connect the angles to the girder.



Ex. 4–9

Allowable unit stresses:

Bearing on effective area of stiffener angles, 21,000 lb per sq in.

Rivets in shear, 12,000 lb per sq in.

Rivets in bearing, 24,000 lb per sq in.

Solution. Since the flange angles have 8-in. outstanding legs, the outstanding legs of the stiffener angles will be made 6 in. The bearing length, after chamfering, will be $5\frac{3}{8}$ in. as the fillet on an 8×8 angle has a $\frac{5}{8}$-in. radius. (The variable dimension of Fig. 4–12 is made $\frac{1}{2}$ in. or greater.) Using four angles under the column, the required thickness will be $\dfrac{400{,}000}{21{,}000 \times 4 \times 5\frac{3}{8}} = 0.88$ in. Use $4 \angle 6 \times 3\frac{1}{2} \times \frac{7}{8}$. In this case, since the angle thickness exceeds $\frac{5}{8}$ in., part of the other leg will bear, but this fact will be ignored in the computation.

Rivets. One rivet in bearing will be good for

$$\tfrac{7}{8} \times \tfrac{11}{16} \times 24{,}000 = 14{,}420 \text{ lb}$$

and in double shear for

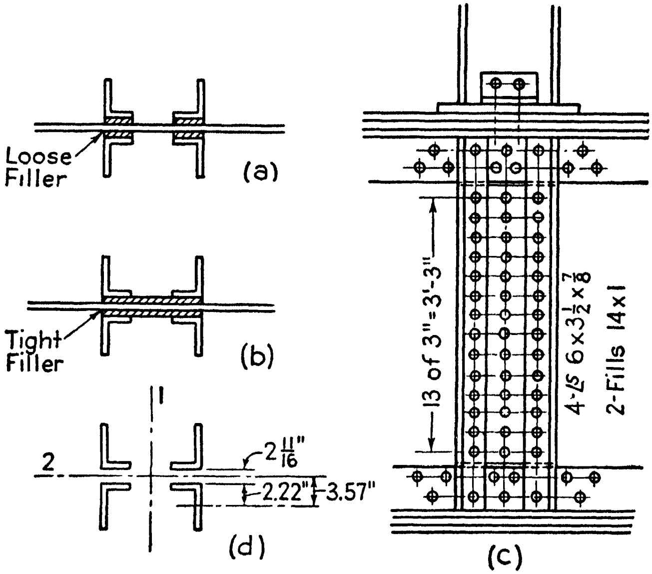
$$2 \times 0.601 \times 12{,}000 = 14{,}420 \text{ lb}$$

(It is a coincidence that these values are equal. Usually one will be different from the other.)

Number required $\qquad \dfrac{400{,}000}{14{,}420} = 28$

The rivets which pass through both stiffener angles and flange angles (see cut) are considered as flange rivets and are not considered as part of the stiffener connections. The required rivets must, therefore, be spaced in the $60\frac{1}{2} - 16 = 44\frac{1}{2}$ in. between flange angles.

Since these are stiffeners which support a concentrated load they will be placed on fillers and not crimped. These fillers may be arranged as shown shaded in the sectional views. Fillers whose widths equal the angle widths are called *loose* fillers. Those which are made wide enough to take extra rivets which do not pass through the angles are called *tight* fillers. Specifications commonly require the number of rivets to be increased 50 per cent



Ex. 4–9

where loose fillers are used because the rivets are loaded at three widely separated points and there is more chance for bending in the rivet than where the filler is tight and is attached to the web by additional rivets.

If loose fillers are used, the number of rivets required in each pair of stiffener angles will be 1.5(28/2) = 21. It is not possible to get this number of rivets in a single row in about 40 in. of depth, and tight fillers must therefore be used. In this case the 50 per cent of extra rivets will be placed in the tight filler as shown.

The angles selected will be tested as a column by means of the formula $P/A = 16{,}000 - 70\,L/r$, 14,000 maximum. Buckling about the axis 1 (perpendicular to the web) will be prevented by the web, and the investigation must be made about the axis in the web. In making this investigation the entire area of the angles which compose the column must be used.

$$\text{Area} = 4 \times 7.55 = 30.20 \text{ sq in.}$$

$$I_2 = 4(26.4 + 7.55 \times 3.57^2) = 490.8 \text{ in.}^4$$

$$r_2 = \sqrt{\frac{490.8}{30.2}} = 4.03 \text{ in.}$$

$$\frac{P}{A} = 16{,}000 - 70\,\frac{60 \times \frac{1}{2}}{4.03} = 15{,}480 \text{ lb per sq in.}$$

Therefore use 14,000 lb per sq in., the maximum allowable. The actual stress is

$$\frac{400{,}000}{30.2} = 13{,}200 \text{ lb per sq in.}$$

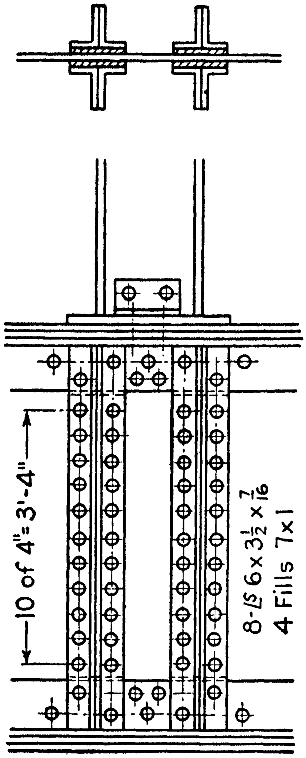


Fig. 4–13

The angles are therefore satisfactory as a column.

In Fig. 4–13 an alternate design for this point is shown, using 8 angles and 4 loose fillers.

Several other methods are in common use for providing bearing area greater than that needed in the foregoing example. One method is to use 8 angles similar to those in Fig. 4–13 but to spread the pairs and place plates equal in width to, or $\frac{1}{2}$ in. narrower than, the outstanding stiffener angle legs between those legs (Fig. 4–14). Another method is to use on each side of the girder an additional $\frac{1}{2}$-in. filler plate *over* the vertical legs of the flange angles and thus push the stiffener angles out far enough to cause their entire area to bear.

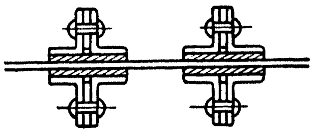


Fig. 4–14

Problem 4–19. Draw the following free bodies with their force systems: the load stiffeners of Fig. 4–11, replacing the tight filler there shown by loose fillers which act simply as washers and carry no stress; the portion of web shown in the same figure neglecting the flange angles; a rivet passing through a pair of stiffeners and the web. For simplicity replace the many rivet holes by two or three holes only. Use isometric views.

Draw the free body consisting of the load stiffeners and tight fillers of Fig. 4–11, assuming the combination to act as though made from a single piece of material, also, as before, the free bodies consisting of the piece of web and a rivet.

Separate the combination of two angles and a tight filler into the three parts and draw each as a free body with its force system.

Problem 4–20. Determine the size of the stiffener angles required under the columns of the girder of Ex. 4–2, p. 86 (P = 225,000 lb). Use the fiber stresses of Ex. 4–10 below. Show on a sketch the number and spacing of the rivets in the stiffeners.

Ans. 4 ∠s $6 \times 3\frac{1}{2} \times \frac{1}{2}$

Problem 4–21. Determine the required size of the stiffener angles at the ends of *the girder of Ex. 4–1, p. 85. Use four angles at each end and assume that the girder bears on the wall for a distance of 12 in. Use the fiber stresses of Ex. 4–9. Show on a sketch the number and spacing of the rivets in the stiffeners.*

Ans. $4 \angle\text{s}\ 5 \times 3\frac{1}{2} \times \frac{11}{16}$

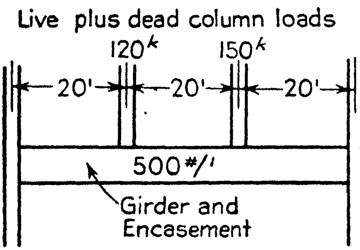
4–12. Web Splice. Large plates are difficult to handle without distortion and bending. As a consequence, web plates are frequently limited to those whose weight does not exceed 3000 lb. This means that in many girders the web must be spliced in spite of the fact that the handbook information would indicate that a single plate of the required length can be rolled.

In the design of the girder the web is assumed to resist the shear and also a portion of the bending moment. The splice, therefore, should be designed to resist the shear at the section where located and also the full share of the bending moment for which the web was designed.[1]

The usual practice is to place a splice plate on each side of the web. These plates are made as deep as the clear distance between flange angles and are often selected to provide as much net area as has the web. In addition, they must carry, at satisfactory unit values, the part of the bending moment resisted by the web.

The rivets which connect the splice plates to the web are usually spaced 3 in. center to center in the vertical rows. These rivets must resist both vertical shear stress and torsional stress. (See Art. 3–5, p. 73, for rivets in torsion.) In computing d and Σd^2 it is usual to measure the vertical distance from the rivet to the mid-height of the web and to neglect the horizontal component of the distance from the rivet to the center of gravity of the group. It hardly need be said that the group on each side of the cut in the web must be figured for *all* the shear and *all* the moment. The example which follows shows the design of a web splice.

Example 4–10. Design a splice for the $54 \times \frac{3}{8}$ in. web of the building girder which has been designed for the location shown with the given allowable stresses.



Ex. 4–10

$s_{\text{shear on net web}}$	12,000 lb per sq in.
s_{tension}	16,000 lb per sq in.
$s_{\text{compression}}$	14,000 lb per sq in.
$s_{\text{bearing on stiffener angles}}$	24,000 lb per sq in.
$s_{\text{rivets in shear}}$	12,000 lb per sq in.
$s_{\text{rivets in bearing}}$	24,000 lb per sq in.

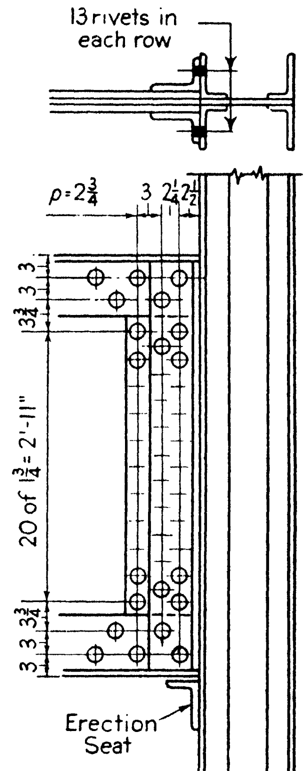
$\frac{7}{8}$-in. diameter rivets.

[1] The web carries maximum bending moment not only at the point of maximum girder moment but also at each point where a cover plate ends. Why?

The girder is made up as follows:

1 web plate $54 \times \frac{3}{8}$ in., $54\frac{1}{2}$ in. b. to b. angles
4 flange angles, $8 \times 8 \times \frac{3}{4}$ in.
2 cover plates, $18 \times \frac{11}{16}$ in. $\times$ 38.6 ft (theoretically)
2 cover plates, $18 \times \frac{5}{8}$ in. $\times$ 28.2 ft (theoretically)

Rivet pitch in flange: 2.7 in. in end thirds; 27.8 in., use 6 in. in center third.
Intermediate stiffeners: $6 \times 3\frac{1}{2} \times \frac{3}{8}$ angles in pairs;
spacing at end = 40.8 in. {125}
Load stiffeners: 4 angles, $6 \times 3\frac{1}{2} \times \frac{3}{8}$ in.
End connection: see sketch. Field rivets through column (at 10,000 lb per sq in. shear) = 26.
Shop rivets through connection angles and tight filler = 30.

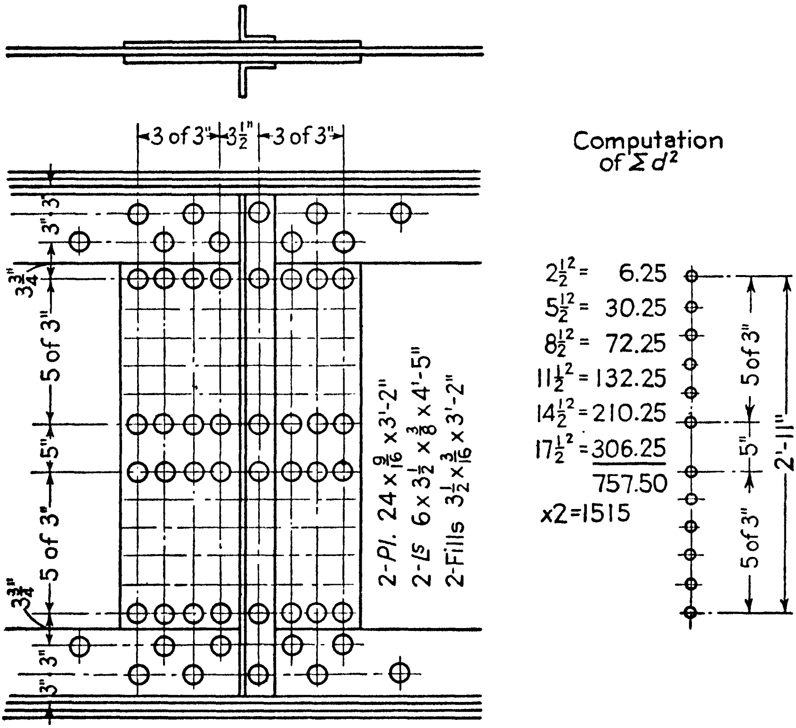


End Connection
Ex. 4–10

Design of Web Splice. This splice will be placed at the center of the girder where the shear is small. The amount to be carried at the splice will be assumed as 30,000 lb, the maximum shear which may occur in the center third. The moment to be carried will be taken as the maximum the web will ever carry, which equals the web equivalent times the maximum flange stress times the effective depth (Eq. 4–2, p. 83); $\frac{1}{8} \times 54 \times \frac{3}{8} \times 16{,}000 \times 53 = 2{,}150{,}000$ in-lb (h had been computed in designing the girder as 52.9 in.; an approximate value is sufficient for this computation).

The splice plates will be adequate in shear if their net section equals that of the plate being spliced, which is $(54 - 14)\,\frac{3}{8} = 15.0$ sq in., the allowance of 14 holes being explained by the sketch. The clear distance between flange angles is $38\frac{1}{2}$ in., assuming no over-run of angles. Using 38-in. splice plates, their thickness as fixed by shear is $(38 - 12)\,2\,t = 15.0$ and $t = 0.29$ in. The first step in fixing the thickness required by moment is to estimate the extreme fiber stress which can be developed at the edge of the plates, which, assuming the neutral axis as approximately at the axis of the girder, in this case equals $16{,}000 \times \frac{19.00}{28.56} = 10{,}650$ lb per sq in. The moment of resistance of two splice plates, $\frac{1}{8} \times 10{,}650 \times 2\,t \times 38^2$ (the $\frac{1}{8}$ being used instead of $\frac{1}{6}$ to allow for rivet holes), must equal 2,150,000 above computed: whence $t = 0.56$ in., calling for $\frac{9}{16}$-in. plates. The shear requirement is less and moment governs.

Each group of rivets through the splice plates on one side of the cut in the web must carry the combined shear and moment computed. The horizontal spacing of rows of rivets will be neglected, which amounts to saying that the strength of the groups equals the sum of the strengths of the vertical rows. It becomes possible to determine the number of rows by determining the maximum stress which would be induced in the extreme rivet were one



Ex. 4-10

row used. The number of rows plainly will equal that stress divided by the allowable stress on a rivet. Assuming one row on each side, spaced as shown, each rivet will carry in shear 30,000/12 = 2500 lb; the extreme rivets will be stressed by the moment

$$r = \frac{Md_1}{\Sigma d^2} = \frac{2,150,000 \times 17.5}{1515} = 24,900 \text{ lb}$$

The computation for Σd^2 is shown in the sketch. The resultant stress is 25,100 lb, which, divided by 7880 lb, the allowable stress on one rivet, gives 3.18. Accordingly four rows like the ones shown are required.

Problem 4-22. Design a web splice to replace that in Ex. 4-10, using rivets spaced at 4 in. on centers, except for a 3-in. space at mid-depth of girder.

Problem 4–23. Assuming that an allowable stress of 20,000 lb per sq in. were given for bending in splice plates, would that change the design in Ex. 4–10? How would decreasing the thickness of the splice plates affect the moment of resistance actually developed by them?

Problem 4–24. Check completely the design of girder given in the data for Ex. 4–10.

4–13. Flange Splices. It is seldom necessary to splice either flange angles or cover plates, except in exp rt work where girders must occasionally be shipped in pieces. When these must be spliced, the splices for the two angles of one flange should be placed on opposite sides of and equally distant from the center line of the girder, Fig. 4–15. The angle splice should be as far as possible from a web splice, and, if practicable, should be located near the end of a cover plate in order that the plate may be extended to serve as splice material.

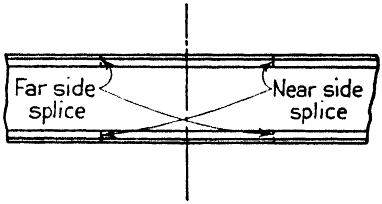


FIG. 4–15

If a cover plate is not available, a second angle, shown cross hatched

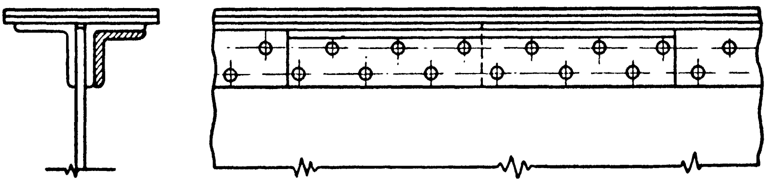


FIG. 4–16

in Fig. 4–16, is used. This angle must be ground to fit the fillet of the angle spliced and, if at a point where looks count, must be cut so that the splice legs do not project beyond those of the other angle.

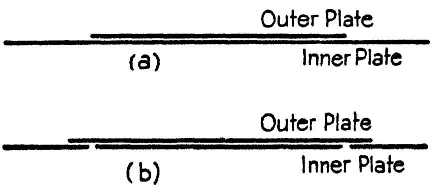


FIG. 4–17

A cover plate splice may usually be made in the manner indicated in Fig. 4–17. Here it is assumed that two plates are required, Fig. 4–17*a*. If the upper plate is extended and spliced to the ends of the lower plate,

Fig. 4–17*b*, at a point beyond the theoretical end of the upper plate so that the excess length of the upper plate may serve as an adequate splice for the lower, the spliced combination becomes equivalent to the long plate of Fig. 4-17*a*.

4–14. Box Girders. Where heavy loads must be carried in bending and available depth is limited owing to architectural or other requirements, recourse must sometimes be had to separate shallow girders placed close together, or to box girders, several forms of which are shown in Fig. 4–18.

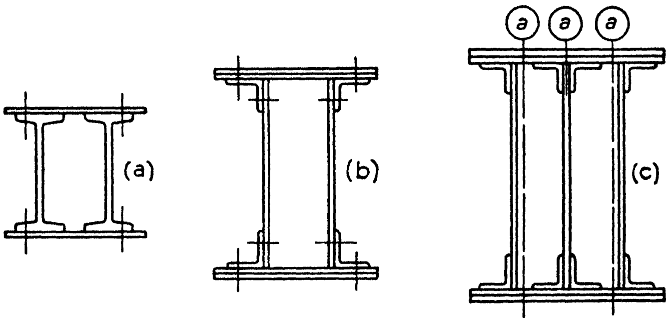


FIG. 4–18

Box girders are best fitted for situations where the load is applied to the tops of the girders in such a way as to ensure equal deflection of the component parts. They should be used with caution in situations where the load comes from other members which frame in from the side unless diaphragms may be arranged between the webs at each load point. Except in the case of large girders where a man may enter from the end, diaphragms may prove difficult to rivet. In such cases the diaphragms should be bolted — perhaps through hand holes cut in the outer webs — but never omitted.

The principles already outlined will serve for design. In the case of the type of Fig. 4–18*c* the design is simplified if imaginary planes *a* are assumed to divide the girder into four equal parts, and rivet computations, etc., are made accordingly. It is obvious, in this case, that the center web should have a thickness equal to the sum of the thicknesses of the side webs, and also that the girder must be assembled by riveting, first, the flange angles to the webs, second, the covers to the center segment, and, last, the covers to the side segments.

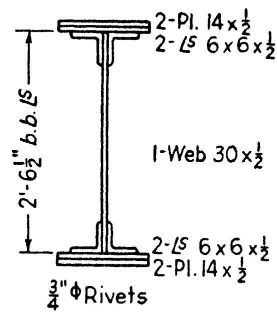
Problem 4–25. Design a 3-web box girder to support a uniform load of 30,000 lb per ft of length applied to the top flange. The span length is 36 ft; the overall depth of the girder, including the rivet heads in the cover plates, must not exceed

48 in. Allowable unit stresses are:

$s_{\text{shear on gross web}}$	13,000 lb per sq in.
s_{tension}	18,000 lb per sq in.
$s_{\text{compression}}$	15,500 lb per sq in.
$s_{\text{rivets in shear}}$	13,500 lb per sq in.
$s_{\text{rivets in bearing}}$	27,000 lb per sq in.

Problem 4–26. (*a*) A girder with the cross section shown is used on a span of 30 ft to carry a uniformly distributed load of 8000 lb per linear foot. Determine the maximum unit stresses in the tension and compression flanges of the girder. (Neglect the dead weight of the girder.)

Ans. s_c = 13,360 lb per sq in.
s_t = 15,325 lb per sq in.



PROB. 4–26

(*b*) Find the theoretical lengths of the cover plates of the tension flange.

Ans. 15.24 ft, 21.58 ft

(*c*) What is the required pitch of the rivets joining the web and the flange angles at the *end* and at the *center* of the girder? Assume that the load is a moving load and is applied directly to the top flange of the girder. Neglect impact. Allowable unit stresses for rivets are: shear, 12,000 lb per sq in.; bearing, 24,000 lb per sq in.

Ans. End 2.18 in., center (7.42 in.) 6 in.

(*d*) Determine the size and spacing of the intermediate stiffener angles near the end of the girder.

Ans. Not needed

Problem 4–27. A girder with a span of 42 ft carries at a point 7 ft from its left end a load of 300 kips and at its mid-point a load of 100 kips. At the point of maximum moment the cross section is composed of

1 web	$60 \times \frac{1}{2}$
4 angles	$6 \times 6 \times \frac{9}{16}$
4 covers	$14 \times \frac{9}{16}$

The rivets are $\frac{7}{8}$ in. in diameter. Neglect dead load.

(*a*) Find the maximum fiber stress in each flange.

Ans. s_c = 13,000 lb per sq in.
s_t = 15,100 lb per sq in.

(*b*) What are the theoretical lengths of the compression flange cover plates?

Ans. 22.8 ft, 27.6 ft

(*c*) Find the required pitch of the rivets joining web and flange angles at the left end of the girder. Allowable unit rivet values are: 12,000 lb per sq in. shear; 24,000 lb per sq in. bearing.

Ans. 2.09 in.

(*d*) Are intermediate stiffeners required in the 21 ft at the right end? If they are, find their size and spacing.

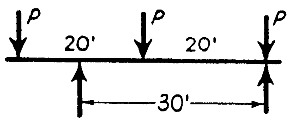
Ans. Two angles $5 \times 3 \times \frac{3}{8}$, 5 ft c.c.

(*e*) Design the stiffeners under the 300-kip load. The bearing on the stiffener angle legs is not to exceed 20,000 lb per sq in.

Ans. Four angles $5 \times 3\frac{1}{2} \times \frac{7}{8}$

Problem 4–28. Girder section:

1 web	$48 \times \frac{1}{2}$
4 angles	$6 \times 6 \times \frac{3}{4}$
4 covers	$14 \times \frac{5}{8}$



PROB. 4–28

Allowable unit stresses:

$s_{\text{shear on gross web}}$	12,000 lb per sq in.
s_{tension}	18,000 lb per sq in.
$s_{\text{compression}}$	16,000 lb per sq in.
$s_{\text{bearing on stiffeners}}$	24,000 lb per sq in.
$s_{\text{rivets in shear}}$	13,000 lb per sq in.
$s_{\text{rivets in bearing}}$	26,000 lb per sq in.

(*a*) Find the allowable column load, *P*. Neglect dead load.

(*b*) Determine the theoretical lengths of the bottom flange cover plates.

(*c*) Find the minimum pitch required to join flange angles and web. Show by a sketch where this pitch must be used.

(*d*) Design the intermediate stiffener angles (size and spacing).

(*e*) Design the stiffeners over the left support.

CHAPTER V

ROLLED BEAM AND PLATE GIRDER DECK RAILROAD BRIDGES

5–1. A *deck bridge* is one in which the floor system is supported in the plane of the top chord or flange, with the ties (or slab) which carry the load resting directly on the top chord, or else upon stringers supported by floor beams, as in a through bridge. The first arrangement is the one generally followed in a deck structure where the main carrying members are rolled beams or plate girders.

5–2. Floors. If bridge ties rest directly on the supporting members the resulting construction is known as an *open floor* (see design sheet BB1, p. 119), a type which is sometimes used in bridges which cross streams or other railroads. For crossings over roads or streets, and generally elsewhere, a *solid floor* is used in modern practice. When the depth available for a floor is severely limited, the ties (in rare cases the rails themselves) are supported directly on the floor. The usual type of solid floor employs ballast with a minimum thickness of 6 in. [**95**][1] under

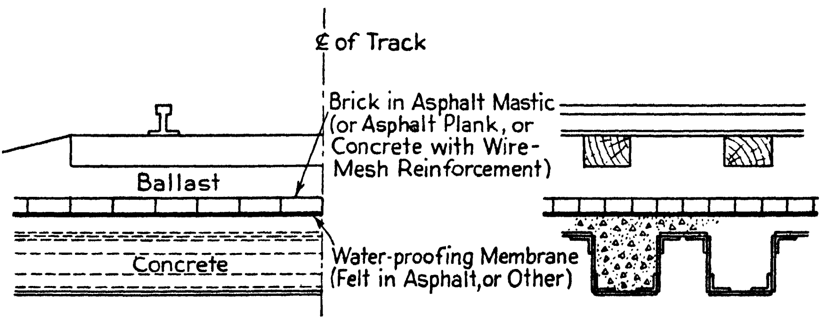


Fig. 5–1

the ties. For such a floor there may be used: (1) a reinforced-concrete slab (see design sheet DG1, p. 127); (2) transverse I-beams, either encased in concrete, or supporting a thin reinforced-concrete slab, or supporting a steel plate with a covering of concrete and waterproofing; or

[1] Bold-face numerals in brackets refer to the 1931 A.R.E.A. Specifications, Appendix D.

(3) the favorite of earlier days, the trough floor, Fig. 5–1, built up from plates and angles.

The difficult matter of waterproofing is thoroughly treated in "The Water Proofing of Solid Steel Floor Railroad Bridges" by S. T. WAGNER, *Transactions*, A.S.C.E., Vol. LXXIX, and the discussion thereof. Experience teaches that, if ample provision is made for drainage, a dense concrete slab with a minimum thickness of 12 in. will be impervious without the use of additional costly waterproofing materials.

5–3. Rolled Beam Railroad Bridge with Open Floor. For many short bridges where underclearance is not limited, rolled beams may be used as the main supporting members with the ties or slab resting directly on them. The specification of the A.R.E.A. permits the use of four to eight rolled beams per track [**99**] where the span length does not exceed 35 ft. [**9**]. Because of the small amount of shop work involved, this will prove an economical type of construction, especially where the depth permitted by the conditions makes possible the use of four beams per track.

Design sheets BB1, 2, 3, and Fig. 5–2 show the design and drawing of a short-span beam bridge. Much of this should be clear without comment, but the following remarks will probably answer most questions.

Ties on bridges are commonly made 10 ft. long and 8 in. wide. Standard thicknesses vary by 2-in. intervals, with 8 in. as a minimum. The design of a tie on p. 141 will make clear that in the present instance a depth of 8 in. is ample.

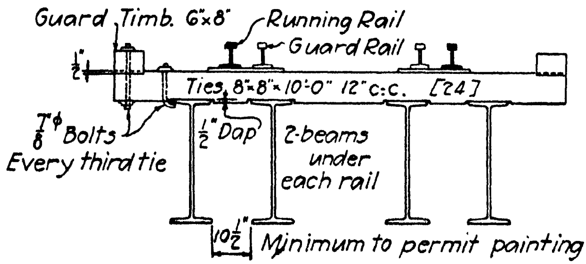
Reference to Fig. 5–2 will show that the dead load used for the beams was the correct value. This was *not* a lucky guess. It was the result of a rough trial design, not shown. Any problem in design involves the weight of the structure itself, and this must be assumed until the design is finished or, sometimes, even until the shop drawings are made. Three sources are available to help in the assumption: previous experience, a trial design, and published tables and charts.[1] A frequently quoted rule is that any error in assumed dead load may be neglected provided it does not affect the design values — in this case the moment of 1,689,500 ft-lb — by more than one per cent.

The live moment and shear were taken from the tables in the Appendix, p. 383. Since the load is E–50, the loads are $\frac{50}{60}$ of those specified in [**20**]. See also [**21**].

[1] Chapter LV, "Weights of Steel Structures," in "Bridge Engineering" by J. A. L. WADDELL, is especially helpful. The young graduate who goes into bridge work would do well to become thoroughly familiar with this work and its companion, "Economics of Bridgework," by the same author.

Beam Bridge with Open Floor — *Design Sheet – BB1*

Data



Single Track 4-Beam Railroad Bridge [99]

Span: 26 ft. c.-c. bearings
Live Load: Cooper E-50
Live Stresses: (See Appendix)
Moment, 812.0 ft. kips per track
Shear, 145.5 kips " "
Specifications: A.R.E.A. 1931 (Printed in Appendix)

Dead Load

Tie: $\frac{8\times 8}{12}\times 10' \times 4.5\#$/F.B.M. [19] = 240 #/ft. of track
Guard Rail Timbers: $2\times\frac{6\times 8}{12}\times 4.5\#$ = 36
Rails and Fastenings: [19] = 150
Beams: 4 estimated = 500
Bracing: " = 50
976 #/ft. of track

Total Stress

	Moment	Shear
Dead: $\frac{1}{8}\times 976\times \overline{26}^2$ =	82,500'#	$976\times\frac{26}{2}$ = 12,700 #
Live	812,000	145,500
Impact [28]	795,000	142,400
	1,689,500'#	300,600 #

Beams: Section

Moment Requirement
Section modulus: $S=\frac{I}{y}=\frac{M}{s}$
Unit compressive stress, ... [48]

$$s_c = 16{,}000 - 150\frac{L}{b} = 15{,}000\#/\square'', \text{ assumed}$$

$$S=\frac{M}{s}=\frac{1{,}689{,}500\times 12}{4\times 15{,}000}= 338 \text{ in.}^3 \text{ required}$$

Minimum Required depth [50]

$$d=\frac{26\times 12}{15}= 20.8''$$

Trial Section 33 WF 125 ← **Use**
S = 385.1 in.³, web thickness = 0.57 in. [57]
flange " = 0.80 "
flange width = 11.5 "

Beam Bridge with Open Floor — **Design Sheet-BB2**

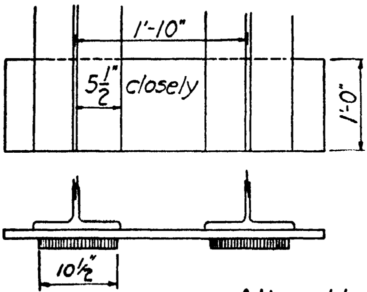
Beams Section (Continued)

Shear Requirement

$$S_s = \frac{300{,}600}{4 \times 0.57 \times 33} = 4{,}000\ ^{\#}/\square'' < 10{,}000\ ^{\#}/\square'' \quad [38]$$

$\therefore$ O.K.

Beams Support



Assume sole plate as 12" long.

Web Stress at Bearing
Column action

$$R = f_b \times t\left(a + \frac{d}{4}\right)$$

$$\therefore f_b = \frac{\frac{1}{4} \times 300{,}600}{0.57\left(12 + \frac{33}{4}\right)} = 6520\ ^{\#}/\square''$$

(See Art. I-10)

$$\text{Allowable } S_b = \frac{18{,}000}{1 + \frac{1}{6000}\left(\frac{33}{0.57}\right)^2} = 11{,}500\ ^{\#}/\square'' > 6{,}520\ ^{\#}/\square''$$

$\therefore$ O.K.

Crushing directly over bearing

$$\frac{\frac{1}{4} \times 300{,}600}{0.57 \times 12} = 11{,}000\ ^{\#}/\square'' < 30{,}000\ ^{\#}/\square'' \therefore \text{O.K}$$

Use — 4-33 WF125

Diaphragm Spacing

Actual unit compression in flange

$$S_c = \frac{1{,}689{,}500 \times 12}{4 \times 385.1} = 13{,}200\ ^{\#}/\square''$$

$$\therefore 13{,}200 = 16{,}000 - 150\,\frac{L}{b} \cdots [48]$$

$$\frac{L}{b} = 18.7$$

$$\therefore L \leqq 18.7 \times 11.5 = 215 \text{ in.}$$

or

$$L \leqq 12 \times 11.5'' = 138 \text{ in.} \cdots\cdots [99]$$

Limit

Sole Plate

Allowable bearing pressure on masonry $= 600\ ^{\#}/\square''$ [38] See sketch above

Required bearing area under one beam

$$\frac{\frac{1}{4} \times 300{,}600}{600} = 125.3\ \square'' \quad 12'' \times 10.5'' \text{ (Shown shaded)} = 126\ \square''$$

Thickness:

Neglect part of plate outside the $10\frac{1}{2}$ in. shaded bearing area in sketch above.

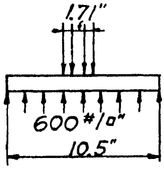
Beam Bridge with Open Floor — **Design Sheet-BB3**

Sole Plate (Continued)

Assume beam spreads load to 3 times web thickness, $3\times 0.57 = 1.71''$

Moment in sole plate per inch of length

$$= 600\times 5.25\left(\frac{5.25-0.85}{2}\right) = 6930''^{\#}$$



Required plate thickness, (d)

$$S = \frac{Mc}{I} = \frac{6M}{bd^2}$$

$$d^2 = \frac{6\times 6930}{1\times 16000} \qquad d = 1.61'' > 1'' \ [134]$$

Plate $12\times 1\frac{5}{8}$ — Use

Lateral Bracing

Lateral Load:

Wind on Structure	[32]	350
Wind on Train	[32]	300
Sway	[33]	250
		900 #/'

End Shear $= 900\times\frac{26}{2} = 11{,}700^{\#}$

Stress in Bracing $= 11{,}700\times\sqrt{2}\ \pm = 16{,}600^{\#}$

Try 1-L $3\frac{1}{2}\times 3\times\frac{3}{8}$ area $= 2.30$ □"

Compression $L = 4.5'\pm$ $r = 0.62$

$$\frac{L}{r} = \frac{54}{0.62} = 87 < 120 \quad [49]$$

Allowable Load $= (15{,}000-50\times 87)(2.3)$
$= 24{,}500 > 16{,}600^{\#}$

Tension

Effective area $= \frac{3}{8}\left[\left(3\frac{1}{2}-1\right)+\frac{1}{2}\left(3-\frac{3}{8}\right)\right]$ [54]
$= 1.33$ □"

Allowable Load $= 16{,}000\times 1.33 = 21{,}300^{\#}$
$> 16{,}600^{\#}$

1-L $3\frac{1}{2}\times 3\times\frac{3}{8}$ — Use

Lateral Bracing Rivets

Rivet Values: Shear, $0.60\times 12{,}000 = 7{,}200^{\#}$; Bearing, $\frac{3}{8}\times\frac{7}{8}\times 24{,}000 = 7{,}880^{\#}$ [38]

Number of rivets required to connect angle to gusset plate

$$\frac{21{,}300}{7200} = 3$$ — Use

Rivets connecting gusset plate to flange

Longitudinal stress $\mp 2\left(21{,}300\times\frac{1}{\sqrt{2}}\right)$
$\mp 30{,}000^{\#}$

$$\text{Rivets} = \frac{30{,}000}{7{,}200} = 4 \text{ closely}$$ — Use

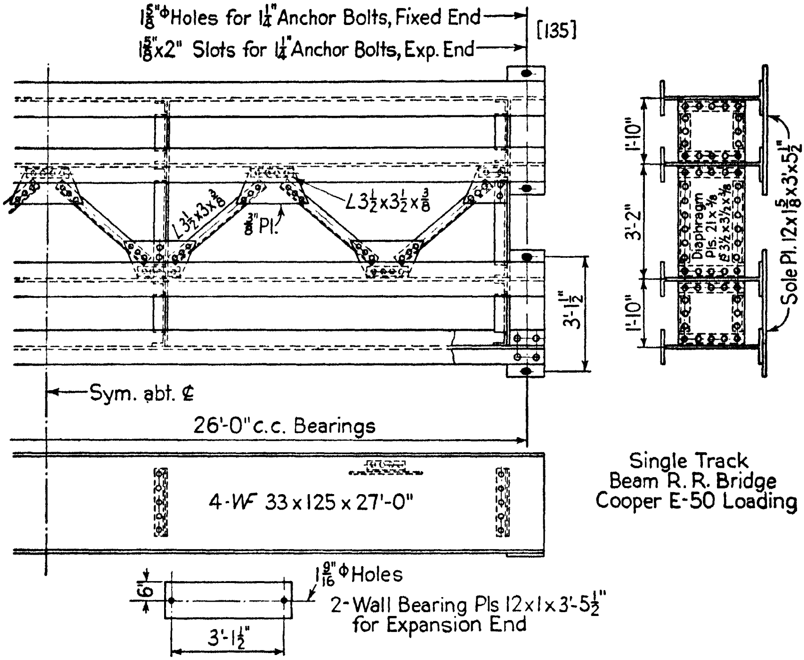


FIG. 5-2

Any of the following beams might be used, the required section modulus being 338 in.³:

$$\begin{aligned} 24 \text{ WF } 140,\ S &= 358.6 \\ 27 \text{ WF } 145,\ S &= 402.9 \\ 30 \text{ WF } 124,\ S &= 354.6 \\ 33 \text{ WF } 125,\ S &= 385.1 \end{aligned}$$

If the depth were limited, as it frequently is in grade crossing eliminations, it might be necessary to use the 24-in. beam. It was assumed, however, that ample depth was available and that a deeper beam might be used. As a general rule the lightest beam would be selected that would furnish the required section modulus. However, since a slight increase in weight furnishes a considerable increase in strength, the 33 WF 125 was used in place of the 30 WF 124.[1]

The beams, which have a flange width of 11½ in., have been spaced 1 ft. 10 in. c.c. in order that the clear width between flanges, 10½ in.,

[1] Neglecting the cost of the abutments and considering only the steel, what is the percentage increase in total cost, the cost varying directly as the weight? What is the percentage increase in strength?

will be great enough to permit a man working from below to get between beams to paint [**51**].

Before final decision could be made as to beam size it was necessary to ascertain whether the section chosen to fill moment and shear requirements could be used without reaction and intermediate stiffeners, as desirable for economy. The A.R.E.A. specifications do not cover the determination of the allowable reaction on a rolled beam, and this matter was decided upon in accordance with the discussion of Art. 1–10, p. 31. Since the web thickness is slightly less than one-fiftieth of the depth between flanges, strict adherence to the specifications [**126**] would require the use of intermediate stiffener angles. However, the web stresses are very low and it was believed that stiffeners (which are practically never used on rolled beams) might safely be omitted.

Fig. 5–2 illustrates a *general drawing.* This gives the sizes of material, and the number of rivets in connections, but does not give all lengths or the exact spacing of rivets. It will be noted that all rivets are shop rivets. This is in accordance with [**132**].

Where expansion must be considered it is common to allow for 1 in. of movement per 100 ft of length, which provides for a range of about 130° F. Provision has been made here for expansion by use of a sliding bearing at one end, since the length is well within the 70-ft limit [**88, 91**]. As a matter of fact, provision for movement is frequently omitted where the span length is 25 ft or less and might perhaps be omitted here.

Anchor bolt holes in the supporting masonry in bridge work are generally drilled *after* the steel is in place. For this reason the holes have been located in the clear where they may be drilled without interference from the beams or bracing. The usual type of anchor bolt is the *swedge bolt,* which is a rod with a thread and nut at one end and with the body of the bolt roughened in some manner. After the bolt is in the hole drilled to receive it, the space between bolt and hole is filled with a thin cement grout.

Problem 5–1

Span feet c.c. Bearings	Loading			
	E–55	E–60	E–65	E–70
18	a	e	i	m
20	b	f	j	n
22	c	g	k	o
24	d	h	l	p

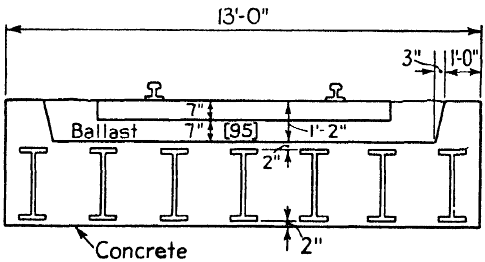
Problem 5–1n is for a 20-ft span and E–70 loading, etc.

Problem 5–1. Design and make a general drawing of a single track beam bridge with open floor. Use six beams under the track. Select the shallowest beams possible.

Problem 5–2				
Span feet c.c. Bearings	Loading			
	E–50	E–55	E–60	E–65
15	a	e	i	m
18	b	f	j	n
21	c	g	k	o
24	d	h	l	p

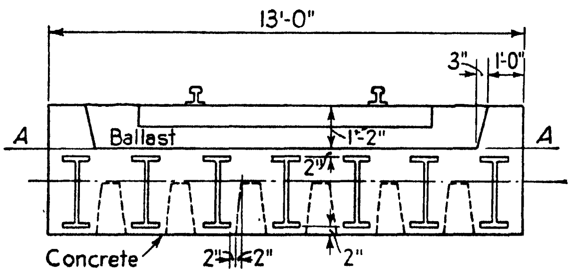
Problem 5–2g is for a 21-ft span with E–55 loading, etc.

Problem 5–2. Design a beam bridge of the type shown. Assume that the concrete stiffens the compression flange so that the beams may be designed for a fiber stress of 16,000 lb per sq in., but otherwise neglect the stress-carrying properties of the concrete. Neglect also the outside beam on each side [27]. A bridge of this type has the advantage that it may be precast, that is, poured or cast away from the site where it is to be used, allowed to cure, and then set in place by a crane. It is ready for use as quickly as the track can be laid.



PROB. 5–2

Problem 5–3. The same data as for Prob. 5–2 except that the compression-carrying property of the concrete is to be considered.[1] Design by means of the trans-



PROB. 5–3

[1] Tests have shown that encasing with concrete greatly reduces the stresses in steel beams. See "Tests on Steel Floor Joist with Concrete Encasement," *Engineering News-Record*, May 5, 1923, p. 800; June 28, 1923, p. 1116. See also "A Practical Method for the Design of I Beams Haunched in Concrete," privately published by

formed section[1] on the solid area in the figure, assuming an allowable unit compressive stress in concrete of 650 lb per sq in. Neglect the concrete above line *A-A*. (First make a trial design neglecting the concrete and assuming a stress of 18,000–20,000 lb per sq in. in the steel.) What would be the effect on the strength of the bridge of omitting the concrete shown in the dotted areas?

5–4. Girder Limits. Section [**9**] gives span lengths of 30 ft to 125 ft as the preferred range for girders. Under special conditions girders are built which are shorter and longer than this range. For example, girders over 150 ft long have been used in American practice, and this length has been exceeded in Europe.[2]

Two reasons limit the number of very long girders used: (1) the point is reached where a truss is both lighter and cheaper, in spite of the fact that the price per pound will generally be higher for a truss than for a girder because of the difference in shop work involved, and (2) economy in the case of a long girder calls for great depth and the limit is reached in the greatest depth which may be shipped from the fabricating shop to the bridge site. This latter problem, if the depth approximates 10 ft, must be discussed at an early stage with the railroad over which shipment is to be made.

5–5. Deck Plate Girder Railroad Bridge with Solid Floor. Sheets DG 1 to DG 8 and Fig. 5–4 show the design computations and general drawing for a single-track deck plate girder railroad bridge with a solid floor of reinforced concrete.

The provision for drainage here made by sloping the slab 2 in. from the center line to the side, and inserting drain pipes at frequent intervals, may be used over a stream but could not be used for a crossing over a street. For such a situation the top of slab is usually given longitudinal slopes to carry the water to the ends of the bridge. With two or more parallel tracks, spans of the kind here shown are often placed side by side. Sometimes it is necessary to provide a walkway on one or both sides of the bridge. For this, Fig. 5–3(*a*) shows the practice of the Reading Co., Fig. 5–3(*b*), the Pennsylvania Railroad.

Since the design is being made for E–65. the loads will be $\frac{65}{60}$ of those of [**20**] and [**25**].

Web Depth. In most cases a considerable range of depths is available for use. The common rule that the depth should be one-eighth of the

Professor R. A. Caughey, Iowa State College; and "Girders, Combined with Concrete or Reinforced Concrete, Subject to Bending," by C. H. Lobban, *Preliminary Publications*, First Congress, International Association for Bridge and Structural Engineering, p. 647.

[1] "Reinforced Concrete Design," by Sutherland and Clifford.

[2] "Solid Web Girder Bridges of Large Span in Steel," by Dr. Ing. L. Karner, *Publications*, International Association for Bridge and Structural Engineering, Vol. 1, 1932, p. 297.

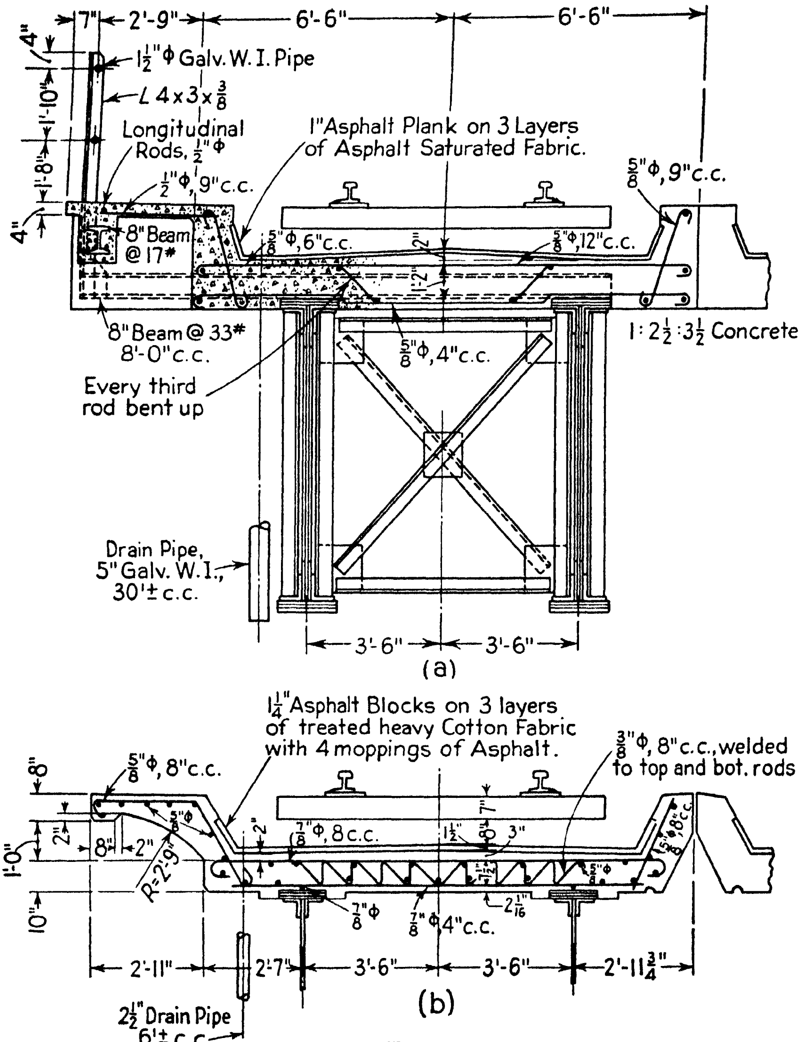


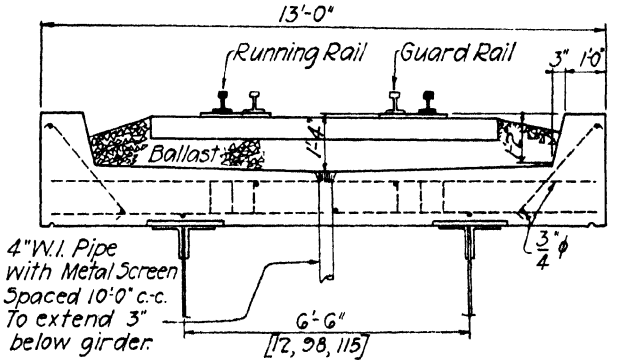
Fig. 5–3

span will usually give a satisfactory girder. A rule sometimes given that the depth should be such as to use the minimum thickness of metal allowed by the specification is seen to be inapplicable here. Another rule, that for greatest economy the area of the web should equal the combined area of the two flanges, will be found by trial to give an absurd girder.

Deck Plate Girder Railroad Bridge with Solid Floor

Design Sheet-DG1

Data



Single-Track : Solid Floor Deck Plate Girder Railroad Bridge

Span : 56'-0" c.-c. bearings

Live Load : Cooper E-65

Live Loads are 6.5 times those given by tables in Appendix

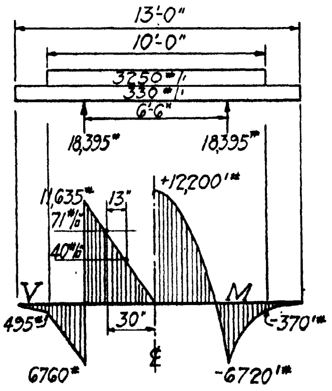
Specifications : A.R.E.A. 1931, (Printed in Appendix)

Working Stress : Reinforced Concrete

Steel S_t = 16,000 #/□"

Concrete S_c = 650 #/□"

Slab



Shear & Moment

Dead Load:

Slab (14")	175 #/□'
Ballast & Ties	140 [19]
Rails & Fastenings	15 [19, 27]
	330 #/□'

Live Load [20, 25]

$15{,}000 \times \frac{65}{60} = 16{,}250$ #/'

Impact 100% = 16,250

32,500 #/'

Deck Plate Girder Railroad Bridge with Solid Floor — **Design Sheet-DG2**

Slab (Continued)

$M = s_c\left(\frac{1}{2} b \times \frac{3}{8} d\right)\frac{7}{8} d = \frac{1}{6} s_c b d^2$ approximately

Thickness

$d^2 = \frac{12,200\times 12}{\frac{1}{6}\times 650\times 12} = 112.8$ $\quad d = 10.6"$

Use 12", overall 14"

Reinforcement

$a_s = \frac{12,200\times 12}{\frac{7}{8}\times 12\times 16,000} = 0.87$ □"/' $\quad$ Use $\frac{3}{4}"\phi$ @6" c.-c. in bottom

Reinforcement

$6"\times\frac{12,200}{6,720} = 10"$ spa. $\quad$ Use $\frac{3}{4}"\phi$ @10" c.-c. in top over girders

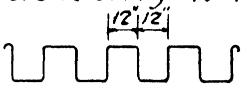
---

Shear at edge of assumed 18" cover plates

$$\frac{11,635\times\frac{30}{39}}{12\times\frac{7}{8}\times 12} = 71 \text{ \#/□"}$$

Stirrups required over $30\times\frac{31}{71} = 13"$, assuming concrete to carry 40 #/□"

Stirrups

Try 

$\frac{3}{8}"\phi$ continuous stirrups placed lengthwise of span

$a = \frac{0.11}{12} = 0.009$ □"/" width $\quad \therefore$ Spacing $= \frac{0.009\times 16,000}{31} = 4.7"$

Use

Use 3 stirrups each side

Girder

Dead Load per linear foot

Slab etc. $330\times\frac{13}{2}$ =	2150 #/'
Girder and bracing	450
	2600 #/'

Shear		Moment	
Dead: $2600\times\frac{56}{2}$ =	72,800#	Dead $\frac{2600\times\overline{56}^2}{8}$ =	1,020,000'#
Live: $46.5\times\frac{65}{10}\times\frac{1}{2}$ =	151,000	Live: $576.1\times\frac{65}{10}\times\frac{1}{2}$ =	1,872,000
Impact	136,700	Impact	1,695,000
	360,500#		4,587,000

Web: Minimum depth [50] $56'\times\frac{12}{12} = 56"$

Depth for minimum thickness [57] $\frac{360,500}{10,000\times\frac{3}{8}} = 96"$

By [120] thickness $= \frac{\sqrt{D}}{20}$

assuming 6" flange angles $D = 96.5 - 12 = 84.5$

$t = \frac{\sqrt{84.5}}{20} = 0.47" > \frac{3}{8}"$ $\quad \therefore \frac{3}{8}"$ cannot be used

Deck Plate Girder Railroad Bridge with Solid Floor	Design Sheet-DG3
Common railway practise: depth $= \frac{Span}{8} = \frac{56\times12}{8} = 84''$ thickness $= \frac{360{,}500}{10{,}000\times84} = 0.43$ Try $\frac{7}{16}''$ With 8" flange angles $D = 84.5-16 = 68.5$ $t = \frac{\sqrt{68.5}}{20} = 0.41''$ $\therefore$ Use $84\times\frac{7}{16}$	Girder (Continued) Web
Flanges — Assume $\frac{56'}{4}$ between cross struts [48] $S_c = 16{,}000 - 150\,\frac{14\times12}{18} = 14{,}600\ ^{\#}/\square''$ $M_R = s\left(A_F + \frac{Web}{8}\right)h$ Assume $h = 83.1''$	
(table below)	Flange
Weights: Web - 125 #/' 4-Ls - 130.8 Cover Pls: - 153.0 408.8 #/' < 450 #/' (assumed), but Bracing, Stiffeners etc., will bring weight close to that assumed.	Check of Weights
(cover plate equations below)	Cover Plate Lengths

		Compression		Tension
$\left(A_F+\frac{Web}{8}\right)$	$\frac{4{,}587{,}000\times12}{14{,}600\times83.1} =$	$45.40\ \square''$	$\frac{4{,}587{,}000\times12}{16{,}000\times83.1} =$	$41.35\ \square''$
$\left(\frac{Web}{8}\right)$ [116]	$\frac{1}{8}\times84\times\frac{7}{16} =$	4.60		4.60
		40.80	$\frac{7}{8}''$ rivets	36.75
(A_F)	2-L $8\times8\times\frac{5}{8}$	19.22	$-\left(4\times1\times\frac{5}{8} = 2.50\right)$	16.72
		21.58		20.03
	Pl. $18\times1\frac{1}{4}$	22.50	$-\left(2\times1\times1\frac{1}{4} = 2.50\right)$	20.00
		Excess = $0.92\square''$	Shortage =	$0.03\square''$

"h" $h = 84.5 - 2\left(2.23 - \frac{22.5(0.62+2.23)}{22.5+19.22}\right)$

$= 84.5 - 2(2.23 - 1.54) = 83.12''$

Use for each flange
2-L $8\times8\times\frac{5}{8}$
2-Pl. $18\times\frac{5}{8}$ [118]

$L' = L\left(0.1 + 0.9\sqrt{\frac{a}{A}}\right) + 3'$ [119]

$L_1' = 56\left(0.1+0.9\sqrt{\frac{10.00}{41.32}}\right)+3' = 33.4'$ bottom flange

$L_1' = 56\left(0.1+0.9\sqrt{\frac{11.25}{46.32}}\right)+3' = 33.4'$ top "

$L_2' = 56\left(0.1+0.9\sqrt{\frac{20.00}{41.32}}\right)+3' = 43.7'$ bottom flange

$L_2' =$ [119] 56'+ top "

Deck Plate Girder Railroad Bridge with Solid Floor — Design Sheet-DG4

Flange Rivets

Shear	End	1/4 Point	Center
Dead	72,800	36,400	0
Live	151,000	$\times\frac{5}{8}=$ 94,300	$\times\frac{2}{7}=$ 43,200
Impact	136,700	89,000	42,100
	360,500	219,700	85,300

Vertical load on flange

Dead: $\frac{2150}{12} = 180$

Live: $\frac{60000}{60}\times\frac{65}{60}\times\frac{1}{2} = 542 \left(> \frac{75000}{84}\times\frac{65}{60}\times\frac{1}{2}\right)$ [20]

Impact: 100% $\quad 542$

1264 #/" of length

$\frac{7}{8}$"ϕ Rivets: Double Shear, - 14,430#

Bearing on $\frac{7}{16}$" web - 9,190# - governs

Horizontal Rivets

Pitch:

$$\text{end},\ p = \frac{9190}{\sqrt{\left(\frac{360{,}500}{83.1}\right)^2+(1264)^2}} = 2.03"$$

$$\tfrac{1}{4}\text{ point},\ p = \frac{9{,}190}{\sqrt{\left(\frac{219{,}700}{83.1}\right)^2+(1264)^2}} = 3.13"$$

$$℄,\ p = \frac{9190}{\sqrt{\left(\frac{85{,}300}{83.1}\right)^2+(1264)^2}} = 5.64"$$

Vertical Rivets

Rivets through cover plate at end:

$$p = \frac{2\times 7710}{\frac{360{,}500}{82\pm}\times\frac{11.25}{35.07}} > 6" \quad \text{Use } 6"\text{pitch}$$

Intermediate Stiffeners

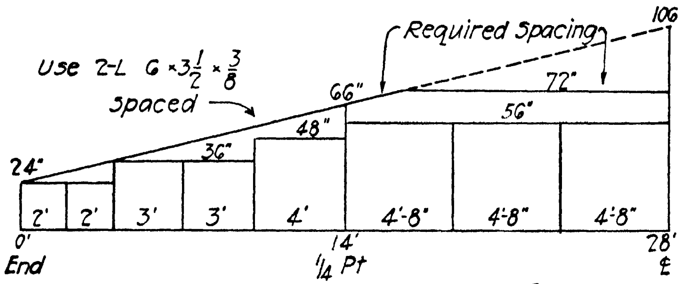
End: $s = \frac{360{,}500}{84\times\frac{7}{16}} = 9800$#/□" $\quad d = \frac{7/16}{40}(12{,}000-9{,}800) = 24"$ [125, 126, 128]

$\frac{1}{4}$ Pt: $s = \frac{219{,}700}{84\times\frac{7}{16}} = 5980$#/□" $\quad d = \frac{7/16}{40}(12{,}000-5980) = 66"$

℄: $s = \frac{85{,}300}{84\times\frac{7}{16}} = 2320$#/□" $\quad d = \frac{7/16}{40}(12{,}000-2320) = 106" > 72"$ limit

Deck Plate Girder Railroad Bridge with Solid Floor — Design Sheet-DG5

Intermediate Stiffeners (Continued)



Stepped line shows spacing furnished

End Stiffeners

Bearing area required $= \frac{360{,}500}{24{,}000}$ [38] $= 15.02\,\square''$ [124]

Thickness required $= \frac{15.02}{4 \times 5\frac{1}{2}} = 0.68''$ using 4 L

Use 4-L $6 \times 3\frac{1}{2} \times \frac{11}{16}$ with tight fillers — **Angles**

Rivets:

Required through angles, limited by double shear, — **Rivets**

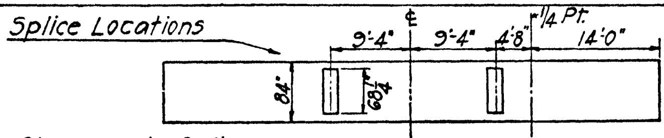
$$\frac{360{,}500}{14{,}430} = 25 \text{ rivets}$$

Required through angles and fillers, limited by web bearing. — **Rivets**

$$\frac{360{,}000}{9{,}190} = 39 \text{ rivets}$$

Web Splice

Splice Locations



Stresses in Splice

Shear: (See rivet pitch computation and text discussion)

$$\frac{1}{3} \times 853{,}000 + \frac{2}{3} \times 219{,}700 = 174{,}900^{\#}$$

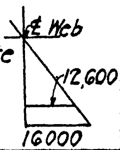
Moment:

$$\left(\frac{1}{8} \times 84 \times \frac{7}{16}\right)(16{,}000)(83.1) = 6{,}110{,}000''^{\#}$$

Splice Plates

Unit bending stress at edge of plate

$$= 16{,}000\,\frac{\frac{1}{2} \times 84.5 - 8}{\frac{1}{2} \times 84.5 + 1.25} = 12{,}600\ ^{\#}/\square''$$



Deck Plate Girder Railroad Bridge with Solid Floor — Design Sheet-DG6

Web Splice (continued)

Splice Plates (Continued)

Thickness: $b = \dfrac{6M}{5\,d^2} = 2t$

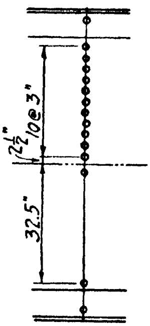
$$t = \frac{1}{2}\left(\frac{6 \times 6{,}110{,}000}{(68.25)^2 \times 12{,}600}\right) = 0.31'' < \tfrac{3}{8}''$$

Net Section: web : $(84''-24'')(7/16'') = 26.25\square''$ (See Fig. 5-5)

splice plates: $(68.25-22)(3/8) = 34.9$ > above

Use 2-$\tfrac{3}{8}''$ plates

Thickness



Rivets

d	d^2
2.5	6.25
5.5	30.25
8.5	72.25
11.5	132.25
14.5	210.25
17.5	306.25
20.5	420.25
23.5	552.25
26.5	702.25
29.5	870.25
32.5	1056.25
	4358.75

$\Sigma d^2 = 2 \times 4358.75 = 8717.5$

Assume one row of rivets

Stress due to shear $= \dfrac{174{,}900}{22} = 7900\#/\text{rivet}$

Stress in outer rivet due to moment

$\dfrac{6{,}110{,}000 \times 32.5}{8717.5} = 22{,}780\#$

Resultant stress

$\sqrt{(7{,}900)^2 + (22{,}780)^2} = 24{,}150\#$

Allowable stress = 9,190# (bearing on 7/16" web)

Number of rows $= \dfrac{24{,}150}{9{,}190} = 3$

Rivets

Top Flange Bracing

Lateral Forces

Wind: [32]

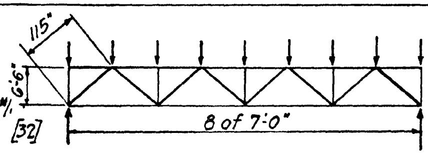
$30 \times 7 \times 1\tfrac{1}{2} = 315$

$< 200 + 150 = 350\#/'$

Wind on train = 300 [32]

Sway = 400

$1050\#/'$ or $7350\#/$panel point



Max. Stress, end diagonal $= 7{,}350\left(\tfrac{7}{2}\right)\sqrt{2} = 36{,}400\#$ approx.

Try 1-L $6 \times 6 \times \tfrac{7}{16}$ @ $5.06\square''$ [49, 65c]

$\dfrac{L}{r} = \dfrac{115}{1.19} = 97$ $\quad \dfrac{P}{A} = 15{,}000 - 50 \times 97 = 10{,}150\#/\square''$

Allowable stress $= 5.06 \times 10{,}150 = 51{,}400\#$
$> 36{,}400\#$ ∴ O.K.

Investigation in tension not needed as inspection indicates area is sufficient.

Use 1-L $6 \times 6 \times \tfrac{7}{16}$

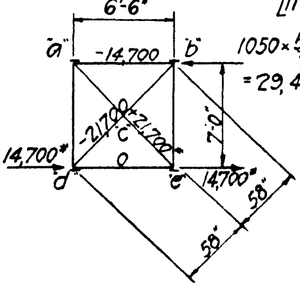
Angle

Deck Plate Girder Railroad Bridge with Solid Floor — Design Sheet-DG7

Top Flange Bracing Rivets

[56] Rivets $= \frac{10,150 \times 5.06}{7,210} = 7.2$ Use 7

End Cross Frame



[112] $1050 \times \frac{56}{2} = 29,400\#$

Assume equal shear distribution as shown.

Try 1-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$

$\frac{L}{r} = \frac{78}{0.69} = 113$

Allowable $\frac{P}{A} = 15,000 - 50 \times 113$

$= 9,350\#/\square''$

Actual $\frac{P}{A}$, member bd, $\left(\frac{L}{r} < 113\right)$

$\frac{21,700}{2.48} = 8750\#/\square''$ } O.K.

Actual $\frac{P}{A}$, member ab, $\frac{14,700}{2.48}$ < above

Tensile Stress allowable [54]

$16,000 \times \frac{3}{8} \times \left(3\frac{1}{2} - 1 + \frac{1}{2} \times 3\frac{1}{8}\right) = 24,300\# > 21,700\#$

Member "de", zero stress

Try minimum angle $3\frac{1}{2} \times 3 \times \frac{3}{8}$ [113]

$\frac{L}{r} = \frac{78}{0.62} = 126 > 120$ [49]

For all members use 1-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$

Angles

Rivets [56] limited by single shear

$9350 \times 2.48 = 23,200 < 24,000\#$ above

Number Reqd. $= \frac{24,300}{7,210} = 4$

Rivets

Intermediate Cross Frame Angles

Bottom chord lateral panel load [112, 113]

$= 150 \times 14 = 2,100\#$ [32]

negligible.

Try minimum angle $3\frac{1}{2} \times 3 \times \frac{3}{8}''$

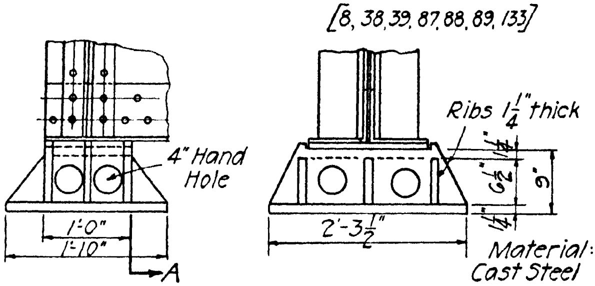
$\frac{L}{r} = \frac{78}{0.62} = 126 > 120$

Use because of low stress.

Deck Plate Girder Railroad Bridge with Solid Floor — *Design Sheet-DG8*

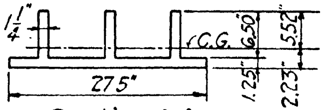
End Bearing Design

[8, 38, 39, 87, 88, 89, 133]



Bearing area = $\frac{360{,}500}{600} = 601\,\square''$

$22 \times 27.5 = 605\,\square''$



Section A-A

Assume casting shown as minimum possible.

Investigate bending on Section A

Bending Moment $= 600(5 \times 27.5)\frac{5}{2} = 206{,}000''\#$

Centroid:

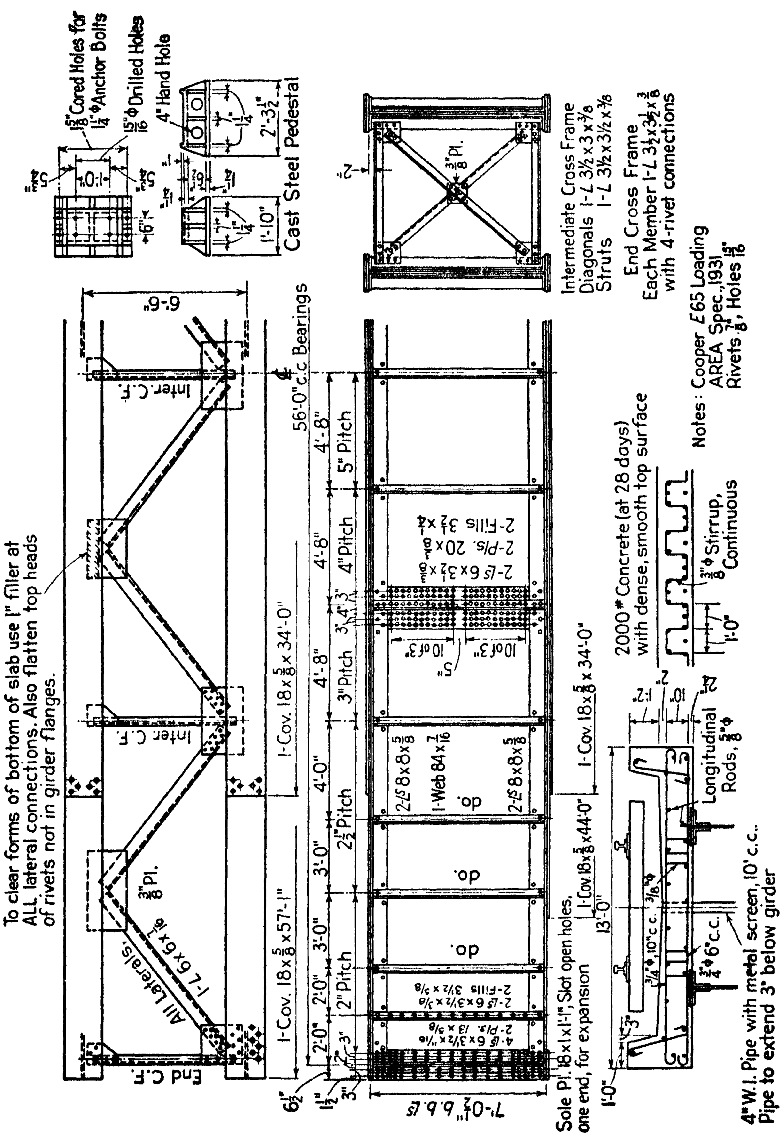
		in² × arm = product
27.5×1.25	=	$34.4 \times 0.625 = 21.5$
$3 \times 6.5 \times 1.25$	=	$24.4 \times 4.5 = 109.8$
		$58.8 \times 2.23 = 131.3$

Moment of Inertia

$$\frac{1}{12} \times 27.5 \times \overline{1.25}^3 = 4$$
$$33.75 \times \overline{1.60}^2 = 89$$
$$3 \times \frac{1}{12} \times 1.25 \times \overline{6.5}^3 = 86$$
$$24.4 \times \overline{2.26}^2 = 125$$
$$304''^4$$

$$S_c = \frac{206{,}000 \times 5.52}{304} = 3740\#/\square''$$

O.K.



To clear forms of bottom of slab use 1" filler at ALL lateral connections. Also flatten top heads of rivets not in girder flanges.
All Laterals, 1-L 6x6x7/16
End C.F.
Inter. C.F.
Inter. C.F.
6'-6"
56'-0" c.c Bearings
1-Cov. 18x5/8x57'-1"
1-Cov. 18x5/8x34'-0"
1-Cov. 18x5/8x34'-0"
1-Cov. 18x5/8x44'-0"
13'-0"
2" Pitch
2 1/2" Pitch
3" Pitch
4" Pitch
5" Pitch
2-Ls 8x8x5/8
1-Web 84x7/16
2-Ls 8x8x5/8
Sole Pl. 18x1x1'-1", Slot open holes, one end, for expansion
Cast Steel Pedestal
4" Hand Hole
Intermediate Cross Frame
Diagonals 1-L 3 1/2x3x3/8
Struts 1-L 3 1/2x3 1/2x3/8
End Cross Frame
Each Member 1-L 3 1/2x3 1/2x3/8
with 4-rivet connections
Notes: Cooper E65 Loading
AREA Spec., 1931
Rivets 7/8", Holes 15/16"
2000# Concrete (at 28 days) with dense, smooth top surface
3/8" Stirrup, Continuous
Longitudinal Rods, 5/8"
4" W.I. Pipe with metal screen, 10' c.c.
Pipe to extend 3" below girder

FIG. 5–4

Greatest economy can be reached only by a careful comparison of a series of structures of varying proportions, with estimates made from completed drawings which take into account differences in stiffeners, bracing, splice plates, etc. To have any meaning, such a comparison should take into consideration difference in volumes of abutments and piers which will follow from differences in girder heights. Suffice it to say that these factors offset one another and that a girder of the proportions adopted will provide a satisfactory structure.

Flanges. In order to arrive at an allowable value for s_c it was necessary to decide on the arrangement of the top chord bracing. This is as shown on sheet DG 6.

Cover Plate Lengths. It is usually assumed that the curve of required flange area and the curve of maximum moment are similar. The curve of maximum moment due to uniform load or to a single moving load is a parabola. The curve of maximum moment due to Cooper loading lies slightly above a parabola. It is not infrequently assumed that this curve is one with a *horizontal section* equal to the maximum moment and of the length $L/10$, joined to two end curves each of which is one-half of a parabola.[1] From such a curve the value of moments at the $L/4$ points in the girders are equal to 0.802 times the maximum. Quarter-point moments computed for a series of spans varying by 10-ft intervals from 40 to 120 ft (the range over which girders would commonly be used) give an average value of 0.775 times the center moment for that at the quarter point with a maximum of 0.786 and a minimum of 0.764. The approximate curve is seen to vary but little from the true curve, and that on the safe side; its use is therefore justified. Using this curve the required length of a cover plate may be obtained from the equation

$$L' = L(0.1 + 0.9\sqrt{a/A})$$

Rivet Pitch. This depends on shear. Computed for the same series of spans as before, the average quarter-point shear is 0.593 times the maximum end shear (maximum 0.621, minimum 0.580), and the average center shear is 0.273 times that at the end (maximum 0.292, minimum 0.257). Since this shear involves only the spacing of rivets and intermediate stiffeners there is ample justification for following a frequently quoted rule that quarter-point shear equals $\frac{5}{8}$ times end shear, and center shear equals $\frac{2}{7}$ times end shear.[2]

Web Splice. The web splice must be designed for both shear and moment. For this purpose it is sufficiently accurate to assume that the

[1] "Design of Steel Bridges," by F. C. Kunz, p. 156.

[2] "Modern Framed Structures," by Johnson, Bryan, and Turneaure, Part III, Ninth Edition, p. 137.

shear varies as a straight line between the values computed for the center line and for the quarter point. At the location chosen for the splice, one-third of the way from the quarter point to the center, the shear equals the quarter-point shear plus one-third of the difference, or as expressed on the design sheet, it equals two-thirds of the quarter-point shear plus one-third of the center shear.

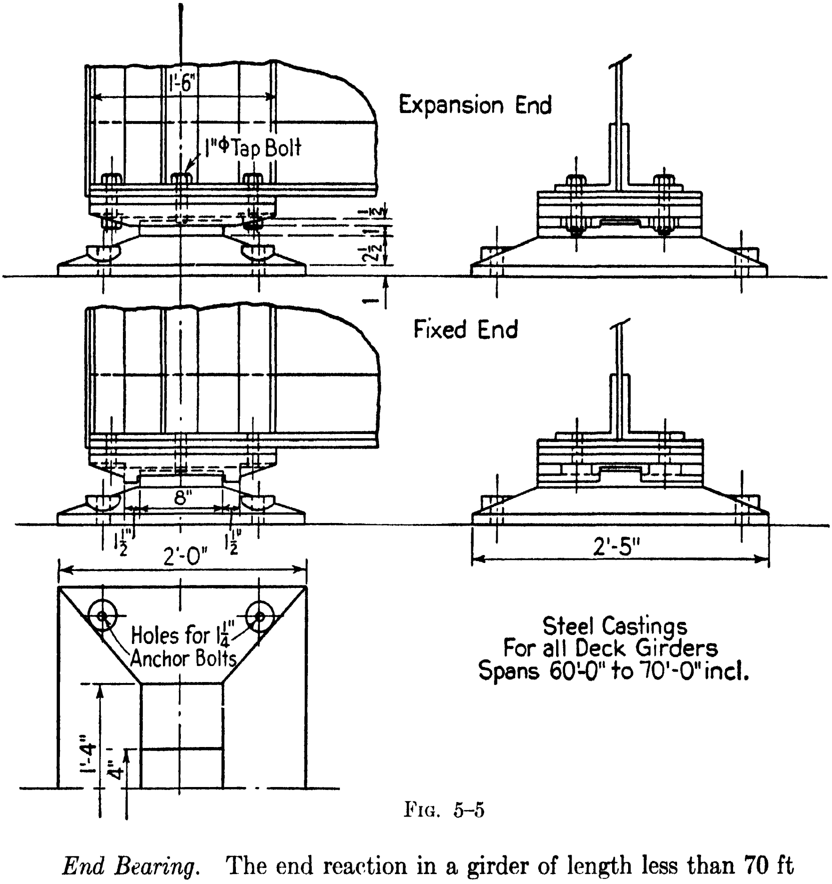


FIG. 5–5

End Bearing. The end reaction in a girder of length less than 70 ft may be distributed to the masonry by means of a bearing plate (see Fig. 5–2) or by means of castings. The kind of casting here used represents a common type. Another type for deck girders from 60 ft to 70 ft long is shown in Fig. 5–5 (P.R.R. standard). Bearings for heavier and longer spans are shown in Figs. 7–6 and 8–4. The most likely plane of weakness in the casting of sheet DG 8 is that marked Section A. The investigation shows low stresses at this section.

Drawing. Fig. 5–4 is a general drawing for the girder as designed.

Problem 5–4				
Span feet c.c. Bearings	Loading			
	E–55	E–60	E–65	E–70
48	a	f	k	p
52	b	g	l	q
56	c	h	—	r
60	d	i	n	s
64	e	j	o	t

Problem 5–4. Design and make a general drawing for a single track deck plate girder bridge with solid floor of the span and loading assigned.

5–6. Deck Plate Girder Railroad Bridge with Open Floor. Plate I shows a *shop drawing* (or *detail drawing*) for a single track deck plate girder bridge designed for Cooper E-55 loading. According to the specifications, this bridge might be completely shop riveted. Instead it has been detailed for shipment knocked down, the pieces being listed in the "required" list. In addition it will be seen that each minor piece of material has an "assembly mark" in lower-case letters. This is to prevent pieces being lost while being cut, punched, etc., prior to assembly. The mark painted on the piece would include not only the letters here shown but also the sheet number and the contract number. The system of marking used is to mark stiffener angles **s**, the first one detailed being **sa**, the second **sb**, etc. The letters commonly used for assembly marks are:[1]

- **a** For all angles not excepted hereinafter.
- **b** For seat angles used to support beams or girders that connect to girders or columns.
- **c** For cap plates, base plates, and splice plates on office-building columns.
- **d** For fillers with two or more lines of holes.
- **f** For fillers with single line of holes.
- **h** For bent angles or plates.
- **k** For stiffener angles or plates fitted at one end only, such as angles under beam seats or stiffener angles at column bases.
- **p** For all plates not excepted elsewhere.
- **s** For stiffener angles fitted at both ends.
- **t** For top angles tying beams or girders to girders or columns.
- **w** For web members and bracing.
- **y** For lattice bars.

[1] American Bridge Co. usage.

Other points to be noted in connection with the drawing are these:

The original, of which this is a reproduction, was drawn to the scale $\frac{3}{4}$ in. = 1 ft 0 in. This is the usual scale for *detail* drawings and should be used in all except the most unusual cases. If the first drawing is to be traced, shop rivets should be shown by a dash or a *free hand* circle. Also, at an early stage in the drawing, make sure the lines are heavy enough to be seen clearly through tracing cloth. There is no justification for the use of a pencil harder than a 3H; a softer one is better.

In general, on the original the first dimension line was at a distance of $\frac{5}{8}$ in. from the view with $\frac{5}{16}$-in. intervals between lines.

Dimensions were placed *on* the dimension lines. That is, the lines were *not* broken, differing therein from machine practice.

Dimensions under 12 in. were given in inches, 12 in. and over in feet and inches.

Gage line for angles may be made different from those given in the handbook ("Steel Construction," Second Edition, p. 134) if there is any advantage to be gained by so doing.

There is a complete absence of cross hatching.

Certain pieces on the drawing are marked *right* and *left*, for example, $\mathbf{sa}^R$, $\mathbf{sa}^L$. The difference between such pieces — gloves and shoes illustrate the point — should be clear without explanation. The one shown is generally marked *R*.

In drawing girder or truss bridges, structural practice is to show the *near* side of the *far* main member.

Plan views of bottom flanges, bottom chords, bottom lateral systems, etc., are taken from above looking down, that is, there are no worm's-eye views.

Problem 5–5				
Span feet	Loading			
	E–50	E–60	E–65	E–70
45	a	f	k	p
50	b	g	l	q
55	c	h	m	r
60	d	i	n	s
65	e	j	o	t

Problem 5–5. Design and make a shop drawing for an open floor single track deck plate girder railroad bridge of the span and loading assigned.

CHAPTER VI

HALF-THROUGH PLATE GIRDER RAILROAD BRIDGES

6-1. A *half-through*[1] plate girder bridge is one which has its floor system in (or near) the plane of the bottom flange of the girders. These girders are commonly joined by a bottom lateral system but are, of course, too shallow to permit the use of top laterals. The floor system may consist of a series of closely spaced floor beams, but the usual construction utilizes both floor beams and stringers.

The girders in a half-through structure are approximately equal in weight to the girders that would be used for a deck structure at the same site. Since the half-through structure requires the addition of a floor system it is heavier and consequently more expensive than the deck structure. The reason for its use is found in the fact that the distance from top of rail to under-clearance line (the lowest point in the steel superstructure) is less than when deck girders are used. Hence, assuming a fixed elevation for the rail, the half-through girder bridge will be found at stream crossings where navigation requirements or the location of the high-water line make a shallow floor system desirable, and over streets and roads where a fixed clearance is needed for traffic and further lowering of the roadway surface would cost more (on account of greater property damage, excavation, etc.) than the additional expense arising from the use of this type of structure. As in the deck bridge, either the solid or the open floor may be used.

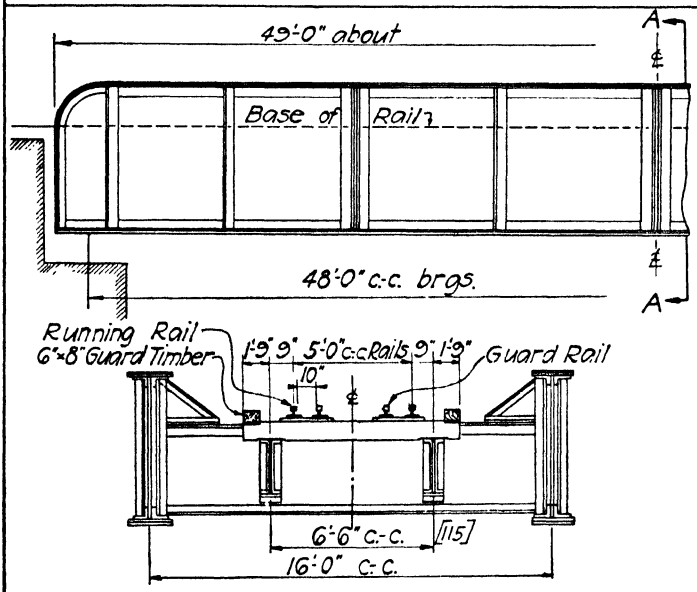
6-2. Half-Through Plate Girder Bridge with Open Floor. Sheets TG 1 to TG 15 and Plate II show the design computations and detail drawing for a single-track half-through plate girder railroad bridge.

Although the drawing shows built-up stringers and floor beams, many modern bridges make use of rolled shapes, and an alternate design has been made on that basis. The computation on sheet TG 3 for the end connection on the built-up stringer was made, keeping in mind the fact that the rivets which go through both the connection angles and the fillers may fail either in bearing on the girder web (since the web thickness is less than the combined thickness of the fillers) or in double shear on planes B, Fig. 6-1. Since the allowable load in bearing on $\frac{1}{2}$-in. metal is less than the double shear value, 10 rivets are needed in this group (12

[1] Sometimes called a *through* plate girder bridge.

Half-Through Plate Girder Railroad Bridge with Open Floor — *Design Sheet-TG1*

Data



SECTION A-A

Single-Track: Half-Through Plate Girder Railroad Bridge.

Span 48'-0" center to center of bearings.
Panels 4 of 12'-0" Stringers 6'-6" c.-c.
Girders 16'-0" c.-c.

Live Load: Cooper E-60

Specifications: A.R.E.A. 1931

Numbers in brackets refer to the A.R.E.A. Specifications.

Ties

Assume ties 8" wide × 10" deep × 10'-0" long, spaced 4" clear between ties, dapped to $9\frac{1}{2}$" over stringers. [24]

Weight — Tie $\frac{8\times10}{12}\times4.5\times10' = 300$ [timber @ $4\frac{1}{2}$#/F.B.M.]

Guards $2\times\frac{8\times6}{12}\times4.5\times1 = 36$

Rail, etc. $= 150$

Dead Load (one tie) $= 486$#

Dead Reaction (one tie) $= 243$#

Half-Through Plate Girder Railroad Bridge with Open Floor	Design Sheet-TG2
One Engine wheel (alternate loading) = 37,500# Live Load per tie = $\frac{37,500}{3}$ = 12,500# Impact [24] 100% = 12,500# L.L. + Impact = 25,000 D.L. < 1% of L.L. + Impact and therefor may be neglected Moment$_{(L.L.+Imp.)}$ = 25,000 × 9" = 225,000"# (See Sect. A-A) $S_{allowable}$ = 2000#/□" [24]; $\frac{1}{6}bh^2 = \frac{225,000}{2000} = 112.5$ $h^2 = \frac{112.5 \times 6}{8} = 84+$, $h = 9.2"$ (use 10") Try 8"×10" tie for Shear, $v = \frac{3}{2} \times \frac{V}{A} = \frac{3}{2} \times \frac{25,000}{8 \times 10} = 469$#/□" >180#/□" allow. However, shear is resisted by a 30" length, not 9" $\therefore v = 469 \times \frac{9}{30} = 140$#/□"	Ties (Continued) 8"×10" 10'0" long spaced 4" clear
Dead Load — Track and Ties 243#/'; Assume Stringer 125; 368#/' use 370 End Shear, 370 × 6 = 2,220# use 2,200# ¼-Pt Shear, 370 × 3 = 1,110# = 1,100# ℄ Shear, = 0 ℄ Moment, $\frac{370 \times 12^2}{8}$ = 6,660 use 6,700'# Summary	Stringer Shear and Moment
Depth commonly $\frac{1}{6}$ to $\frac{1}{7}$ of span length, will use 24" depth here. [38] Shear, $t = \frac{108,100}{10,000 \times 24} = 0.45"$ ∴ use Web 24 × ½ [120] $t \geq \frac{\sqrt{D}}{20} = \frac{\sqrt{24.5-12}}{20} = 0.18" < \frac{1}{2}"$ used ∴ O.K. (assuming 6" vertical angle legs)	Web 24 × ½

Summary

Loading	End Shear	Quarter-Point Shear	Center Shear	Moment
Live	53,100#	34,500#	18,750#	120,000'#
Impact	52,800 [28]	34,300	18,750	119,400
Dead	2,200	1,100	0	6,700
Total	108,100#	69,800#	37,500#	246,100'#

Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG3

Flange
4-L 6×4×½

Assume 6"×4" Flange Angles, Web ½"

[116] [48] $S_c = 16{,}000 - 150\,\frac{12\times12}{8.5} = 13{,}460\ \#/\square''$

$M_R = S(A_F + \frac{Web}{8})h$ Assume $h = 20.5''$

Compression Flange	Area	Tension Flange	Area
$\frac{246{,}100\times12}{20.5\times13{,}460} =$	10.70□"	$\frac{246{,}100\times12}{20.5\times16{,}000} =$	9.00□"
$\frac{1}{8}$ Web $= \frac{1}{8}\times24\times\frac{1}{2}$	1.50		1.50
Area required in Ls	9.20		7.50
2-L 6×4×½ =	9.50	−(2-1"φ out = 2×1×½) =	8.50
Excess	0.30	Excess	1.00

Check: $h = 24\frac{1}{2} - 2\times1.99 = 20.52''$ O.K.

Alternate Design
1-24 WF 93

Alternate Design

Try 24 WF 93 Flange-10" wide

$16{,}000 - 150\,\frac{12\times12}{10.0} = 13850\ \#/\square''$

Required Section Modulus $= \frac{246{,}100\times12}{13{,}850} = 213.2$

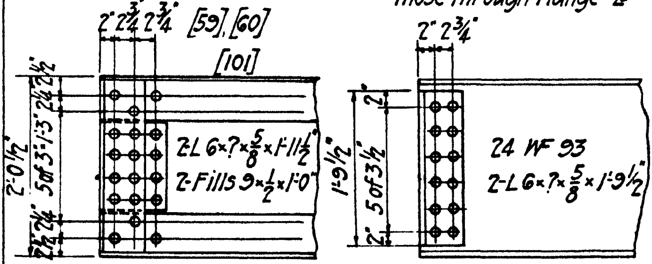
24 WF 93 = 224.7
Web = 0.481" O.K.

Stringer End Connection

[58] ⅞"φ rivet: double shear = 14,430# [38]
bearing on ½" web = 10,500# [38]

$\frac{108{,}100}{14{,}430} =$ 8 rivets thru Ls

$\frac{108{,}100}{10{,}500} =$ 10+ rivets thru web: may count those in filler, neglect if possible those through flange Ls

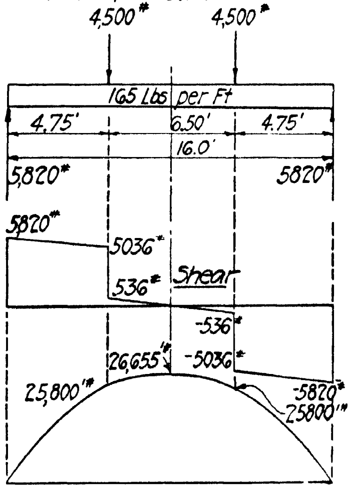


Half-Through Plate Girder Railroad Bridge with Open Floor	Design Sheet-TG4
Rivet Pitch Approximate Method $m = \frac{37{,}500 \text{ [20, Fig.3]}}{3 \times 12'' \text{(tie space)}} + 100\% \text{ impact} = 2{,}080 \text{\#/}''$ [24] [12] [Bearing value: $\frac{1}{2}$" web] End pitch, $p = \frac{10{,}500}{\sqrt{\left(\frac{108{,}100}{20.5}\right)^2 + (2{,}080)^2}} = \frac{10{,}500}{5670} = 1.85''$ $\frac{1}{4}$-Pt. pitch, $p = \frac{10{,}500}{\sqrt{\left(\frac{69{,}800}{20.5}\right)^2 + (2{,}080)^2}} = \frac{10{,}500}{3990} = 2.63''$ ℄ pitch, $p = \frac{10{,}500}{\sqrt{\left(\frac{37{,}500}{20.5}\right)^2 + (2{,}080)^2}} = \frac{10{,}500}{2770} = 3.79''$	Stringer Rivet Pitch End, 1.85" $\frac{1}{4}$-Pt., 2.63" ℄, 3.79"
Exact End Pitch for Comparison: Increase in flange stress per inch $= \frac{108{,}100}{20.5} \times \frac{9.5}{9.5 + 1.5} = 4{,}550 \text{\#/}$ $p = \frac{10{,}500}{\sqrt{(4{,}550)^2 + (2{,}080)^2}} = \frac{10{,}500}{5{,}010} = 2.10''$	Exact End $p = 2.10''$
Stringer Web Stiffeners : [12G] $24\frac{1}{2}'' - 12'' = 12\frac{1}{2}'' < 50 \times \frac{1}{2}''$ ∴ no stiffeners are needed	Stiffeners none reqd.
Weights 1 - Web $24 \times \frac{1}{2}$ = 40.8 #/' 4 - L $6 \times 4 \times \frac{1}{2}$ = 64.8 Assume Average of 5 rivets per foot, top and bottom = 4.8 110.4 #/' Main Material = 110.4×12 = 1,325 # End Conn. { 4-L $6 \times 6 \times \frac{5}{8} \times 1'\text{-}11\frac{1}{2}''$ = 190 4-Pl. $9 \times \frac{1}{2} \times 1'\text{-}0''$ = 61 48 rivet heads = 12 } 1,590 # **Check Dead Load Shears and Moment** *End Shear* Track: $243^{\#} \times 12 \times \frac{1}{2}$ = 1,458 # Stringer $1590 \div 2$ = 795 End Shear 2,253 # Assumed 2,200 Excess 53 # O.K. *℄ Moment* Track: $\frac{243 \times 12^2}{8}$ = 4,370 '# Stringer $1325 \times 12 \times \frac{1}{8}$ = 1,988 " $\frac{1590 - 1325}{2} \times \frac{4}{12}$ = 43 6,401 '# Assumed 6,700 Excess 299 # O.K.	Check on Dead Weight

Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG5

Floor Beam

Moments & Shears



Moment

Dead End Shear

Assume 165 #/' dead load
Floor Load 486 #/' (TG1)

$486 \times 12 = 5832^{\#}$
2 Stringers = 3168
$9000^{\#}$

End Reaction
$= 9000 \times 1/2 = 4500^{\#}$
$165 \times 8' = 1320$
$R = 5820^{\#}$
use $5800^{\#}$

Dead Center-Line Moment

℄ Moment
$\frac{1}{8} \times 165 \times \overline{16}^2 = 5280'^{\#}$
$4500 \times 4.75 = 21,375$
$M = 26,655'^{\#}$
use $26,700'^{\#}$

Summary Moments and Shears

Summary Loading	End Shear	Moment at Center Line
Live	70,000 #	332,500'#
Impact	68,700 [28]	326,000
Dead	5,800	26,700
Total	144,500 #	685,200 '#

1-Web 40 × 3/8 × 7/16 See Sheet № TG6

Depth $= 6" + \frac{1}{2}" + 2'\text{-}0\frac{1}{2}" + 3" + \frac{1}{2}" + 6" = 3'\text{-}4\frac{1}{2}"$ (See sketch, Sheet TG8)

Web: $t = \frac{144,500}{10,000 \times 40} = 0.361"$ [38]

$t \mathrel{\overset{>}{=}} \frac{\sqrt{D}}{20} = \frac{\sqrt{28.5}}{20} = 0.267$ [20] Use Web $40" \times \cancel{\frac{3}{8}}\ \frac{7}{16}"$

Flange

Flange: [116], (use 4" outstanding legs to keep floor beam width small, - no cover plates)

$S_c = 16,000 - 150 \frac{6.5 \times 12}{\cancel{8\frac{3}{8}}\ 8\frac{7}{16}} = \cancel{14600}\ \#/\square"$ [48]
14612 allowable

$M_R = s\left(A_F + \frac{Web}{8}\right)h$

Assume $h = 36.3"$

Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG6

Flange (continued)

Flange Area $M_R = Fh = s\left(A_F + \frac{Web}{8}\right)h$

Compression	Area □"	Tension	Area □"
$\frac{685{,}200 \times 12}{\cancel{14{,}600} \times 36.3} =$ 14,612	15.5~~3~~1	$\frac{685{,}200 \times 12}{16{,}000 \times 36.3} =$	14.15
$\frac{1}{8}$ of Web $= \frac{1}{8} \times 40 \times \cancel{\frac{3}{8}}\frac{7}{16} =$	~~1.88~~ 2.18		~~1.88~~ 2.18
	~~13.65~~ 13.33		~~12.27~~ 11.97
2-L 6×4×3/4 =	13.88	−(2-1"φ out − 2×1×3/4) −	12.38
excess	0.~~23~~ 0.55	excess	~~0.11~~ 0.41

Check, $h = 40.5" - 2 \times 2.08" = 36.34"$ O.K.

End rivet pitch, 7/8" rivets [58]

value [38], bearing on ~~3/8~~ 7/16" web = ~~7880~~ 9190 #/rivet

double shear = 14,430 #/rivet

Flange stress increment $\frac{144{,}500}{36.34} = 3980$ #/"

$p = \frac{\cancel{7880}\ 9190}{3980} = \cancel{1.98"}\ 2.31"$

same, - flg. rivet pitch allowing for web equivalent

Tension flange, $p = \cancel{1.98}\ 2.31 \times \frac{12.38 + \cancel{1.88}\ 2.18}{12.38} = \cancel{2.28}\ 2.72"$

Stiffeners — Unsupported web depth, [125]

$= 40.5" - 2 \times 6" = 28.5"$

$50 \times 3/8" = 18.75"$ [126]

needed. Spacing is

(a) 6'

(b) 40"

(c) $\frac{3/8}{40}(12{,}000 - 9{,}600) = 22.5"$

too close, increase web to 7/16

(c) $\frac{7/16}{40}\left(12000 - \frac{144500}{40 \times 7/16}\right) = 40.9"$ O.K.

Note changes above incident to this increase of web thickness

---

Floor Beam (Continued)

Flange 4-L 6×4×3/4 4" leg O.S. d = 40½"

End rivet pitch = ~~1.98"~~ 2.31"

Exact, p = ~~2.28"~~ 2.72"

Stiffeners 2-L 3½×3½×3/8 40.9" c-c. [128]

Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG 7

Stringer Connection

Stringer Connection:

(1) End reaction, one stringer = (TG1) (1) 108,100#

(2) Dead reaction, 2 stringers = 4500# (TG5)

L.L. floor beam reaction = 70,000#

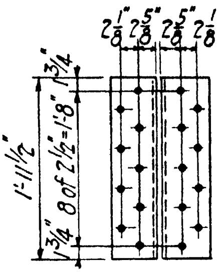
Impact = 68,700

143,200 (2) 143,200

(1) 7/8" rivet, single shear = 7220#, $\frac{108,100}{7,220}$ = 15 rivets

(2) 7/8" rivet, bearing 7/16" web = 9,190#, $\frac{143,200}{9,190}$ = 16 rivets

Rivets ← governs



2-L 6 × 6 × 5/8 × 1'-11 1/2"

← **Angles**

Floor Beam Alternate

Alternate Design Depth = 2" + 24" + 3" + 2" = 31" minimum

Try 36" $f_c = 16,000 - 150\,\frac{12 \times 6.5}{12}$

= 15,025#/□" allowable

$\frac{685,200 \times 12}{15,025}$ = 547 in^3 required S

Use 36 WF 167, S = 569, web 0.67" O.K.

36 WF 167

Alternate Stringer End Connection

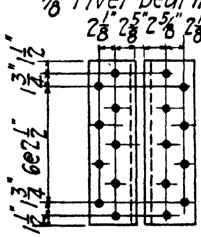
Alternate Stringer Connection:

7/8" rivet bearing on 0.67" web = 0.875 × 0.67 × 24,000 = 14,060#

$\frac{108,100}{7220}$ = 15 rivets in single shear ← governs

$\frac{143,200}{14,060}$ = 11 rivets in bearing

Rivets ←



2-L 6 × 6 × 5/8 × 1'-9 1/2"

Angles ←

Floor Beam End Connection

Floor Beam End Connection to Girder

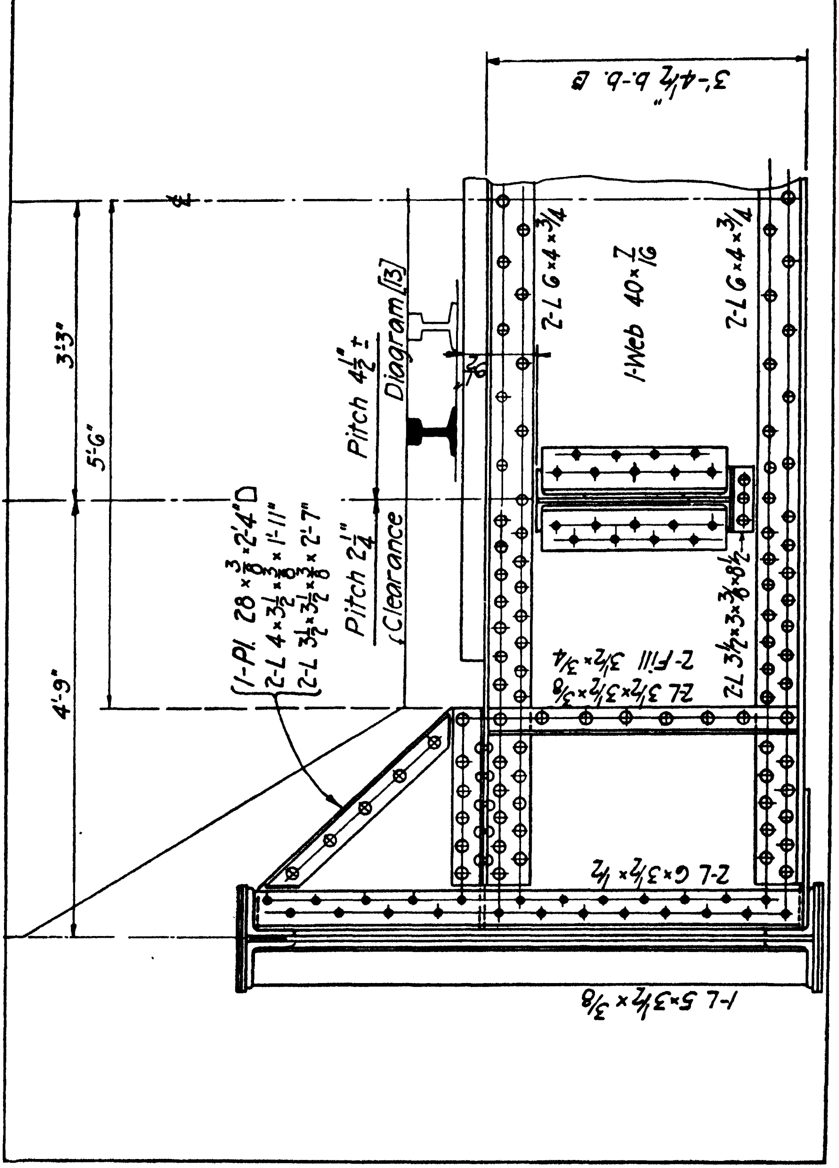
End Reaction = 144,500#

7/8" φ rivet, Bearing on 7/16", 9190#

$\frac{144,500}{9190}$ = 16 rivets in bearing $\frac{144,500}{7220}$ = 20 rivets in single shear

Half-Through Plate Girder Railroad Bridge with Open Floor — *Design Sheet-TG8*

Floor Beam



Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG9

Floor Beam — *Floor Beam: Check of Weight*

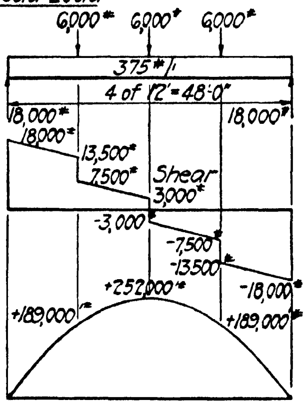
Weight	Lbs.
1-Web $40 \times \frac{7}{16} \times 15'\text{-}9\frac{3}{4}''$	= 940
4-L $6 \times 4 \times \frac{3}{4} \times 15'\text{-}0''$	= 1,416
4-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8} \times 3'\text{-}3''$	= 113
4-Fills $3\frac{1}{2} \times \frac{3}{4} \times 2'\text{-}4''$	= 83
4-L $3\frac{1}{2} \times 3 \times \frac{3}{8} \times 0'\text{-}8\frac{1}{2}''$	= 22
2-Pl. $28 \times \frac{3}{8} \times 2'\text{-}4''$ ◸	= 165
4-L $4 \times 3\frac{1}{2} \times \frac{3}{8} \times 1'\text{-}11''$	= 70
4-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8} \times 2'\text{-}7''$	= 88
Rivet heads 200 rivets ±	= 93
	2,990#

Assumed weight = 2,640#

$\frac{2990-2640}{2} = +175\#$, end reaction error, no revision necessary.

End Reaction = 5,820 + 175 = 5,995

Dead Load — *Girder: Dead Load Moments & Shear*



Moment

Dead End Shear

Panel Load 5995#
use 6000#

375 × 24 = 9,000#
6000 × 3 × 1/2 = 9,000
End R = 18,000#

Dead Shear at End = 18,000#
" " 1/4 Pt = 13,500#
" " ℄ 3,000#

Dead Load Moment

18,000 × 12 = 216,000'#
-375 × 12 × 6 = - 27,000
1/4-Pt. Mon = 189,000'#

$12 \times \frac{7500+3000}{2} = +63,000'\#$

℄ Mom. 252,000'#

Summary — *Summary Moments & Shears*

	Panel Shear		Moment	
Loading	End	Intermediate	Quarter Pt.	Center Line
Live	85,800	41,300	1,030,000	1,331,000
Impact	79,700	39,900	956,000	1,237,000
Dead	18,000	7,500	189,000	252,000
Total	183,500 Lbs.	88,700 Lbs.	2,175,000 Ft.Lb.	2,820,000 Ft.Lb.

Half-Through Plate Girder Railroad Bridge with Open Floor

Design Sheet-TG 10

Web Depth = 72"

End Shear 183,500#

$$\frac{183{,}500}{10{,}000} = 18.35\ \square'' \text{ required gross area [38]}$$

$$\frac{18.35}{72} = 0.26'' \qquad t \geq \frac{\sqrt{D}}{20} \qquad [120]$$

$$= \frac{\sqrt{60.5}}{20} = 0.389''$$

Use Web $72 \times \frac{7}{16}$

Web $72 \times \frac{7}{16}$

Flanges

$$S_t = 16{,}000\ \#/\square'' \ [38] \qquad S_c = 16{,}000 - 150\frac{l}{b} \ [48]$$

$$= 16{,}000 - 150 \times \frac{12 \times 12}{14}$$

$$= 14{,}460\ \#/\square''$$

$$M_R = s\left(A_F + \frac{Web}{8}\right)h$$

Assume $h = 71.6''$

Flange Area

Compression	Area □"	Tension	Area □"
$\frac{2{,}820{,}000 \times 12}{14{,}460 \times 71.6}$ =	32.72	$\frac{2{,}820{,}000 \times 12}{16{,}000 \times 71.6}$ =	29.56
$Web = \frac{1}{8} \times 72 \times \frac{7}{16}$ =	3.94		3.94
	28.78		25.62
2-L $6 \times 6 \times \frac{5}{8}$ =	14.22	$-(4-1''\phi \text{ out} = 4 \times 1 \times \frac{5}{8} =)$	11.72
	14.56		13.90
1-Pl. $14 \times \frac{5}{8}$, 1-Pl. $14 \times \frac{9}{16}$ =	16.62	$-(2-1''\phi \text{ out} = 2 \times 1 \times 1\frac{3}{16}'' =)$	14.25
Excess =	2.06	Excess =	0.35

Flange 4-L $6 \times 6 \times \frac{5}{8}$

2-Pl. $14 \times \frac{5}{8}$

2-Pl. $14 \times \frac{9}{16}$

[118] [119]

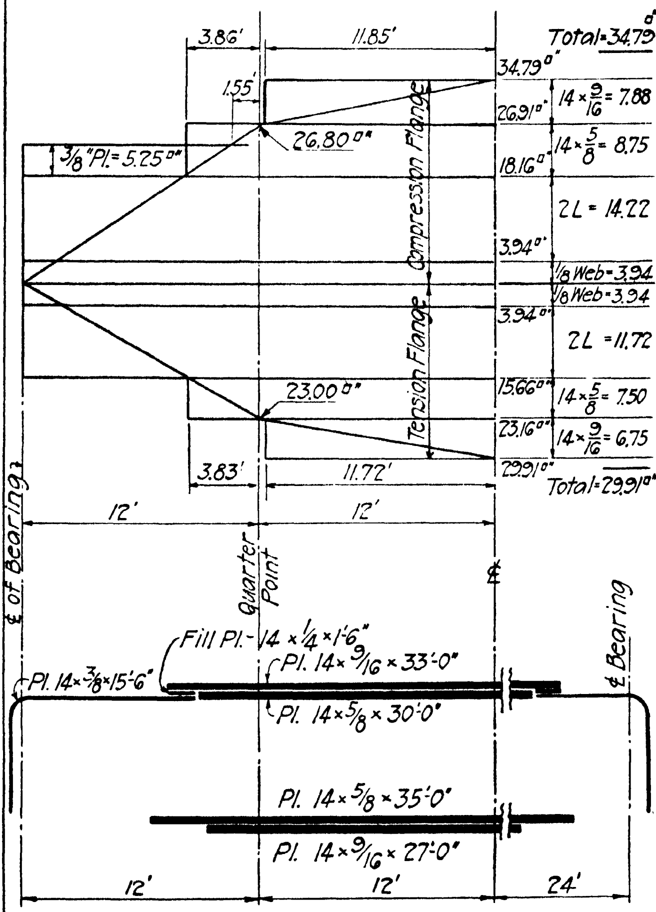
Check h: (taking moments about angle centroid)

$$\frac{16.62(1.73 + 0.59)}{14.22 + 16.62} = 1.25''$$

$$h = 72.5 - 2(1.73 - 1.25) = 71.54'' \quad \text{O.K}$$

Half-Through Plate Girder Railroad Bridge with Open Floor

Design Sheet-TG 11



Girder Cover Plates

- 1-Pl. 14 × 9/16 × 27' bottom
- 1-Pl. 14 × 9/16 × 33' top
- 1-Pl. 14 × 5/8 × 35' bottom
- 2-Pl. 14 × 3/8 × 15'-6" bent, top
- 2-Fill Pl. 14 × 1/4 × 1'-6" top

Area required at 1/4 Pt., $34.79 \times \frac{2{,}175{,}000}{2{,}820{,}000} = 26.80\,\square''$ compr.

$29.91 \times \frac{2{,}175{,}000}{2{,}820{,}000} = 23.00\,\square''$ tens.

Cut off 5/8" pl. $12 \times \frac{26.80 - 18.16}{26.80} = 3.86'$ from 1/4 Pt. Compr.

$12 \times \frac{23.00 - 15.66}{23.00} = 3.83'$ " " " Tens.

Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG12

[125]

(a) - 6'-0"

(b) - $72\frac{1}{2}-12" = 60\frac{1}{2}"$

(c) - $d = \frac{7/16}{40}\left(12{,}000 - \frac{183{,}500}{72 \times 7/16}\right) = 67\frac{1}{2}"$ (End Panel)

Girder Stiffeners

End Panel 2 Pairs

Inter. Panel 2 Pairs

2-L $5 \times 3\frac{1}{2} \times \frac{3}{8} \times 5'\text{-}11\frac{1}{4}"$

Flange Rivet Pitch:

End Panel, $p = \frac{9190 \times 71.54}{183{,}500} = 3.58"$

Intermediate Panel,

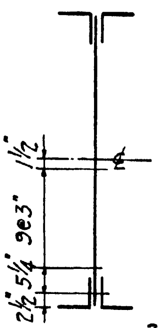
$p = \frac{9190 \times 71.54}{87{,}800} = 7.58" > 6"$ [59]

Rivet Pitch

End Panel 3.58"

Int. Panel 6.00"

Web Splice



d	d²
1½	2.25
4½	20.25
7½	56.25
10½	110.25
13½	182.25
16½	272.25
19½	380.25
22½	506.25
25½	650.25
28½	812.25

$\Sigma = 2992.5$

$2992.5 \times 2 = 5985$

$S = \frac{Mc}{I} = \frac{M}{\frac{1}{6}bh^2}$

$I_{net} = \pm \frac{3}{4} I_{gross}$

Net Area of Web $= (72-22)\frac{7}{16} = 21.9\,\square"$

Assume 2-3/8" splice pls.

net area $= 2(60-20)\frac{3}{8} = 30\,\square"$

∴ OK. for shear

Moment carried by web

$= 3.94 \times 71.54 \times 16{,}000 = 4{,}510{,}000"\#$

$S_{spl.\,pl.} = \left(\frac{4}{3}\right) \times \frac{4{,}510{,}000}{\frac{1}{6} \times \frac{3}{4} \times 60.5^2} = 13{,}140\,\#/\square"$ O.K.

$S_{allowable} = \frac{30.25}{36.25} \times 16{,}000 = 13{,}360\,\#/\square"$

Assume one row of rivets each side of splice

Shear $= \frac{88{,}700}{20} = 4435\#$ per rivet

Torsion $= \frac{4{,}510{,}000 \times 28.5}{5985}$

$= 21{,}500\#$

Load on extreme rivet $= \left[(21{,}500)^2 + (4{,}435)^2\right]^{1/2} = 21{,}950\#$

Required: $\frac{\text{Load Actual}}{\text{Load Allowable}} = \frac{21{,}950}{9190} = 2.39$ rows

9190 ← bearing value

∴ Use 3 rows each side of splice

Web Splice

2-Pl. $18 \times \frac{3}{8} \times 5'\text{-}0\frac{1}{2}"$

[123]

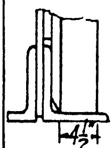
Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG 13

End Stiffeners

End Shear = 183,500#

Assume 2/3 on inner pair

$183{,}500 \times 2/3 = 122{,}300\#$



$\frac{122{,}300}{24{,}000} = 5.10\,\square''$ required bearing area [38]

$\frac{5.10}{2 \times 4.5} = 0.568''$

∴ Use 2-L $5 \times 3\frac{1}{2} \times \frac{9}{16}$

bent pair 2-L $5 \times 3\frac{1}{2} \times \frac{3}{8}$

Rivets in bearing on web: in double shear

$\frac{122{,}300}{9190} = 14$ rivets total $\qquad \frac{122{,}300}{2 \times 7200} = 9$ rivets thru Ls

Girder End Stiffeners: 2-L $5 \times 3\frac{1}{2} \times \frac{9}{16} \times 5'\text{-}11\frac{1}{4}''$; 2-L $5 \times 3\frac{1}{2} \times \frac{3}{8} \times 6'\text{-}5\frac{1}{2}''$ bent and crimped

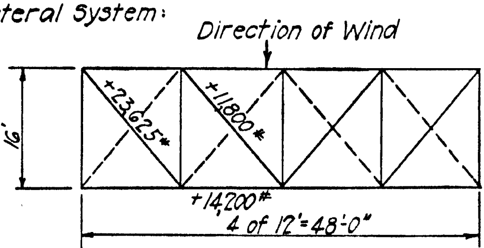
1-Web $72 \times \frac{7}{16} \times 49'\text{-}0''$	5250#
2-L $6 \times 6 \times \frac{5}{8} \times 49'\text{-}0''$	2373
2-Ls $6 \times 6 \times \frac{5}{8} \times 47'\text{-}0''$	2275
2-Pl. $14 \times \frac{3}{8} \times 15'\text{-}6''$	554
1-Pl. $14 \times \frac{9}{16} \times 27'\text{-}0''$	724
1-Pl. $14 \times \frac{9}{16} \times 33'\text{-}0''$	883
1-Pl. $14 \times \frac{5}{8} \times 30'\text{-}0''$	892
1-Pl. $14 \times \frac{5}{8} \times 35'\text{-}0''$	1,040
19-L $5 \times 3\frac{1}{2} \times \frac{3}{8} \times 5'\text{-}11\frac{1}{4}''$	1,112
6-L $6 \times 3\frac{1}{2} \times \frac{1}{2} \times 5'\text{-}11\frac{1}{4}''$	545
23-Fill $3\frac{1}{2} \times \frac{5}{8} \times 5'\text{-}0''$	854
4-L $5 \times 3\frac{1}{2} \times \frac{9}{16} \times 5'\text{-}11\frac{1}{4}''$	361
4-L $5 \times 3\frac{1}{2} \times \frac{3}{8} \times 6'\text{-}0''\pm$ bent	250
4-Pl. $18 \times \frac{3}{8} \times 5'\text{-}0''$	460
4-Fill $3\frac{1}{2} \times \frac{1}{4} \times 5'\text{-}0''$	60
2-Pl. $14 \times \frac{1}{4} \times 1'\text{-}6''$	36
Rivets	600 ±
	18,209

$\frac{18209}{49} = 372\#/' < 375\#/'$ assumed

Girder Weight 372#/'

Half-Through Plate Girder Railroad Bridge with Open Floor — Design Sheet-TG 14

Lateral System:



Wind on Girder = 350 [32]
" " Train = 300 [32]
Sway 400 [33]
1,050#/ft.

1050 × 12 = 12,600#/panel moving load

Stress in end diagonal = $12{,}600 \times \frac{3}{2} \times \frac{5}{4} = +23{,}625^{\#}$

Stress in Intermediate diagonal = $12{,}600 \times \frac{3}{4} \times \frac{5}{4} = +11{,}800^{\#}$

Chord Stress, max. = $12{,}600 \times \frac{3}{2} \times \frac{3}{4} = +14{,}200^{\#}$

Area of Diagonals

$\frac{23{,}625}{16{,}000} = 1.48^{\square''}$

1-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$, $\frac{3}{8}''(3\frac{1}{2} - 1 + \frac{1}{2} \times 3\frac{1}{8}) = 1.53^{\square''}$ [54]

O.K. use throughout

Value of 1-L $1.53 \times 16{,}000 = 24{,}500^{\#}$

$\frac{24{,}500}{7{,}200}$ = 3+ rivets in single shear to develop 1-L

Use 4 rivets in End Panel
" 3 " " Intermediate Panel

Laterals: 1-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$

Increased Stress in Tension Flange

Wind 14,200#

Tractive effort $\frac{6000 \times 36 \times 0.10}{2}$ = 10,800 [37]

25,000#

$\frac{25000}{29.91} = 836^{\#}/\square''$; Total = $16{,}836^{\#}/\square'' < 20{,}000^{\#}/\square''$ [46]

Stringer End Stiffeners:

Assume ½ stringer reaction to each pair

$\frac{108{,}100}{24{,}000} \times \frac{1}{2} = 2.25^{\square''}$

use 2-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$ each pair

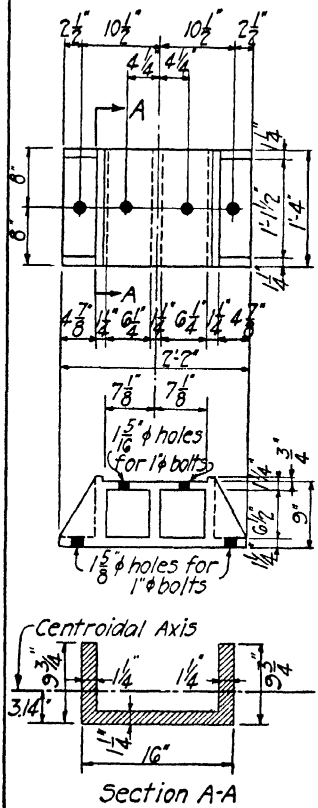
⅞" ϕ rivets bearing on ½" web = 10,500#/rivet

$\frac{108{,}100}{10{,}500} \times \frac{1}{2}$ = 5+ rivets in each pair

Stringer End Stiffeners: 2-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$ each pair 5+ rivets

Half-Through Plate Girder Railroad Bridge with Open Floor	Design Sheet-TG 15

Girder End Pedestal



↑ Vertical End Reaction = 183,500#

→ Wind End Reaction = 12,600#

→ Tractive Effort = 10,800#

Stresses

Bearing,

Steel, $s_b = \dfrac{183{,}500}{16\times 1\frac{1}{4}\times 3} = 3060$ #/□" OK

Concrete

$s_b = \dfrac{183{,}500}{16\times 26} = 440$ #/□" O.K.

Shear,

Longitudinal

$s_s = \dfrac{10800}{3\times 16\times 1\frac{1}{4}} = 180$ #/□" O.K.

Transverse

$s_s = \dfrac{12{,}600}{3\times 16\times 1\frac{1}{4}} = 210$ #/□" OK

Bending at A-A

Location of neutral axis (centroid)

Area × Arm = Moment

$2\times 9\frac{3}{4}\times 1\frac{1}{4} = 24.35 \times 9\frac{3}{4}\times\frac{1}{2} = 118.90$

$13\frac{1}{2}\times 1\frac{1}{4} = 16.88 \times 1\frac{1}{4}\times\frac{1}{2} = 10.55$

$41.23 \qquad 129.45$

$\bar{y} = \dfrac{129.45}{41.23} = 3.14"$

I about neutral axis

$\frac{1}{12}\times 1\frac{1}{4}\times\left(9\frac{3}{4}\right)^3\times 2 = 198.0$

$24.35\times(1.74)^2 = 73.7$

$\frac{1}{12}\times 13.5\times\left(1\frac{1}{4}\right)^3 = 2.2$

$16.88\times(2.51)^2 = 106.2$

380.1 in.^4

$M = \frac{1}{2}\times 440\times\left(4\frac{7}{8}\right)^2\times 16 = 83{,}400$ '#

$s_c = \dfrac{83{,}400\times(9.75-3.14)}{380.1}$

$= 1450$ #/□" O.K.

are shown in the sketch). Similarly, the rivets on planes A may fail either by double shear or by bearing on $2 \times \frac{5}{8} = 1.25$-in. metal. The allowable load in double shear is less, and 8 rivets are both needed and used. In a connection of this kind it is common to neglect eccentricity.

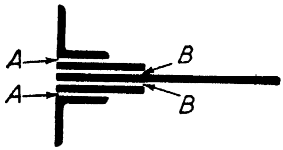


FIG. 6–1

In the design of the floor beam a web depth of 40 in. was used. Many shops carry their stock material in this range of widths in multiples of 6 in. and a 42-in. plate might better be used if conditions permitted the under-clearance line to be lowered the necessary 2 in.

It is seen, sheet TG 11, that $14 \times \frac{3}{8}$ in. plates were used for top flange cover plates over the bearings and carried down the ends of the girder. This requires a cover plate splice, but is done because the shop finds it easy to bend a relatively short plate and especially difficult to space properly two bends in a single plate. The stress which passes into the $\frac{9}{16}$-in. plate at the splice may be assumed to remain in that plate, and the $\frac{5}{8}$-in. plate may be considered as the added plate which begins at the end of the $\frac{3}{8}$-in. plate.

In figuring the required spacing for the intermediate stiffeners on the girders, it was found, sheet TG 12, that the greatest distance allowable was $60\frac{1}{2}$ in. This meant that two pairs must be used in each panel, which worked out well in connection with the girder web splice. A web splice is usually placed at a pair of stiffeners, and the arrangement adopted allowed three plates of closely equal lengths to be used for the web.

The handbook indicates that it is possible to get a single plate $72 \times \frac{7}{16} \times 49$ ft 0 in. In spite of this, a web splice is shown since it might be convenient to purchase the material from a mill that was unable to furnish the 49-ft length and also since some fabricating shops prefer to splice a plate of this thickness, the weight of which exceeds 3000 lb.

On sheet TG 14 the increase in tension flange stress due to lateral and longitudinal forces has been computed. Generally this may be neglected as the allowable increase of [**46**] prevents its influencing the design.

On Plate II it will be seen that both the girder web and the floor beam web were treated as though their thickness were $\frac{1}{2}$ in. This was done to keep thirty-seconds of inches out of the dimensions, and does not

introduce inaccuracies in excess of the usual tolerances. Instead of the assembled drawing of Plate II it is customary in practice to make separate detail drawings for individual beams, girders, etc. This alternative practice is illustrated in the drawings of Chapter X for an office-building frame.

Problem 6–1. Make a drawing similar to that on Sheet TG 8 using the alternate rolled beam stringer and floor beam.

Problem 6–2. Design and make a detail drawing for the span and loading assigned by the instructor.

Suggested spans: 40, 42, 44, 46, 48, 50, and 52 ft.

Suggested loadings: Special A, B, C, D, E (Fig. 6–2); Cooper E–55, E–60, E–65, E–70.

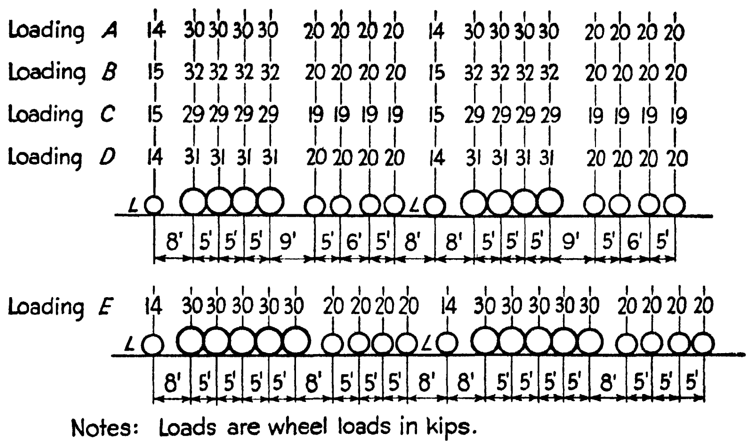


Fig. 6–2

(*Note.* The 48-ft span and E–60 loading have been used in Art. 6–2.)

CHAPTER VII

THROUGH RIVETED TRUSS HIGHWAY BRIDGE

7–1. Trusses replace plate girders in bridge construction primarily when the span length becomes too large for the economical use of the girder. Some of the other considerations which the engineer must keep in mind in deciding whether to use truss or girder are the limitations imposed by shipment, the method and cost of erection, the spacing and cross-bracing of the main carrying members, the stiffness of the structure and its liability to vibration on passage of load, ability of the structure to stand up well under over-stress, liability to accident, ease of maintenance, deterioration in use, and ease of inspection. Beyond the question as to what type of main member to use for a given short span is the broad problem of the determination of the most economical, efficient, and sightly structure for any crossing. In addition to the lesser considerations which have been enumerated, the engineer comes to a decision whether to build a suspension bridge or cantilever, arch or simple bridge, to use steel or reinforced concrete, by study of: foundation conditions, the length or lengths of span imposed, required over- and under-clearance, the desirable elevation of the roadway, the sort of live load to be carried, first cost, and the cost of operation and maintenance. These are matters where decision is possible only on the basis of extensive experience, and the most cursory of attention only is usually given them in undergraduate instruction. The student should consult Dr. J. A. L. WADDELL's monumental works, "Bridge Engineering" and "Economics of Bridgework,"[1] for discussion of the reasons determining the type of structure in any given situation.

The span length usually given as that marking the upper limit of plate girder use and the lower limit for trusses for bridges is 100 ft with a 20 or 30 per cent or more increase possible depending on local conditions at the bridge site and the variation of individual judgment

[1] Published by John Wiley & Sons, New York, 1916 and 1921.

A valuable summary of the problem is given by Dr. WADDELL in the *Journal*, Western Society of Engineers, October, 1927, "Suitability of the Various Types of Bridges for the Different Conditions Encountered at Crossings." The student should consult also "Structural Design in Steel," SHEDD (Wiley, 1934), Chapter VII; and "Analysis and Design of Steel Structures," FULLER and KEREKES (Van Nostrand, 1936), Chapter VIII.

in estimating the requirements imposed by the various considerations above enumerated.[1]

Shorter trusses are often used, generally as pony trusses, that is, as a half-through structure with no transverse bracing connecting the two top chords. It is impossible to estimate accurately the strength of an unsupported top chord, and best practice forbids the use of the pony truss unless the top chord is stiffened by bracket extensions on the floor beams.

Modern practice tends to the use of riveted trusses for long as well as for short spans on account of the greater stiffness of a riveted truss over that of one with eye bars and pin connections. The economy of pin connections for long spans has largely disappeared with advances in shop practice and with the necessity of using stiff pieces instead of eye bars for certain members carrying tensile stress. Today, in any case, one would hardly use a pin structure for a highway span under 250 or 300 ft.

The manufacture of the modern wide-flanged rolled section has made possible considerable economy in truss construction, and these sections are increasingly used as truss members.

7–2. Floor Types. The floor of a highway bridge consists of a wearing surface resting on a structural slab which is supported by a system of beams supported in turn by the main members. The slab and beams are usually made of wood, steel, or reinforced concrete. Wood floors are very unsatisfactory for modern heavy loads, owing to their lack of stiffness, the softness of wood, and to the present-day difficulty of securing plank and timber of suitable quality. The most common floor for small steel bridges consists of a reinforced-concrete slab resting on longitudinal steel stringers, which are supported by transverse steel floor beams spanning between the main members at panel points. To provide a wearing surface, the structural slab may be made $\frac{3}{4}$ in. or more thicker than required for strength, the top addition being made rich and dense, or a separately placed layer of rich concrete may be used, 3 in. or so in thickness. For large bridges with wide roadways, the lightest and simplest support for the slab consists of transverse joists resting on longitudinal stringers which are carried by the panel point floor beams. The wearing surface is often a bitulithic or asphalt pavement $\frac{3}{4}$ in. thick, placed directly on the structural slab, or a wood block pavement, in this country laid with a $\frac{1}{2}$- to 1-in. sand cushion on the concrete, in Europe often laid directly on the carefully surfaced slab. Brick, stone block, and thick asphalt block pavements with sand cushions are not favored for bridges on ac-

[1] For a description of a 5-span continuous plate girder bridge with a center span of 217 ft., see *Engineering News-Record*, May 19, 1938, pp. 707 and 725.

count of their weight. Recently, thin asphalt blocks (1 in. thick) have been developed, cemented directly to a steel plate floor.

Floors made with buckle plates[1] have been common but are going out of use because of their weight, cost, and a lack of rigidity which, with heavy loads, permits the breaking up of the block pavement common in this construction. Buckle plates are so called from the dome-like indentations made by a die in the originally flat plate, the purpose of which is to supply stiffness. The plates are riveted to the flanges, top or bottom, of the supporting beams, more efficiently, except as to amount of filler material, with the buckles turned down so that the plates are in tension. Concrete is poured on the plates and forms an inert filler on which the pavement is laid.

The perpetual search for lighter bridge floors has led to several notable new types, three of which are especially to be noted. The battle-deck floor (so called from its resemblance to naval construction) consists of a series of rolled steel beams spaced usually 24 in. on centers, with longitudinal steel plates $\frac{1}{4}$ in. narrower than the beam spacing, laid along the tops of the beams and welded to them by an automatic electric welding machine which lays a continuous weld along the center line of the top of each beam in the $\frac{1}{4}$-in. gap between adjoining plates. The plates act both to carry the load transversely to the beams and longitudinally as extensions to the compression flanges of the beams. The strength of the resulting tee beam may be computed on the assumption that the width of the top flange approaches the beam spacing.[2] The economy of this floor comes from the fact that, due to the distributing action of the plates, the beam directly under a wheel carries only a little more than one-half of the wheel load.

Structural shapes (angles, I- and H-sections, channels, tees and zees) made of aluminum alloys have been developed recently and may be used for bridge floors along exactly the same lines as steel and with great savings of weight. Aluminum structural alloys weigh about 35 per cent as much as steel, but weight cannot be saved in this proportion since somewhat lower fiber stresses must be used than for steel and since due regard must be paid to the stiffness of the construction, the modulus of elasticity for aluminum alloy being about one-third that of steel. The first notable example of the use of aluminum for a bridge floor was in the reconstruction of the Smithfield Street bridge in Pittsburgh, Pennsylvania, in 1933, described in the *Engineering News-Record* of Nov. 23 and Dec. 28 of that year.

[1] For details see any structural-steel handbook.

[2] "Structural Behavior of Battle-Deck Floor System," LYSE and MADSEN, *Proceedings*, A.S.C.E., Jan., 1938.

Limited use has been made of steel mesh floors, with the interstices of the grid left open or filled with concrete. That used for the floors of the bascule leaves of the University Bridge, Seattle, Washington (*Engineering News-Record,* Oct. 12, 1933; Sept. 20 and Oct. 4, 1934), consists of an open grid composed of a series of $2\frac{1}{2}$ in. by $\frac{3}{16}$ in. steel bars with the $2\frac{1}{2}$ dimension vertical, spaced $2\frac{1}{2}$ in. on centers horizontally, connected by sets of double curved zigzag bars, $1\frac{1}{2}$ in. by $\frac{3}{16}$ in., riveted with top edges of longitudinals and cross-bars flush to provide the floor surface for traffic, no filler being used. In position on the bridge, the mesh is supported directly by 6-in. channels which rest on the longitudinal bridge stringers.

7–3. Load Distribution. The live loads brought to a bridge by the highway are given in detail for design purposes in the specifications of the American Association of State Highway Officials (hereafter referred to by initials and by bold-face numerals in brackets) in Appendix C [**5.2.3–10**].

Problem 7–1. Plot the formula for sidewalk live-load reduction of the A.A.S.H.O. specifications [**5.2.12**] for sidewalk widths of 5 ft and 10 ft and loaded lengths to 500 ft. Plot also the corresponding formula of the specification of the A.S.C.E. *Transactions,* 1924, p. 1273: $P = \left(25 + \frac{2250}{L}\right)\left(\frac{W+5}{W}\right)$, with the same limit of 100 lb per sq ft for P. Why are the load requirements more severe for the shorter spans? Note that considerable density of crowding is required to produce a concentration of 100 lb per sq ft; the usual crowd weighs less than half as much. Why does the severity of the loading decrease with increase of sidewalk width? Why is the reduction in load intensity specified in [**5.2.10**] justified? How do these reductions affect the factor of safety decided upon?

The ordinary highway-bridge floor slab is continuous either over a series of longitudinal stringers and their (transverse) supporting floor beams, or over a series of floor beams with longitudinal beams along the slab edges, and consequently consists of a series of continuous panels each supported on four sides. The problem of determining the load and stress distribution in a panel supported on the four edges,[1] with length not more than 50 per cent in excess of width, is very complicated, and present design rules are not in an altogether satisfactory or final state, particularly when the adjoining panels differ in size. However, in bridge floors, the panels are usually so long in proportion to width (one and a half times or more) that each panel may be considered to be supported on the two long edges only. The usual design practice considers such a

[1] See "Moments and Stresses in Slabs," WESTERGAARD and SLATER, *Proceedings,* American Concrete Institute, 1921; "Distribution of Wheel Loads and Design of Reinforced Concrete Bridge Floor Slabs," ERPS, GOOGINS, and PARKER, *Public Roads,* October, 1937, pp. 149–167.

slab as a continuous beam over the length or width of the bridge, or from expansion joint to expansion joint, and computes the bending moments as 80 per cent of the maximum [**5.7.5**] produced by the loading in a simple beam, considering this to be much simpler than an exact solution, as by the three-moment equation, and sufficiently accurate. This is the procedure recommended by the 1931 A.A.S.H.O. specification which considers slabs so designed as safe in shear without further investigation [**5.3.2,** last paragraph].

When a slab is thus designed as a continuous beam, the first problem is the determination of the width of slab underneath a wheel which may be considered as effective in carrying that load with uniform stress distribution [**5.3.2**]. This "effective width" becomes the width of loaded beam designed, and when supported on stringers [**5.3.2,** Case II] the entire width of the panel is made the same in thickness and reinforcement. When the main support consists of the floor beams of the bridge [**5.3.2,** Case I] it is possible for two trucks to be side by side with adjoining wheels 3 ft apart on centers [**5.2.6,** Fig. 4] in which case their effective widths will overlap. In this situation, the combined effective width becomes the width of beam supporting two wheels, and the slab under the central parts of the roadway where this can occur requires greater strength than the outer regions.

Various formulas have been proposed to give the effective width of slab under a wheel, based on experiment and theory. *Bulletin* 80 of the Engineering Experiment Station, University of Ohio, "Concentrated Loads on Slabs," by Professor Clyde T. MORRIS, gives some information concerning these and also a bibliography through 1932. This should be consulted by any desiring to go more deeply into this matter. The paper is valuable in particular as giving a digest of Professor WESTERGAARD'S study, "Computation of Stresses in Bridge Slabs Due to Wheel Loads," which appeared in *Public Roads*, March, 1930, and which furnishes the basis of the more exact method of load distribution given in the A.A.S.H.O. specification of 1935. Figs. 13 and 16 of *Bulletin* 80 give closely the percentages of increase from additional loads as determined in the Westergaard paper.

Problem 7-2. Plot the variation of effective width with varying span for cases I and II, A.A.S.H.O., 1931 [**5.3.2**]. For Case II, consider a single wheel at mid-span. Plot also the effective width for a single wheel when two wheels are within 3 ft of each other on centers, Case I. Plot also the 1935 A.A.S.H.O. formula [**5.3.2**].

Problem 7-3. As a rough check on the reduction in simple load moment specified by A.A.S.H.O. [**5.7.5**] compute the maximum positive and negative bending moments in a continuous beam of four 6-ft spans with a load of 16 kips at the center of one interior span and another in the center of the adjacent end span.

Ans. The maximum bending moment developed is 16.7 kip-ft at the support between the loads: the 80 per cent rule gives 19.2 kip-ft.

Problem 7–4. Compute the length of simple span for which the maximum bending moment due to a single concentrated live load will equal the maximum produced by two similar loads spaced 3 ft apart.

Ans. 5.1 ft

Problem 7–5. Compute the ratio of the maximum bending moments per foot strip of slab produced on a 6-ft simple span by two 16-kip loads (H20) 3 ft apart computed by these two methods: (*a*) by the rule of A.A.S.H.O., 1931 [**5.3.2,** Case II]; (*b*) considering the two loads supported on a beam of width equal to the average effective width by the formula of **5.3.2,** 1931.

Ans. 0.85

Similarly compute the moment ratio for a 7-ft span.

Ans. 0.70

Problem 7–6. Plot the percentage increase per load given by A.A.S.H.O., 1935 [**5.3.2,** Case II]. For comparison, plot also the curve given by computing the maximum moment due to two loads on the element, distant D apart, considering the slab as a simple beam.

Note. For the second curve put the equation giving the moment into terms of D/S. For $D/S = 0$ and 0.6 the two curves will coincide (practically): for $D/S = 0.1$ and 0.3 the values for the second curve are 81 per cent and 44 per cent respectively.

The A.A.S.H.O. rules for the distribution of wheel loads to stringers and floor beams [**5.3.1**] provide for the effect of a continuous floor in spreading the support of any concentrated load over more than one beam even when that load rests on the floor directly above a beam. In this case the deflection of floor and beam directly under the load results in smaller deflections of the immediately adjacent beams and the consequent distribution of load reaction between at least the three. (Sketch the deflection and reactions on a two-span continuous beam with a load in the center.) For a shorter distance between beams, less reaction is developed on the beam immediately under the load.

Problem 7–7. Two continuous beams are identical in dimensions and loading except that one is uniformly one-half as stiff as the other. Do the two sets of reactions differ? Does difference of stiffness explain the different fractions of wheel load specified for each support for floors of different material [**5.3.1**]? Is the continuity of a plank or strip (made of boards laid edgewise) floor as complete as that of a concrete slab? With a non-continuous floor will there be any distribution to adjacent beams of a load over one beam? Explain.

7–4. Truss Weights. Various formulas have been proposed for estimating in advance of design the weight of a given span complete: floor system, bracing, and trusses. Although all these are based on the actual weights of existing structures, they are necessarily approximate and the actual weight should always be computed and the design revised if need be.

For the design given in this chapter, the first estimate of weight was by a formula devised by Dr. C. W. HUDSON and originally presented by Professor C. M. SPOFFORD in his "Theory of Structures":

$$A_1 = \frac{L + I + D_1 + D_2}{s_t} \qquad 7\text{-}1$$

$$A_2 = 5\,A_1$$

$$W = \tfrac{50}{3}\,A_1 \qquad 7\text{-}2$$

where A_1 = area in square inches of heaviest bottom chord section,
L = maximum live stress in bottom chord,
I = impact stress in the same member,
D_1 = dead stress in same member due to known weight of floor,
D_2 = dead stress in same member due to weight of truss and bracing, making a trial assumption of that weight,
s_t = allowable unit stress in tension,
A_2 = area of steel in square inches in one truss and its bracing per linear foot of span,
W = weights in pounds per foot of span for one truss and its bracing.

The relation $A_2 = 5\,A_1$ is obtained on the assumption that the area to be taken for top chord, and that for web members, is 25 per cent greater in each case than for the bottom chord, that the area to cover details equals that for bottom chord, and that for bracing one-half as much. Since a bar of steel 1 sq in. in cross-sectional area and 1 ft long weighs closely $\tfrac{10}{3}$ lb (the figure commonly used is 3.4), $W = \tfrac{10}{3}\,A_2$, which leads directly to Eq. 7-2. It is clear that this formula can be used for railway and highway bridges alike. Evidently, this formula is a rational one with empirical coefficients giving the usual relations observed in actual bridges between the areas of the various parts. By changing these coefficients the equation can be made to fit other classes of structures.

Formed on an entirely different basis is the following equation for highway bridges given by URQUHART and O'ROURKE in their "Steel Structures":

$$w = \frac{w_1}{9} + L \qquad 7\text{-}3$$

where w = weight in pounds per foot of one truss and its bracing,
w_1 = the load — dead, live, and impact — in pounds per foot brought to each truss by the floor system,
L = span in feet.

A similarly devised formula for highway bridges with 16-ft roadways, recommended by JOHNSON, BRYAN, and TURNEAURE in "Modern

Framed Structures," is:

$$w = 0.05\,L\sqrt{p} + 50 \qquad \text{7–4}$$

where p = the live load in pounds per linear foot. Here w includes the weight of floor beams but does not cover that of stringers, slab, or wearing surface. For other widths of roadway add or subtract an amount per foot equal to $0.2\,L$ for each 2-ft change of width.

A well-devised formula for truss weights will give very good results for any structure designed in accordance with the same specifications and in the same general style as those actual structures used as basis for the formula. With differences in specification and arrangement, any formula will give poor results.

7–5. Example of Highway Bridge Design. In the pages which follow is given the complete design of a through riveted truss highway bridge with drawings of the principal elements. The specifications followed are those of the A.A.S.H.O., 1931, excerpts from the structural-design portions of which are reprinted in the Appendix of this book, along with parallel portions from the 1935 edition of these same specifications in certain places where important changes have been made. The headings of the paragraphs which follow refer to the design sheets. Reference to any provision of the specification is by the numeral designation in square brackets both on the design sheets and below.

(RT 1) The 1931 Specification provides for a 50 per cent larger working stress for dead load than for live and at the same time requires that a 100 per cent increase of live load shall not produce, in combination with dead load, unit stresses higher than those specified for dead loads alone [**5.4.1, 5.4.2**]. It thus becomes mathematically inevitable that a structure designed by this specification shall be proportioned to carry the dead load plus twice the live load at the fiber stresses specified for dead load. The 1935 Specification follows the usual practice of giving a single stress limit for each kind of stress for both live and dead loading.

Problem 7–8. A tension member is subjected to a dead load, D, and a live load, L. Show that the area required by Art. **5.4.2,** 1931 Specification, is less than that required by Art. **5.4.1.**

Problem 7–9. Plot curves to show the relation of working stress in tension to the ratio of total live to total dead stress in the member as specified by the 1931 and the 1935 A.A.S.H.O. Specifications. (Take L/D as $\frac{1}{2}$, 1, 3, 5, 10.)

Suggestion. The unit stress desired is that in A (area) $= \dfrac{L + D}{s}$. By the 1931 rules, we have for $L = 2\,D$, $A = \dfrac{D + 2\,L}{24{,}000} = \dfrac{5\,D}{24{,}000}$; also, it is desired that $A = \dfrac{L + D}{s}$, which equals $\dfrac{3\,D}{s}$. Accordingly, $s = 24{,}000 \times \frac{3}{5} = 14{,}400$ lb per sq in.

Note. The use of an increased working stress in tension (except for $L/D = \frac{1}{2}$ in this case) is consistent with modern tendencies. Improved material and closer analysis of stress effects justify this.

Problem 7–10. Plot for comparison the column formulas of the 1931 and the 1935 A.A.S.H.O. Specifications; give three curves for the 1931 formula, for $L/D = 0$, 1 and 5.

1931
$$\frac{2L + D}{A} = \frac{16{,}000}{1 + \dfrac{(L/r)^2}{13{,}500}} \quad \text{with} \ \frac{L}{r} \geqq 40$$
$$\leqq 120$$

1935
$$\frac{L + D}{A} = 15{,}000 - \frac{1}{4}\left(\frac{L}{r}\right)^2 \qquad \frac{L}{r} < 140$$

(RT 3) Floor Beams. The provision for the reduction of live load with increased width of bridge [**5.2.10**] is justified by the decreased likelihood that the loads of several traffic lanes will be simultaneously in position for maximum stress effect. Even with two lanes only, increase in width decreases the number of times that maximum stress effects may be expected.

On this computation sheet, the reduction is not shown applied to the first found live stresses. Since the second bending moment is decidedly larger with reduction included, it was not thought worth while to reduce the stress superseded.

Instead of considering stringer reactions in computing floor-beam stresses, the simpler process is usual of assuming the dead load of the stringers and deck uniformly distributed on the floor beam with the live load applied directly.

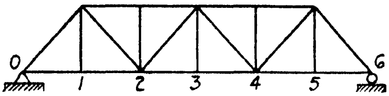
(RT 5) Live Panel Loads. The student should not be slowed up by the necessity of reading three articles in the specifications before being able to explain a very innocent-appearing parenthesis: $(640 \times \frac{10}{9} \times 0.98)\ 20$. Each item in turn is explained by the corresponding article, and the 20 is the panel length. Remember that, if both lanes are loaded alike, the floor-beam reaction equals the panel load, equals the load on one lane.

(RT 6) Live Stress. The live stress in U_1U_3 equals the live moment at L_2 divided by the truss depth. That moment here is due to a single concentrated movable load of 19,600 lb (see previous computation for live panel loads) placed at L_2 and equals $\frac{8}{9}$ of the moment produced by the same load at L_3.

Problem 7–11. Write the equation for the locus line giving the value of the bending moment under a single concentrated load as it crosses a simple span. Rewrite the equation for rectangular axes through the vertex of the curve, expressing it in terms of offsets and tangent distances. $(O_1 : O_2 = D_1^2 : D_2^2)$. Verify the value discussed in the paragraph above.

Riveted Truss Highway Bridge *Design Sheet–RT 1*

Data



Class AA Bridge
Live Load = H 20
[5.1.17, 5.2.8]

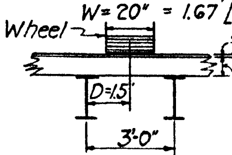
Span: 120'-0" c.-c. bearings
Panels: 6 of 20'-0" Roadway 20'-0" [5.1.9]
Specifications: American Association of State Highway Officials. 1931

Numbers in brackets refer to the specifications, reprinted in the Appendix

Slab

Floor: Reinforced Concrete Slab on longitudinal stringers, spaced 3'-0" c.-c.

$W = 20'' = 1.67'$ [5.2.6]



Dead Load

$\frac{3}{4}''$ Tar Mat	=	10#/□' [4.7.22-24 Etc.]
8" Slab	=	100 " " (assumed)
		110#/□' Total

Live Load: Rear Wheel weighs $\frac{0.8 \times 40{,}000}{2} = 16{,}000$# [5.2.6]

[Note: neglect Art. 5.2.10]

Effective width of slab carrying wheel [5.3.2 II]

$E = 0.7(2D+W) = 0.7(2 \times 1.5 + 1.67) = 3.27'$

Live Load per ft. of slab $= \frac{16{,}000}{3.27'} = 4{,}900$#/1

Bending Moment

Dead: $0.8 \times \frac{1}{8} \times 110 \times \overline{3}^2 \times 12 = 1{,}200$ "# [5.7.5]

Live $0.8 \times 4900 \times \frac{36}{4} = 35{,}300$

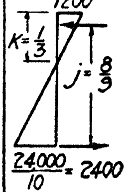
Impact: $\frac{50}{L+125} = \frac{50}{128} = 39\% = 13{,}800$ [5.2.13]

Add Live + Impact Moments $= 49{,}100$ [5.4.1, 2nd paragraph]

Total Mom. 99,400"#

Slab Depth: Allowable unit stresses [5.4.7, 5.4.8]

$n = 10$, $f_c = 1{,}200$#/□" $f_s = 24{,}000$#/□" $f_c' = 3{,}000$#/□"



$R = \frac{M}{bd^2} = \frac{\frac{1}{2} f_c b k d j d}{bd^2} = \frac{1200}{2} \times \frac{1}{3} \times \frac{8}{9} = 178$

Required $d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{99{,}400}{178 \times 12}} = 6.82''$ for slab with tension reinforcement only.

(RT 6) Diagonals; Live Stress. Continual reference to the previous computation sheet with its figures and panel loads is essential to quick checking here. Note that the horizontal component of end post stress is equal to the L_0L_2 stress just obtained; for maximum shear in panel 1–2 place the live panel load for shear, 28.3 kips, at L_2; the uniform load effects are found by aid of the influence line.

(RT 7) Lateral System. The neglect of the actual framing in assuming a Pratt truss evidently is of no account since the minimum lateral panel load is found to be required. Approximate preliminary computations are of great value in exploring the way to go and should be used freely. They are invaluable as a means of developing judgment. No precise computation of any magnitude should ever be made without parallel approximate figures to ensure that the set-up is correct.

(RT 11) Top Chord. The horizontal compression chord of a bridge is subjected to the stresses due to the bending caused by its own dead weight and accordingly falls under the provisions of the article dealing with combined stress [**5.6.7**]. (See Art. 2–9.) In this bridge, it is planned that the top chord shall be a continuous member over the four panels of its length, and reference to the curve of bending moments for such a continuous beam with uniform load shows that the positive moment in any span is materially reduced from the $WL/8$ value for a simple beam, not attaining a magnitude equal even to two-thirds of that value. The specification accordingly is very conservative in taking three-fourths of $WL/8$. The negative moment reaches a higher relative value but its effect is at a point where the column is restrained from buckling.

(RT 12) Member U_2L_2. Choice here lay between using a heavier section to satisfy the L/r requirement or, as decided upon, using the same section here as for all the other verticals on the ground that the slight excess in slenderness ratio is of small import in a member carrying only a small dead-load stress.

(RT 13) The top lateral strut consists of the top and bottom chords of the small Warren truss placed between the tops of opposite posts of the two main trusses to serve as sway bracing, the function of which here is to equalize the deflections of the two trusses under load on one traffic lane only and thus to give transverse stiffness to the structure as a whole. When only one lateral truss is used, the sway bracing acts like a portal to carry the top chord transverse load to the lower chord level, or the reverse with the lower laterals omitted. Since the top and bottom lateral trusses deflect horizontally unequally under their unequal loadings, the sway bracing is brought into play in reducing the differential.

Riveted Truss Highway Bridge	*Design Sheet – RT 2*

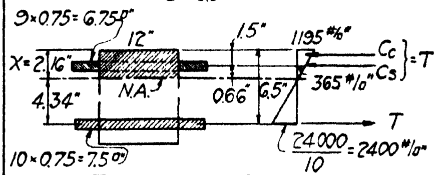
Since it is not practicable to use bent rods with 3 ft spacing of stringers use double reinforcement, equal steel, straight and continuous in both top and bottom. Since reinforced concrete design tables are not given in this text an approximate solution will be made using the transformed section. | *Slab (Continued)*

With tension reinforcement only the slab should 6.82 + 1.5 = 8.32" thick. Steel area should be

$$A_S = \frac{99,400}{24,000 \times \frac{8}{9} \times 6.82} = 0.68\ \square''/'$$

Try 8" slab: d = 6.5'   Steel area about $0.68 \times \frac{6.82}{6.5} = 0.72\ \square''/'$

Use $\frac{3}{4}''\phi$ *@ 7" c-c.* $A_S = 0.75\ \square''/'$



Transformed Section

Neutral Axis

$$\frac{12\bar{X}^2}{2} + 6.75(X - 1.5) = 7.5(6.5 - X)$$

whence $X = 2.16''$

Moment of Resistance

		arm	M ''#
$C_c = 1195 \times 12 \times 2.16 \times \frac{1}{2}$	= 15,490	1.44"	22,300
$C_s = 365 \times 6.75$	= 2465	0.66"	1630
	17,955		
$T = 2400 \times 7.5$	= 18000#	4.34"	78,000

M_R = 101,930

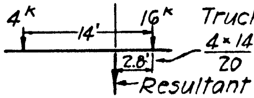
Bending Moment = 99,400"#

Use 8" Slab, $\frac{3}{4}''\phi$ *@ 7" c.-c. top & bottom* | *Use*

Stringer spacing, 3'-0"   Span = 20'-0" | *Inside Stringer*

Proportion of wheel load on stringer [5.3.1a]

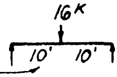
$$= \frac{S}{4.5} = \frac{3}{4.5} = \frac{2}{3}$$



$\frac{4 \times 14}{20} = 2.8'$

Loading [5.2.6]

∴ For maximum moment place at ℄.



Dead Load:

Slab: 8"	100 × 3 =	300 #/'	Sheet RT1
Tar Mat: $\frac{3}{4}''$	10 × 3 =	30	
Beam		50 ±	
Total		380 #/'	

Problem 7-12. Compute the stresses in the several members of the sway frame at U_3 due to full live load on one traffic lane only.

Fig. 7-1 shows a "stress sheet" for the bridge designed in this article giving a summary of the stresses and the sections for the main truss members and the lateral systems. The arrangements of members at typical joints are shown in Figs. 7-2 to 7-6. These are "general drawings"; that is, they are drawn to scale but are not fully dimensioned.

In making the drawing for a joint it is usual to locate the rivets and then sketch the gusset plate around them. This will give the plate outline, but there still remains the matter of plate *thickness*. It may be questioned whether the use of the beam formula for the determination of this thickness is valid, but, to date, no better method has been proposed. For example, Fig. 7-3, the gusset plate at joint L_2 may be examined. This plate is called upon to act also as a splice plate, and it appears that a critical section might be the vertical one which passes through the two shop rivets at the left end of chord member L_2L_4. Furthermore, if failure occurred here, the failing section would probably slant to the right and pass through the rivet holes which match with the floor-beam connection. Hence, based on the solid section of the plate, it would probably be proper to assume that the stress, due to the loss of section in rivet holes, would be increased 30 per cent, approximately. At this section the plate depth is 29 in. Resolving the stress in U_1L_2 into components where it crosses the section, the horizontal component, 138,000 lb, is found to have an eccentricity of 2.75 in. The bottom chord stress has an eccentricity of 9.25 in., and the amount of this stress may be assumed to be $\frac{5}{8}$ of the total in L_1L_2 ($\frac{5}{8} \times 188{,}000 = 118{,}000$) since a separate splice plate has been provided for the horizontal legs. Therefore, based on a $\frac{5}{8}$-in. gusset-plate thickness, the plate stress will be

$$
\begin{aligned}
s &= \frac{P}{A} + \frac{Mc}{I} \\
&= 1.3\left[\frac{138{,}000 + 118{,}000}{2 \times \frac{5}{8} \times 29} + \frac{(138{,}000 \times 2.75 + 118{,}000 \times 9.25)\, 14.5}{2 \times \frac{1}{12} \times \frac{5}{8} \times 29^3}\right] \\
&= 20{,}300 \text{ lb per sq in.}
\end{aligned}
$$

Since the main members have been designed for 24,000 lb per sq in., this appears to be a safe stress in the gusset.

Problem 7-13. Check the adequacy as regards number and spacing of rivets, gusset-plate thicknesses, etc., of the joints shown in Figs. 7-2 to 7-6.

7-6. Camber. If the lengths used in computing stresses in a truss were used in fabricating the members, the truss lower chord would

Riveted Truss Highway Bridge — *Design Sheet—RT3*

Inside Stringer (Continued)

__Moment__

Dead $380 \times \frac{20^2}{8} = 19{,}000'^{\#}$

Live $\left(\frac{2}{3} \times 16{,}000\right) \times \frac{20}{4} = 53{,}300$

Impact $\frac{50}{L+125} = \frac{50}{145} = 0.345 = 18{,}400$ [5.2.13]

100% Increase $L+I = 71{,}700$ [5.4.1]

$M = 162{,}400'^{\#}$

Required S

$S = \frac{162{,}400 \times 12}{24{,}000} = 81.2$ ← top flange encased [5.4.2]

Try 18 WF 47 $S = 82.3$ ← Use

__Shear__ [5.3.1]

Live: $16 + \frac{6}{20} \times 4 = 17{,}200^{\#}$

Impact 0.345 = 5,930

Dead $380 \times \frac{20}{2} = 3{,}800$

100% Increase $L+I = 23{,}130$

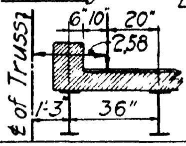
Total $= 50{,}060^{\#}$

$S_s = \frac{50{,}060}{18 \times 0.35} = 7{,}950^{\#}/\square'' < 15{,}000^{\#}/\square''$

$\therefore$ O.K. [5.4.2]

Outside Stringer

__Loading__ [See Fig 4, 5.2.6]

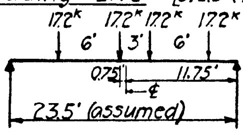


$\frac{20}{36}$ of 1 wheel on outside stringer [5.3.1b]

Use 18 WF 47 ← Use

Floor Beam

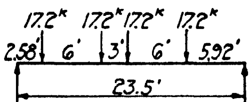
__Loading: Live__ [5.2.9 (1)]



{Since rear wheels @ 16k may come on floor beam in positions shown assume effect of front wheels at same points as an approximation. See stringer live end shear.

$\text{Max. M} = \frac{17.2^k \times 4(11.75 - 0.75)^2}{23.5} - 17.2 \times 6$

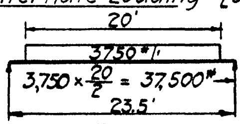
$= 251{,}500'^{\#}$



$\text{Max. V} = 4 \times 17.2 \times \frac{13.42}{23.5} = 39{,}300^{\#}$

See outside stringer loading above

__Alternate Loading__ [5.2.9(2), 5.2.10]



$\frac{17.2^k \times 2}{9} \times 0.98 = 3{,}750^{\#}/'$

$\text{Max. M} = 37{,}500(11.75 - 5) = 253{,}100'^{\#}$

$\text{Max. V} = 37{,}500^{\#}$

__Impact Stress__ [5.2.13]

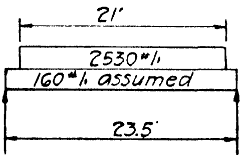
Impact Allowance $= \frac{50}{40 + 125} = 30.3\%$

deflect below the horizontal line joining the end pins and would give the effect of sagging. As this would prove unsightly, the truss is given a *camber* (that is, is arched above the horizontal) such an amount that the chord will become straight when the truss carries dead load, plus one-half of live load, plus impact. The necessary shortening of tension members and lengthening of those in compression may be exactly computed, but usually, for trusses of the lengths considered in this volume, the following less exact rule is applied. The vertical and lower-chord members are given their nominal lengths, the top chords are increased in length by $\frac{1}{8}$ in. for each 10 ft of nominal length, and the diagonals are given the required lengths for diagonals in the altered panels. It will be seen that under this arrangement the panels are trapezoidal, instead of rectangular, and that fitting them together will give a curved effect, the falsework and blocking on which the truss is erected being given the same curve. Camber, obviously, is a matter for consideration at the time of preparation of shop drawings and has no relation to design.

Riveted Truss Highway Bridge — Design Sheet–RT4

Floor Beam (Continued)

Dead Load



Dead Stringer Reaction = 3,800#

Equivalent uniform load

$= \frac{3800 \times 2}{3} = 2{,}530$ #/'

Max. M: $\frac{1}{8} \times 160 \times \overline{23.5}^2 =$ 11,050'#

$2530 \times 10.5(11.75 - 5.25) =$ 172,500

183,550'#

Max. Shear:

$160 \times \frac{23.5}{2} =$ 1,880#

$2530 \times \frac{21}{2} =$ 26,600#

28,480#

Stress Summary

	Moment	Shear
Dead:	183,600'#	28,500#
Live: [5.4.1]	253,100	39,300
	253,100	39,300
Impact 30.3%	76,700	11,900
	76,700	11,900
	M = 843,200'#	V = 130,900#

[5.4.2] S required: $\frac{843{,}200 \times 12}{24000} = 422$

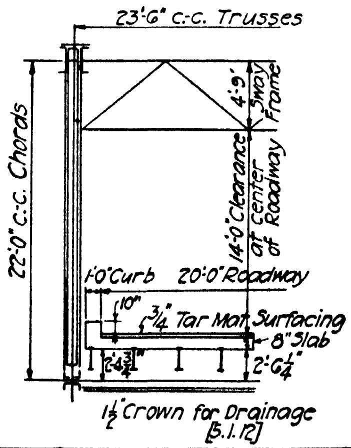
Try 33 WF 141 ← **Use**

S = 446.8

Shear:

$S_s = \frac{130{,}900}{33 \times 0.65} = 6{,}550$ #/□" < 15,000 #/□" ∴ O.K.

Clearance



See 5.1.9
AASHO 1931

Sketch to give truss depth (22'-0")
Some details revised later.
See RT9

Riveted Truss Highway Bridge — Design Sheet—RT5

Dead Panel Loads

Truss Loads

Truss Weight (See text for formula)

$$A_1 = \frac{L+I+D_1+D_2}{St}, \quad A_2 = 5A_1, \quad W = \frac{50}{3}A_1$$

$$L: M = \frac{1}{8}\times 640 \times \frac{10}{9}\times 0.98 \times \overline{120}^2 + \frac{1}{4}\times 18{,}000 \times \frac{10}{9}\times 0.98 \times 120$$

$= 1{,}843{,}000'^{\#}$ [5.2.7b; 5.2.9(2); 5.2.10]

I: allowance $= \frac{50}{120+125} = 20.4\%$

D_1: $M = \frac{1}{8}\times\frac{28500}{20}\times\overline{120}^2 = 2{,}570{,}000'^{\#}$

D_2: $M = \frac{(250\pm)\ \overline{120}^2}{8} = 450{,}000'^{\#}$

Summation

L	1,843,000
	1,843,000
I	376,000
	376,000
D_1 =	2,570,000
D_2 =	450,000
	7,458,000

Chord Stress $= \frac{7{,}458{,}000}{22' \text{ assumed}} = 339{,}000^{\#}$

$A_1 = \frac{339{,}000}{24{,}000} = 14.13\square''$

$W = \frac{50}{3}\times 14.13 = 236^{\#}/'$ Assume $240^{\#}/'$

Dead Panel Loads

Top Chord $= \frac{240\times 20}{2} = 2400^{\#}$ ←

Lower Chord: Truss $2400^{\#}$
Floor Beam $28{,}500$
$30{,}900^{\#}$ ←

Live Panel Loads [5.2.7(b), 5.2.9(2) 5.2.10]

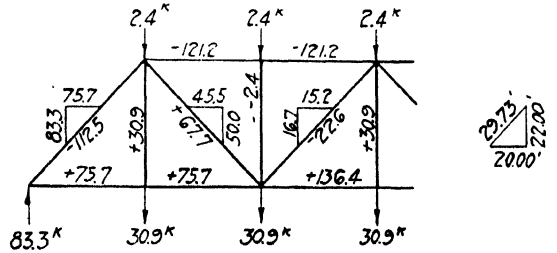
Live Panel Loads

Uniform Load: $[640\times\frac{10}{9}\times 0.98]\,20 = 14000^{\#}$ ←

Excess Load: Moment: $18000\times\frac{10}{9}\times 0.98 = 19{,}600^{\#}$ ←

Shear: $26000\times\frac{10}{9}\times 0.98 = 28{,}300^{\#}$ ←

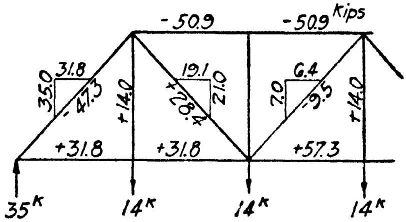
Truss Dead Stresses



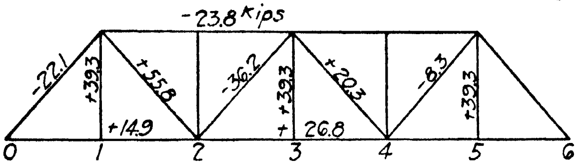
Riveted Truss Highway Bridge — *Design Sheet—RT 6*

Truss: Live Stresses

Full Uniform Live Load



Excess and Partial Uniform Live (See Computation following)



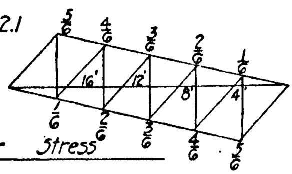
Chords: L_2L_4: $\frac{PL}{4}\times\frac{1}{d} = \frac{19{,}600\times 120}{4}\times\frac{1}{22} = +26{,}750$

$U_1U_3 = 26.75\times\frac{8}{9} = -23{,}800^{\#}$ $L_0L_2 = 26.75\times\frac{5}{9} = 14{,}900^{\#}$

Diagonals

End Post: $14.9\times\frac{29.73}{20.0} = -22.1$

Uniform load per ft $= \frac{14{,}000}{20} = 700^{\#}/'$



Panel	Kips	Shear	Stress
1-2	$\frac{4}{6}\times 28.3 = 18.9$ $\frac{1}{2}\times 96\times\frac{4}{6}\times 700^{\#} = 22.4$	+41.3	$\times\frac{29.73}{22} = +55.8\ U_1L_2$
2-3	$\frac{3}{6}\times 28.3 = 14.2$ $\frac{1}{2}\times 72\times\frac{3}{6}\times 700^{\#} = 12.6$	+26.8	× " = −36.2 U_3L_2
*3-4	$\frac{2}{6}\times 28.3 = 9.4$ $\frac{1}{2}\times 48\times\frac{2}{6}\times 700^{\#} = 5.6$	+15.0	× " = +20.3 U_3L_4
*4-5	$\frac{1}{6}\times 28.3 = 4.7$ $\frac{1}{2}\times 24\times\frac{1}{6}\times 700^{\#} = 1.4$	+6.1	× " = −8.3 U_5L_4

*Here loaded length is less than 60' but [5.2.7a] is neglected

Riveted Truss Highway Bridge — Design Sheet—RT 5

Truss Loads

Dead Panel Loads

Truss Weight (See text for formula)

$$A_1 = \frac{L + I + D_1 + D_2}{St}, \quad A_2 = 5A_1, \quad W = \frac{50}{3}A_1$$

$$L: M = \frac{1}{8} \times 640 \times \frac{10}{9} \times 0.98 \times \overline{120}^2 + \frac{1}{4} \times 18{,}000 \times \frac{10}{9} \times 0.98 \times 120$$

$= 1{,}843{,}000'$# [5.2.7b; 5.2.9(2); 5.2.10]

$$I: \text{ allowance} = \frac{50}{120+125} = 20.4\%$$

$$D_1: \quad M = \frac{1}{8} \times \frac{28500}{20} \times \overline{120}^2 = 2{,}570{,}000'\#$$

$$D_2: M = \frac{(250 \pm)\ \overline{120}^2}{8} = 450{,}000'\#$$

Summation

L	1,843,000
	1,843,000
I	376,000
	376,000
D_1 =	2,570,000
D_2 =	450,000
	7,458,000

$$\text{Chord Stress} = \frac{7{,}458{,}000}{22' \text{ assumed}} = 339{,}000\#$$

$$A_1 = \frac{339{,}000}{24{,}000} = 14.13\,\square''$$

$W = \frac{50}{3} \times 14.13 = 236\#/$, Assume $240\#/$

$$\text{Top Chord} = \frac{240 \times 20}{2} = 2400\# \longleftarrow$$

Lower Chord: Truss 2400#
Floor Beam 28,500
30,900# ⟵

Dead Panel Loads

Live Panel Loads [5.2.7(b), 5.2.9(2) 5.2.10]

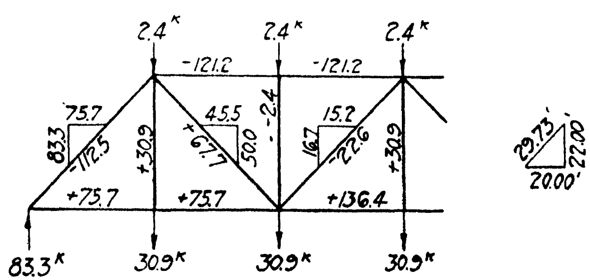
Uniform Load: $[640 \times \frac{10}{9} \times 0.98]\,20 = 14000\#$ ⟵

Excess Load: Moment: $18000 \times \frac{10}{9} \times 0.98 = 19{,}600\#$ ⟵

Shear: $26000 \times \frac{10}{9} \times 0.98 = 28{,}300\#$ ⟵

Live Panel Loads

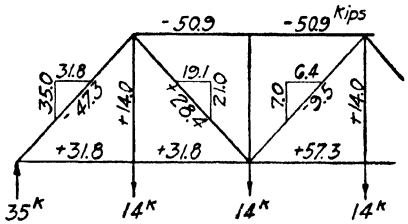
Truss Dead Stresses



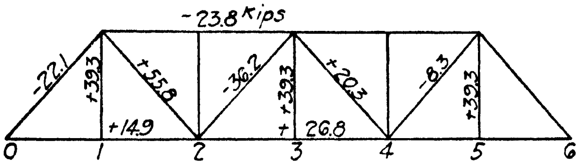
Riveted Truss Highway Bridge — Design Sheet—RT 6

Truss: Live Stresses

Full Uniform Live Load



Excess and Partial Uniform Live (See Computation following)



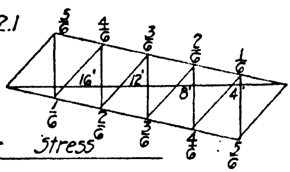
Chords: L_2L_4: $\frac{PL}{4}\times\frac{1}{d} = \frac{19,600\times 120}{4}\times\frac{1}{22} = +26,750$

$U_1U_3 = 26.75\times\frac{8}{9} = -23,800^{\#}$ $\quad L_0L_2 = 26.75\times\frac{5}{9} = 14,900^{\#}$

Diagonals

End Post: $14.9\times\frac{29.73}{20.0} = -22.1$

Uniform load per ft. $= \frac{14,000}{20} = 700^{\#}/'$



Panel		Kips	Shear	Stress
1-2	$\frac{4}{6}\times 28.3 =$	18.9	+41.3	$\times\frac{29.73}{22} = +55.8\ U_1L_2$
	$\frac{1}{2}\times 96\times\frac{4}{6}\times 700^{\#} =$	22.4		
2-3	$\frac{3}{6}\times 28.3 =$	14.2	+26.8	$\times$ " $= -36.2\ U_3L_2$
	$\frac{1}{2}\times 72\times\frac{3}{6}\times 700^{\#} =$	12.6		
*3-4	$\frac{2}{6}\times 28.3 =$	9.4	+15.0	$\times$ " $= +20.3\ U_3L_4$
	$\frac{1}{2}\times 48\times\frac{2}{6}\times 700^{\#} =$	5.6		
*4-5	$\frac{1}{6}\times 28.3 =$	4.7	+6.1	$\times$ " $= -8.3\ U_5L_4$
	$\frac{1}{2}\times 24\times\frac{1}{6}\times 700^{\#} =$	1.4		

*Here loaded length is less than 60' but [5.2.7a] is neglected

Riveted Truss Highway Bridge   Design Sheet – RT 7

Verticals

Live Floor Beam Reaction = 39.3
See RT 3

Truss Live Stress (Continued)

Truss Impact Stress

Bar	Max. Live Stress	Loaded Length	Impact Factor [5.2.13]	Impact Stress
L_0L_1 L_1L_2	+46.7	120	0.204	+9.5
L_2L_3	+84.1	120	0.204	+17.2
U_1U_2 U_2U_3	−74.7	120	0.204	−15.3
L_0U_1	−69.4	120	0.204	−14.2
U_1L_2	−55.8	96	0.226	+12.6
L_2U_3	−36.2	72	0.254	−9.2
U_3L_4	+20.3	48	0.289	+5.9
L_4U_5	−8.3	24	0.336	−2.8
U_1L_1 U_3L_3 U_5L_5	+39.3	40	0.303	+11.9

Lateral Systems Loads

[5.2.15] For simplicity assume truss to be Pratt instead of Warren type in order to equalize lateral panel loads.

Assume top chord and end post 18" wide, other members 12" wide.

Top Chord Load: $20 \times 1.5 = 30^{\square'}$
$11 \times 1 = 11$
$\frac{1}{2} \times 30 \times 1 = \underline{15}$
$56 \times 1\frac{1}{2} = 84^{\square'}$
$84 \times 30 = 2{,}520$ #/panel

By specification: minimum $= 150 \times 20 = 3000$ #/panel

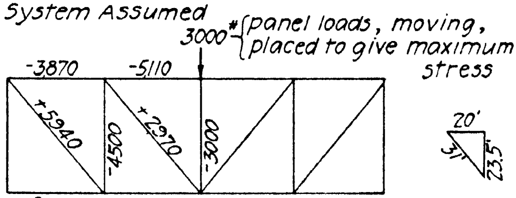
Bottom Chord Load: $20 \times 4 = 80$ (floor & chord)
$11 \times 1 = 11$
$\frac{1}{2} \times 30 \times 1 = \underline{15}$
$106 \times 1\frac{1}{2} = 159$
$159 \times 30 = 4770$ #/panel
Traffic $200 \times 20 = \underline{4000}$
8,770 #/panel

or minimum $300 \times 20 = 6000$ #/panel

Riveted Truss Highway Bridge — Design Sheet—RT 8

Top Lateral System

Tension System Assumed

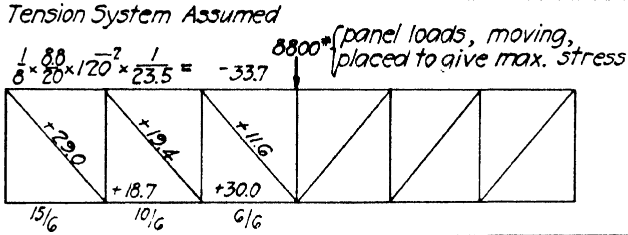


$\frac{1}{8} \times \frac{3000}{20} \times \overline{80}^2 \times \frac{1}{23.5} = 5{,}110\#$

Bottom Lateral System

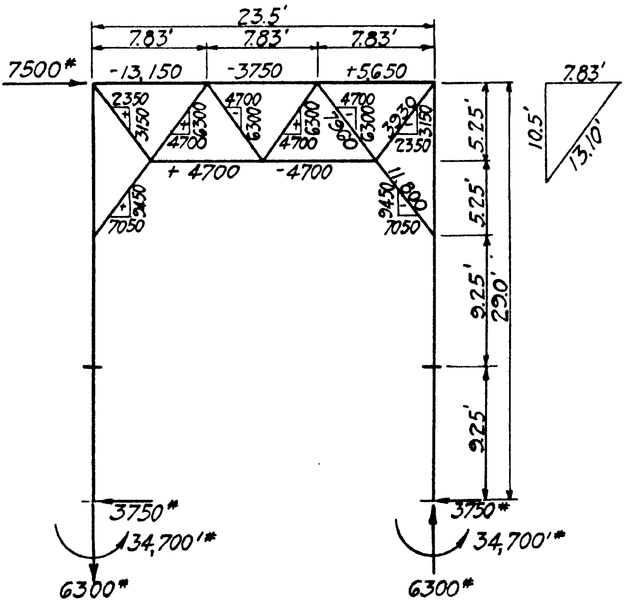
Tension System Assumed

$\frac{1}{8} \times \frac{8.8}{20} \times \overline{120}^2 \times \frac{1}{23.5} =$ -33.7

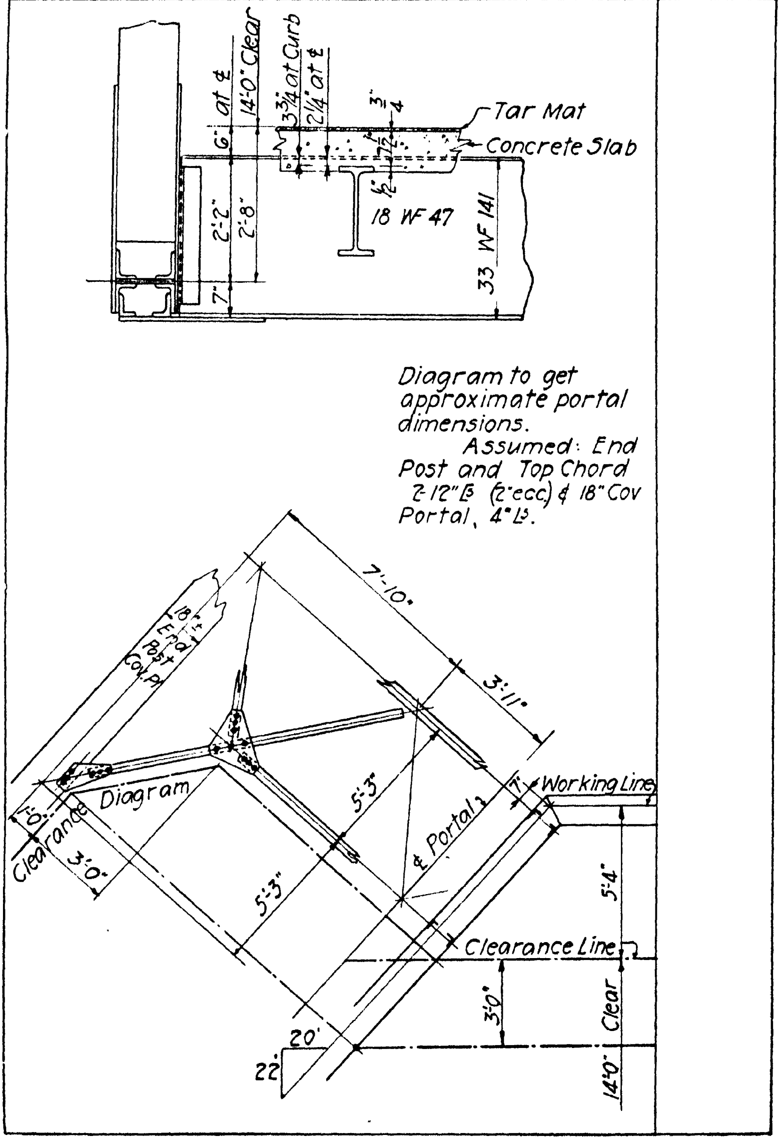


Portal Stresses

Clearances are estimated on sheet RT 9



Riveted Truss Highway Bridge Design Sheet – RT 9



Riveted Truss Highway Bridge — *Design Sheet – RT 10*

U_1L_2 $+204{,}500^{\#}$ — Minimum $\frac{L}{r} = 200$ [5.6.10]

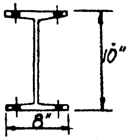
Diagonals U_1L_2 — 10 WF 41

$$\text{Area} = \frac{204{,}500}{24{,}000\,[5.4.1]} = 8.53^{\square''} \quad r = \frac{29.73\times 12}{200} = 1.78$$

Minimum thickness of material $= \frac{5}{16}''$ [5.6.14]

To prevent need of excessive number of rivets in connections will use $7/8''\phi$ rivets.

Try 10 WF 41



Net area $= 12.06 - 4(0.56\times 1) = 9.82^{\square''} > 8.53^{\square''}$ ∴ O.K

$r = 1.99 > 1.78$

L_2U_3 $-113{,}400$, $+29{,}800$ [5.6.6] — Max. $\frac{L}{r} = 120$ [5.6.10]

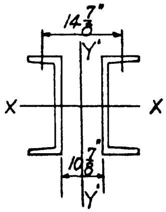
L_2U_3 — 2-[12×25, $11\frac{3}{8}$ b-b, Single Lacing $2\frac{1}{2}\times\frac{7}{16}$

$$\text{Min. } r = \frac{29.73\times 12}{120} = 2.97$$

Trial Section][

$7/8''\phi$ rivets require 12"[, Minimum thickness of metal requires 2-12 [25 [5.6.14]

Spacing: U_1L_2 10"
2 Gussets 1.25
Clearance 0.13"
11.38" back to back



Data:
12 [25 $A = 7.32^{\square''}$ $I_x = 143.5$ $r_x = 4.43$
$\bar{x} = 0.68''$ $I_y = 4.5$ $r_y = 0.79$

$I_y' = 2\left[4.5 + 7.32\,(5.69+0.68)^2\right] > 2\times 143.5$ above

∴ X axis governs

$$\frac{L}{r} = \frac{29.73\times 12}{4.43} = 80.5$$

$$\text{Allowable } \frac{P}{A} = \frac{24{,}000}{1+\frac{(80.5)^2}{13{,}500}} = \frac{24{,}000}{1.48} = 16{,}200^{\#}/_{\square''}$$

$$\text{Actual } \frac{P}{A} = \frac{113{,}400}{2\times 7.32} = 7{,}740^{\#}/_{\square''} < 16{,}200^{\#}/_{\square''}$$

∴ O.K.

Lacing: [5.6.35]

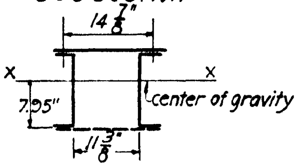
$$\frac{14\frac{7}{8}''}{\cos 30^\circ} = 17.15 \quad \frac{17.15}{40} = 0.43''$$

Use single lacing $2\frac{1}{2}\times\frac{7}{16}$

Riveted Truss Highway Bridge — *Design Sheet - RT 11*

$U_1\ U_3$ $-301{,}200^{\#}$

Use Section



Minimum [for 7/8" rivets in flanges, 5/16" thick web is 12 [25 [5.6.14]

Cover Plate thickness

$$= \frac{14\,^7/_8{}''}{40} = 0.37 \quad [5.6.15]$$

Try $18 \times \frac{3}{8}$

	A Area	a Arm	Aa	$d_{c.g.}$	I_x	x	I_y
2-[12×25	14.64	6.00	87.84	1.95	287.0 55.6	6.12	9.0 595.0
1-Pl. $18\times\frac{3}{8}$	6.75	12.19	82.30	4.24	0 121.4	0	182.3 0
	21.39□"	7.95"	170.14		464.0		786.3

$$r_x = \sqrt{\frac{464.0}{21.39}} = 4.66 \qquad \frac{L}{r} = \frac{20\times 12}{4.66} = 51.5$$

$$\text{Allowable } \frac{P}{A} = \frac{24\,000}{1+\frac{(51.5)^2}{13{,}500}} = \frac{24\,000}{1.197} = 20{,}050\ {}^{\#}/\square''$$

$$\text{Actual } \frac{P}{A} = \frac{301{,}200}{21.39} = 14{,}100\ {}^{\#}/\square'' < 20{,}050\ {}^{\#}/\square'' \quad \therefore \text{O.K.}$$

Check for combined stress [5.6.7]

$$M = \frac{3}{4}\times\frac{1}{8}\left(21.39\times 3.4\right)\overline{20}^2 = 2{,}730\ '^{\#}$$

Bending Stress $s = \frac{(2730\times 12)4.43}{464} = 310\ {}^{\#}/\square''$

Lateral Stress $\frac{5110}{21.39} = 240$

Truss Chord Stress $14{,}100$

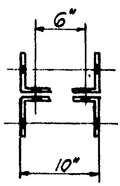
Total $14{,}650\ {}^{\#}/\square''$

$14{,}650\ {}^{\#}/\square'' < 20{,}050$ allowed $\therefore$ O.K.

Top Chord
$U_1U_2 - U_2U_3$

2-[12×25
1-Pl. $18\times\frac{3}{8}$

$L_0L_1 - L_1L_2$ $+188{,}100$



$$\text{Required } \frac{188{,}100}{24{,}000} = 7.85\square''$$

4-L $5\times 3\frac{1}{2}\times\frac{3}{8}$ 1" ϕ holes out as shown

$$A = 4\left(3.05 - 2\times 1\times\frac{3}{8}\right) = 9.20\square'' \text{ furnished}$$

$r = 2.40''$

$$\frac{L}{r} = \frac{20\times 12}{2.4} = 100 < 200 \quad [5.6.10]$$

$\therefore$ O.K.

Bottom Chords

$L_0L_1 - L_1L_2$

4-L
$5\times 3\frac{1}{2}\times\frac{3}{8}$

Riveted Truss Highway Bridge Design Sheet—RT 12	
L_2L_3 + 339,000 Required $\frac{339000}{24,000} = 14.14\,\square''$ 4-L $5 \times 3\frac{1}{2} \times \frac{5}{8}$ with 1-1"ϕ hole in each leg give - $14.68\,\square''$	Bottom Chords (Continued) L_2L_3 4-L $5 \times 3\frac{1}{2} \times \frac{5}{8}$
Stay Plates [5.6.34] Thickness: $t \mathrel{\bar{=}} \frac{6}{50}$ Use $\frac{5}{16}''$ Length: $l \mathrel{\bar{=}} (1\frac{1}{4} \times 6) + 3$ Use 12"	Stay Plates $9 \times \frac{5}{16} \times 1'\text{-}0''$ 3' clear betwn pls.
$U_1L_1 - U_3L_3$ + 133,300 Required $A = \frac{133,300}{24,000} = 5.56\,\square''$ Use 10 WF 41 $A = 12.06 - 4(0.56 \times 1) = 9.82\,\square''$ $r = 1.99$ $\frac{L}{r} = \frac{22 \times 12}{1.99} = 133 < 200$ ∴ OK.	Verticals $U_1L_1 - U_3L_3$ 10 WF 41
U_2L_2 - 2400# Use 10 WF 41 $\frac{L}{r} = \frac{22 \times 12}{1.99} = 133 > 120$ [5.6.10] But free length less than 22', say 20'. $\frac{L}{r} = \frac{20 \times 12}{1.99} = 120$ O.K.	U_2L_2 10 WF 41
L_0U_1 P (Live + Dead) 279,700 P (Portal) 6,300 286,000 M (Portal) 34,700'# Try top chord section (Sheet RT 11) $\frac{L}{r} = \frac{29.73 \times 12}{4.66} = 77$ Allowable $\frac{P}{A} = \frac{24,000}{1 + \frac{(77)^2}{13500}} = \frac{24,000}{1.439} = 16,670\,\#/\square''$ Actual $\frac{P}{A} = \frac{286,000}{21.39} = 13,400\,\#/\square''$ Bending $s = \frac{(34,700 \times 12)9}{786.3} = 4,760$ 18,160 #/□" > 16,670 #/□" [5.6.7] ∴ Too Small	End Post L_0U_1

Riveted Truss Highway Bridge — Design Sheet – RT 13

L_0U_1 (Continued)

Try 2-[12×30, 1-Pl. 18×$\frac{3}{8}$

	A Area	a Arm	Aa	$d_{c.g.}$	I_x	x	I_y
2-[12×30	17.58	6.0	105.48	1.71	322.4 51.4	6.07	70.4 713.0
Pl. 18×$\frac{3}{8}$	6.75	12.19	82.30	4.48	0.0 135.5	0	182.3 0.0
	24.33	7.71	187.78		509.3		905.7

$$r = \sqrt{\frac{509.3}{24.33}} = 4.57 \qquad \frac{L}{r} = \frac{29.77\times 12}{4.57} = 78$$

$$\text{Allowable } \frac{P}{A} = \frac{24000}{1+\frac{(78)^2}{13{,}500}} = \frac{24{,}000}{1.450} = 16{,}550\ \#/\square''$$

$$\text{Actual } \frac{P}{A} = \frac{286{,}000}{24.33} = 11{,}770\ \#/\square''$$

$$\text{Bending} \quad s = \frac{34{,}700\times 12\times 9}{905.7} = \underline{4{,}140}$$

$$15{,}910\ \#/\square'' < 16{,}550\ \#/\square'' \quad \therefore \text{O.K.}$$

End Post L_0U_1 (Continued) — 2-[12×30, 1-Pl. 18×$\frac{3}{8}$

Strut, −4500 # $L = 23.5'$

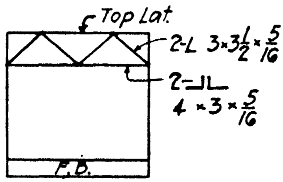
Compare U_2L_2

Allowable $\frac{L}{r} = 140$ [5.6.10]

4-L 4×3×$\frac{5}{16}$ $r = 2.02$

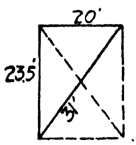
$$\frac{L}{r} = \frac{23.5\times 12}{2.02} = 140 \quad \therefore \text{O.K.}$$

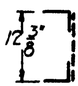
Sway Frames at U_2, U_3, U_4 [5.6.66]



Top Laterals Struts — 4-L 4×3×$\frac{5}{16}$

Diagonals +5940 #





Max $\frac{L}{r} = 200$ will govern

$31\times\frac{1}{2} = 15.5, \quad \frac{15.5\times 12}{200} = 0.93$ reqd 'r'

Try 2-L $3\frac{1}{2}\times 3\times\frac{5}{16}$, Laced $r_y = 1.10$

$$r_x = \sqrt{\frac{I}{A}} = \sqrt{\frac{I_0 + Ad^2}{A}} = d+$$

$d = 6.19 - 0.81 = 5.38$ ∴ OK for X-axis for which $L = 31'$

Top Laterals Diagonals — 2-L $3\frac{1}{2}\times 3\times\frac{5}{16}$

+29,000 #

$$\frac{29000}{16{,}000} = 1.82\ \square'' \text{ reqd. A} \quad [5.4.1.2]$$

2-L  $3\frac{1}{2}\times 3\times\frac{5}{16}$, (2-1"φ holes out of each)

$= 2(1.93 - 2\times 1\times 0.31) = 2.62\ \square''$

Support at mid-length by hanger from stringer.

Bottom Laterals — 2-⌐L $3\frac{1}{2}\times 3\times\frac{5}{16}$

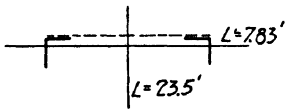
Riveted Truss Highway Bridge — Design Sheet – RT 14

Top Strut – 13,150

One axis $L = 7.83'$
Other " $L = 23.5'$

$\frac{7.83 \times 12}{140} = 0.67$ reqd. r — $L\ 3 \times 3 \times \frac{5}{16}$, $r = 0.92$ (r min for rivet in ea. leg)

Try $2\text{-}L\ 3 \times 3 \times \frac{5}{16}$



$\frac{L}{r} = \frac{7.83 \times 12}{0.92} = 102$

Allowable $\frac{P}{A} = \frac{24,000}{1 + \frac{(102)^2}{13,500}} = \frac{24,000}{1.77} = 13,560\ \#/\square''$

Actual $\frac{P}{A} = \frac{13,150}{2 \times 1.78} = 3,700\ \#/\square''$ Use.

Since section is minimum, use for all other portal members.

Portal: 2-L $3 \times 3 \times \frac{5}{16}$ laced

Reaction

Dead (gross) =	105,000 ±		105,000
Live: 700×60 + 28,300 =	70,300	×2 =	140,600
Impact: $\frac{70,300 \times 50}{245}$ =	14,400	×2 =	28,800
	189,700#		274,400#

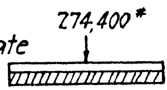
End Bearing

Bearing on Masonry

[5.4.6] $\frac{189,700}{600} = 316\ \square''$ reqd.

$13\frac{1}{2} \times 24 = 324\ \square''$

Bed Plate



$$d^2 = \frac{\frac{274,400}{2} \times 3\frac{3}{8}}{\frac{24,000 \times 24}{6}}$$

$$d^2 = \frac{137,200 \times 3\frac{3}{8}}{96,000} = 4.83 \qquad d = 2.2''$$

Use $24 \times 2\frac{1}{2} \times 1'\text{-}1\frac{1}{2}''$

Arbitrary

Top Plate $10 \times 3 \times 1'\text{-}4''$ (Cut from 20×3 std.)

3" ϕ Pin, (Use Recess Nut for 4 pin — Pg. 114 Carnegie

12 WF 190 × 1'-4" Reduce to 10" width.

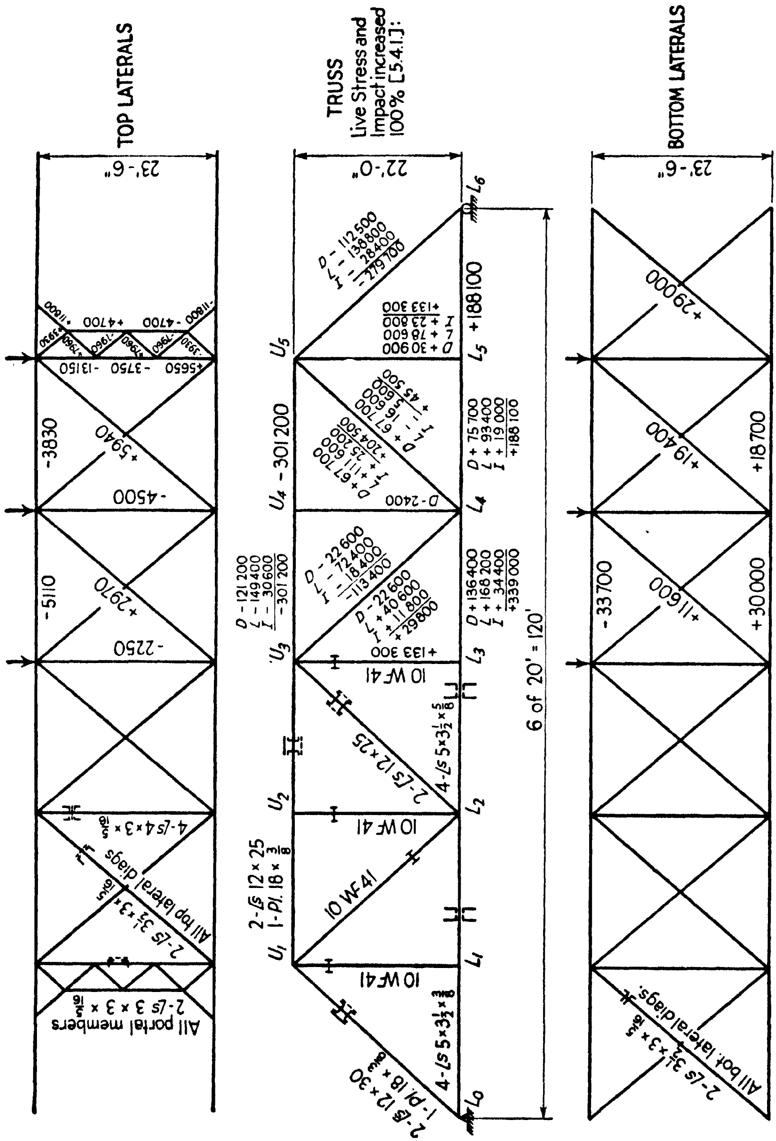
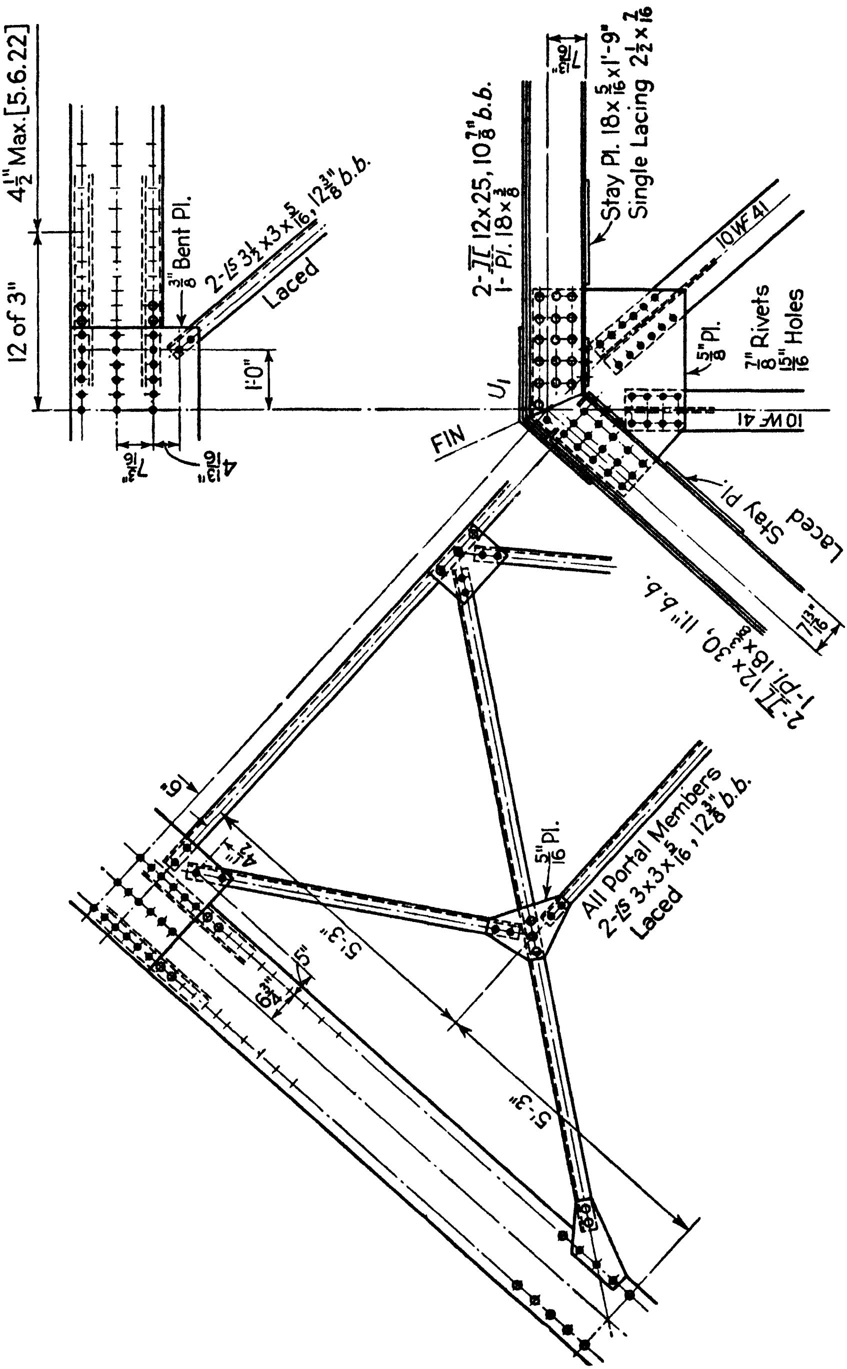

TOP LATERALS
TRUSS
Live Stress and Impact increased 100% [5.4.1.]:
BOTTOM LATERALS
6 of 20' = 120'

Fig. 7–1



4½" Max. [5.6.22]
12 of 3"
3/8" Bent Pl.
2-∟s 3½x3x5/16, 12⅜" b.b.
Laced
1'-0"
4 13/16"
U1
FIN
2-][12x25, 10⅞" b.b.
1-Pl. 18x⅜
Stay Pl. 18x5/16x1'-9"
Single Lacing 2½x7/16
10WF41
5/8" Pl.
7/8" Rivets
15/16" Holes
10WF41
Stay Pl.
Laced
7 13/16"
2-][12x30, 11" b.b.
1-Pl. 18x⅜
All Portal Members
2-∟s 3x3x5/16, 12⅜" b.b.
Laced
5/16" Pl.
5'-3"
5'-3"
6¾"
5"

FIG. 7-2

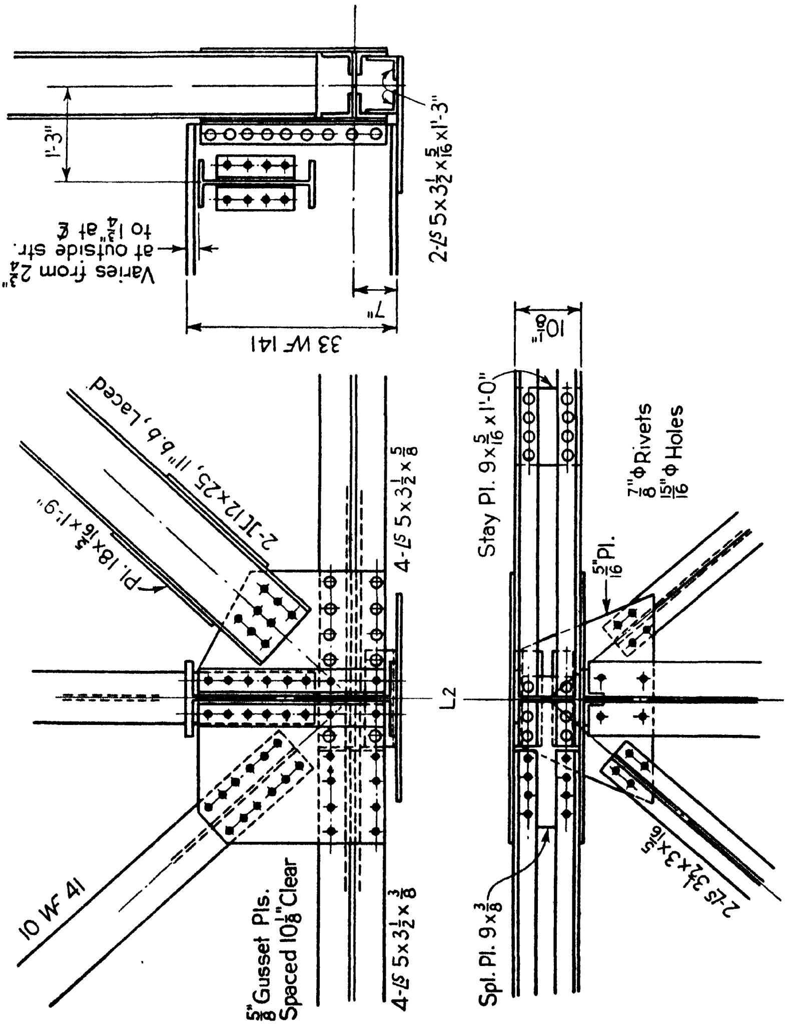

33 WF 141
2-∠s 5×3½×5/16×1'-3"
Varies from 2¾" at outside str. to 1¾" at ℄
2-][12×25, 11" b.b., Laced
Pl. 18×5/16×1'-9"
4-∠s 5×3½×5/8
4-∠s 5×3½×3/8
10 WF 41
5/8" Gusset Pls. Spaced 10⅛" Clear
L2
Stay Pl. 9×5/16×1'-0"
Spl. Pl. 9×3/8
7/8" Φ Rivets
15/16" Φ Holes
5/16" Pl.
2-∠s 3½×3×5/16

FIG. 7–3

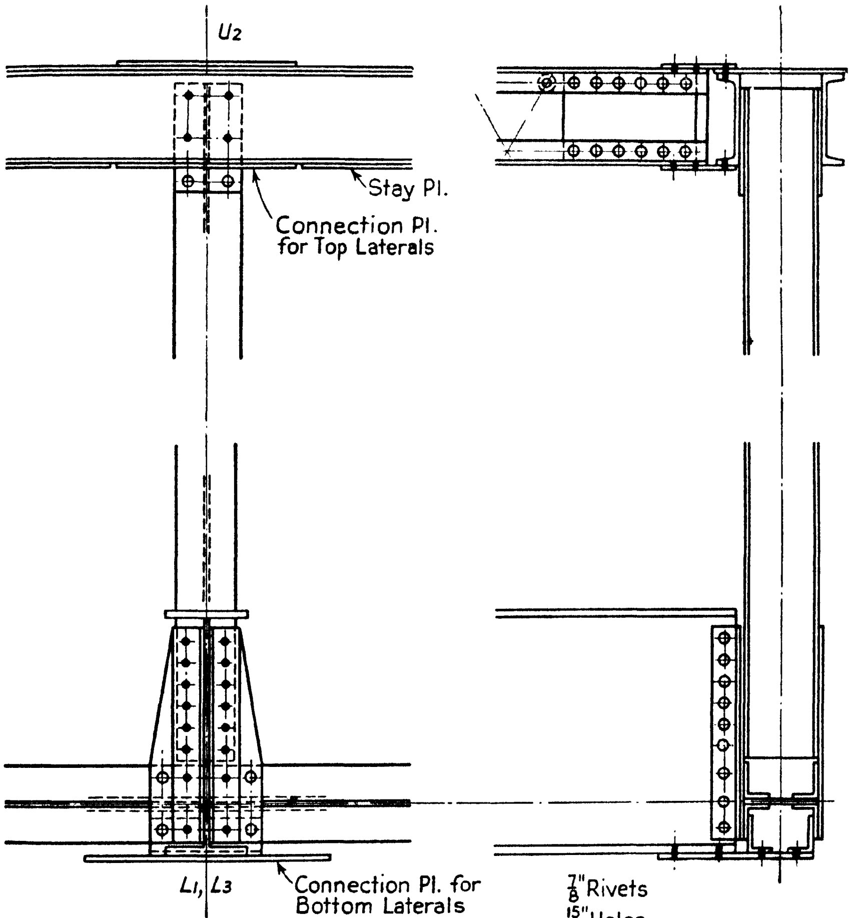
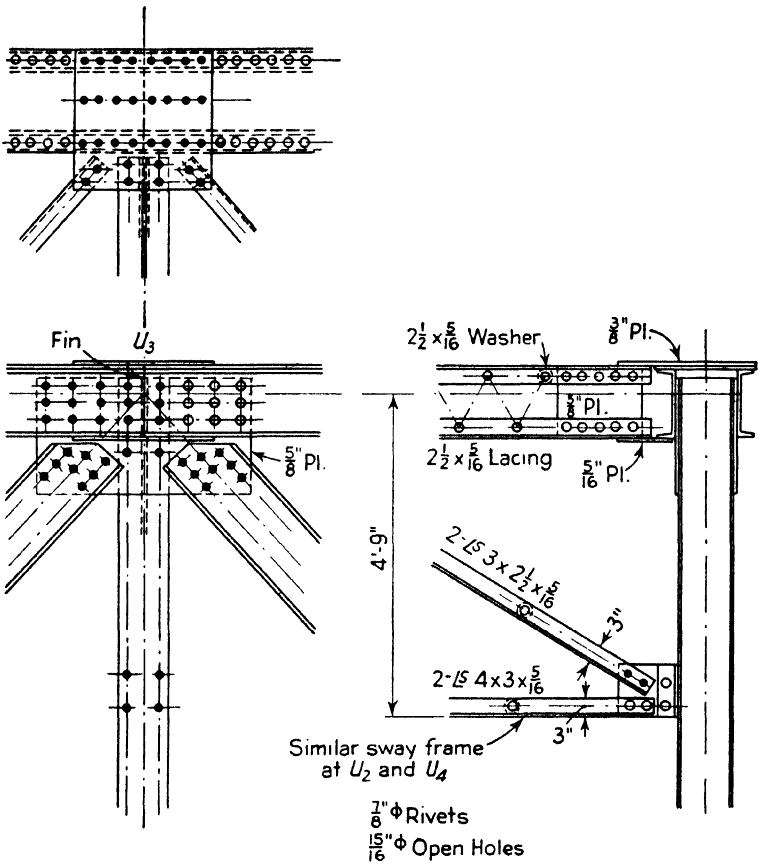

U2
Stay Pl.
Connection Pl.
for Top Laterals
L1, L3
Connection Pl. for
Bottom Laterals
7/8" Rivets
15/16" Holes

FIG. 7–4



Fin
U_3
$\frac{5}{8}$" Pl.
$2\frac{1}{2}\times\frac{5}{16}$ Washer
$\frac{3}{8}$" Pl.
$\frac{5}{8}$" Pl.
$2\frac{1}{2}\times\frac{5}{16}$ Lacing
$\frac{5}{16}$" Pl.
4'-9"
2-∠s $3\times2\frac{1}{2}\times\frac{5}{16}$
3"
2-∠s $4\times3\times\frac{5}{16}$
3"
Similar sway frame at U_2 and U_4
$\frac{7}{8}$"Φ Rivets
$\frac{15}{16}$"Φ Open Holes

FIG. 7–5

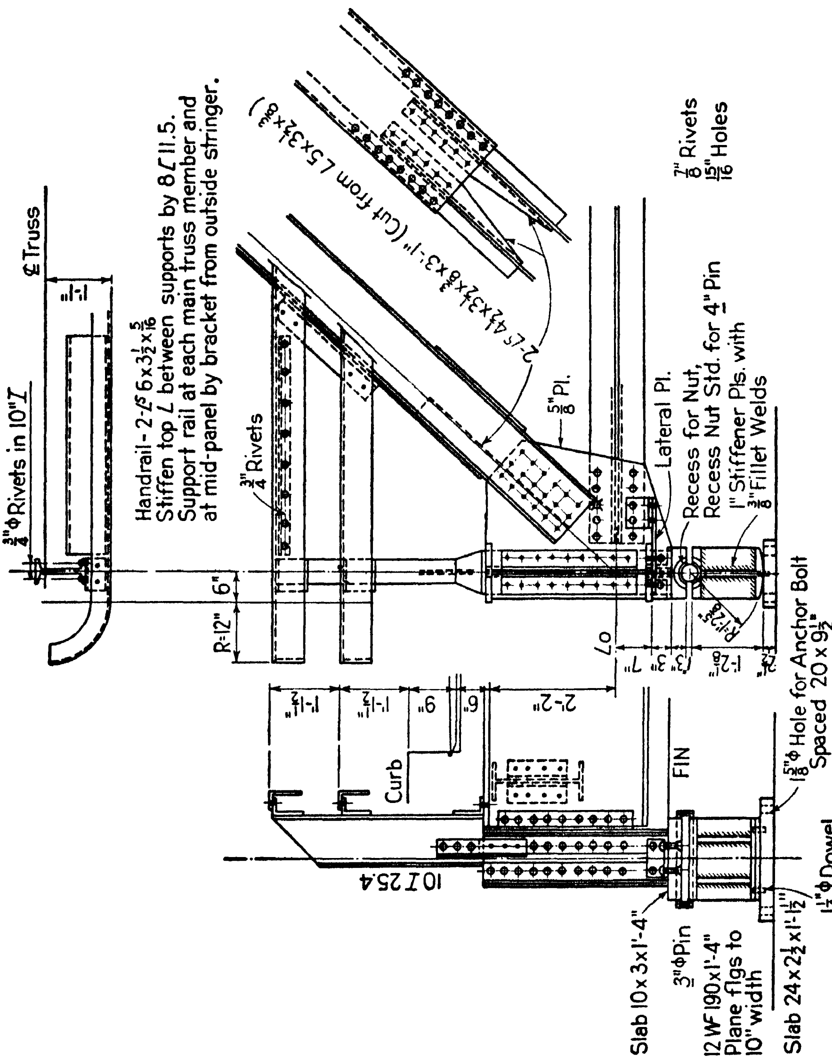

℄ Truss
Handrail - 2-⌐s 6x3½x5/16
Stiffen top ⌐ between supports by 8[11.5.
Support rail at each main truss member and
at mid-panel by bracket from outside stringer.
¾"Φ Rivets in 10"I
¾" Rivets
R=12"
6"
2-⌐s 4½x3½x⅜x3'-1" (Cut from ⌐5x3½x⅜)
⅝" Pl.
Lateral Pl.
Recess for Nut,
Recess Nut Std. for 4" Pin
1" Stiffener Pls. with
⅜" Fillet Welds
⅞" Rivets
15/16" Holes
L0
1'-1½"
9"
6"
2'-2"
Curb
10 I 25.4
FIN
Slab 10x3x1'-4"
3"Φ Pin
12 WF 190x1'-4"
Plane flgs to
10" width
Slab 24x2½x1'-1½"
1¼"Φ Dowel
1⅝"Φ Hole for Anchor Bolt
Spaced 20x9½"

FIG. 7-6

CHAPTER VIII

PIN-CONNECTED BRIDGES

8–1. It has generally been believed that there is a limit of span length below which a riveted truss — because of less vibration and in spite of some increase in weight and erection cost — is more satisfactory and above which a pin-connected truss should be used. This limiting span length has gradually increased until at the present time it is doubtful that a pin-connected railroad bridge would be constructed if its span were less than, say, 500 ft. This means that relatively fewer pin-connected spans are being built. However, it is essential that the student have some knowledge of this type of structure, and this may be obtained best by study of a *short* span, such as the one of this chapter.

8–2. Pins. As denoted by the name, a distinguishing feature of the pin-connected truss is that the connections at the joints are made by means of pins. These are cylinders of steel which are inserted in holes in the members and which carry their loads in shear and bending. A study of the sketches on sheets PCB 11 and 12, and of Figs. 8–3 and 8–4, will aid in an understanding of the function of the pins. The Carnegie " Pocket Companion " lists nuts for pins up to 24 in. in diameter. However, the majority of pins used are under 12 in. in diameter. Common practice is to vary pin diameters by quarter inches, and to limit the number of sizes per truss to three.

To hold the pins in position, use is made of recessed, or Lomas, nuts which attach to threaded ends of reduced diameter. Nuts of this type are illustrated in the steel handbooks.

8–3. Eye Bars. Another feature of the pin-connected truss which is noteworthy is the use of eye bars for the majority of the tension members. It will be evident that a member of uniform cross section, free from rivet holes, is ideal for carrying tension, provided a satisfactory end connection can be made. In the case of the eye bar, a head is upset — specifications prohibit welding — at each end of a member of rectangular cross section and a hole is bored for the pin, Fig. 8–1. In general, the thickness of the head is the same as the thickness of the bar itself; however, a slight over-run is permitted [**531**].[1] Usually a truss member is

[1] In this chapter, bold-face numerals in brackets refer to articles in the 1935 A.R.E.A. Specifications reprinted (with two exceptions) in Appendix D.

composed of a number of eye bars. In order that the distance between end holes may be identical, the bars to form a single truss member are clamped together and the holes are bored while the bars are so held [532].

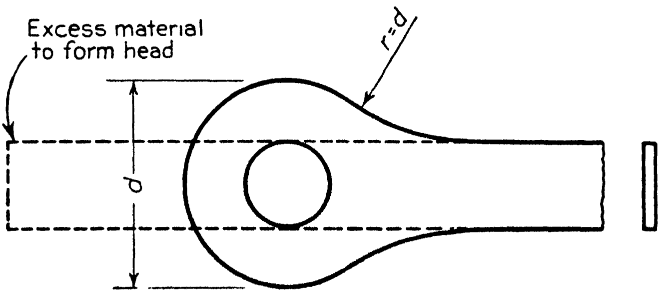


FIG. 8-1

For safety it is essential that the head detail be of such proportions that failure under test will be certain to occur in the body of the bar. To ensure such failure, specifications commonly demand that the area cut by a section passed through the center of the pin hole, and normal to the axis of the bar, shall be 35 per cent greater than the area of the bar itself. Actually, there is no designing of head shapes by the individual engineer as no one wishes to stand the expense of providing the equipment to form special heads. Consequently, use is made of the bars which the manufacturers are prepared to furnish and which are listed in the handbooks.

8-4. Pin-Connected Railroad Bridge. Sheets PCB 1 to PCB 24 show the necessary design computations for a 224-ft span. Much of the information necessary for the understanding of these sheets is given directly on them. In some cases, however, additional comment appears desirable as follows.

Problem 8-1. Show that the cross-tie given on sheet PCB 1 is adequate for the loads of the problem.

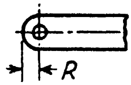
(PCB 2) The assumed depth of the stringer will be seen to be one-seventh of its span. The common range, when one stringer is used per rail, is $\frac{1}{6}$ to $\frac{1}{8}$ and is found satisfactory from the standpoints of rigidity, economy, and depth for end connection. The thickness of the end connection angles is fixed at $\frac{5}{8}$ in. by experience, which indicates that thinner angles will crack in the fillet at the junction of the legs.

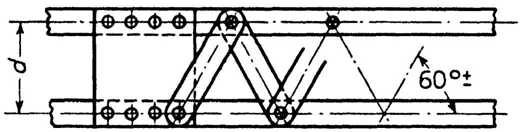
Problem 8-2. Check the value of quarter-point and center-line flange rivet pitch in the stringer. These were based on the assumptions of Chapter V that quarter-point live shear equals five-eighths, center shear two-sevenths, of live end shear.

(PCB 7) Comparison with earlier specifications will show here a much lower coefficient for the impact than has been customary.

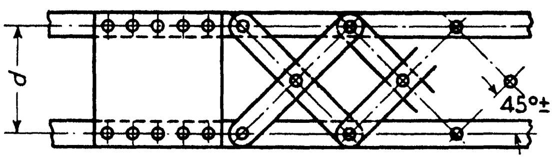
(PCB 8) The sketches on this sheet give a dead stress of +9.1 in bar U_4L_4. This is different from the value of -16.8 given on PCB 6. If the reason for this change is not clear, the student should consult Ex. 4-10 of "Structural Theory."

FLAT BAR LACING
SINGLE RIVET
[401, 417, 420, 421, 524]

Flange Width	Rivet Diameter	Width of Bar	R	
$2\frac{1}{2}$ to 4"	$\frac{3}{4}$"	$2\frac{1}{4}$, $2\frac{1}{2}$"	$1\frac{5}{16}$"	
3 to 5"	$\frac{7}{8}$"	$2\frac{3}{4}$, 3"	$1\frac{9}{16}$"	
$3\frac{1}{2}$ to 5"	1"	3	$1\frac{9}{16}$"	



May be used if d = 15" or less



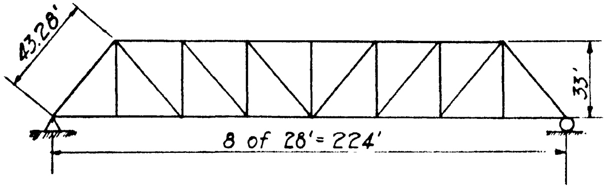
Must be used, with ONE rivet bars, if d is greater than 15"

FIG. 8-2

Also, it will be noted in the top sketch that a counter in panel 2-3 is not needed. However, account must be taken of **[305]**. In order to do this the ratio of dead load to total load in each chord member has been computed and recorded on the sketch. This ratio is least, 0.252, in bars L_0U_1 and L_0L_2, and **[305]** is applied on the basis of these members.

Pin-Connected Truss Railroad Bridge — Design Sheet — PCB1

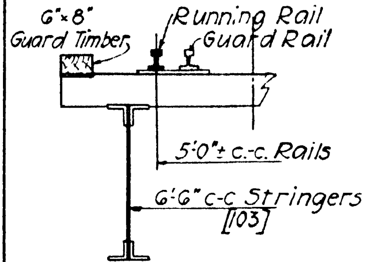
Data



Single-Track Pin-Connected Railroad Bridge
Span: 224'-0" c.-c. bearings.
Live Load: Cooper E-72
Specifications, A.R.E.A. 1935 (See Appendix.)

Ties
8"×10"×10'-0"
14" c.-c.

Use ties 8" wide × 10" deep × 10'-0" long, spaced 6" clear [109]



Weight of Floor

Ties: $\frac{8\times10}{12}\times60\times10\times\frac{12}{14} = 285^{\#}$ [202]

Guard Timbers: $\frac{6\times8}{144}\times60\times2 = 40$ ["]

Rails, Etc. $= 200$ ["]

Weight per ft. track $= 525^{\#}$

Weight per ft of stringer $= \frac{525}{2} = 265^{\#}$

Stringer

Moment and Shear

Assume stringer and bracing at 275 lb. per ft.
Track 265
Dead Load 540

Impact [206]

$$\frac{100}{6.5} + (100 - 0.6\times28) = 15.4 + 100 - 16.8 = 98.6\%$$

Stringer	Moment		Shear	
Live	$182{,}700\times7.2\times\frac{1}{2}$	$= 658{,}000'^{\#}$	$= 30200\times7.2\times\frac{1}{2}$	$= 108{,}700^{\#}$
Impact	[206] 98.6%	$= 650{,}000$	98.6%	$= 107{,}100$
Dead	$\frac{1}{8}\times540\times\overline{28}^2$	$= 52{,}900$	$540\times\frac{28}{2}$	$= 7{,}600$
		$1{,}360{,}900'^{\#}$		$223{,}400^{\#}$

To increase the unit stress one-third in L_0L_2 the increase in total load must be $609.1 \times \frac{1}{3} = 203.0$ kips. The old value for live + impact is 455.4 kips. Therefore, the live load must be increased $203.0/455.4 =$ 44.6 per cent. The second sketch on the sheet shows the increased live and impact loads in the web members.

Under these increased loads it is seen that a counter is *almost* needed in panel 2–3. This fact illustrates the reason for [**305**]. Frequently the increased loading makes necessary a counter where none is required by the regular loading, thereby providing a structure which can carry (at higher stresses) an increase over the regular design loads without requiring troublesome and costly field alterations. In the present case the counter is so nearly required that it will pay to insert a member with minimum section. This can be done at slight expense and may save trouble in the later life of the bridge.

(PCB 9) The designing of members which commences on this sheet has been done in the order: verticals, eye bars (diagonals and bottom chords), upper chords, and end post. In studying the design it will be noted that the clearances follow directly, one after the other, as a consequence of this logical order of design.

Problem 8–3. Design the stay plates and lacing bars for member U_2L_2. See [**420, 421**] and Fig. 8–2.

Ans. Stay plate $12 \times \frac{3}{8}$
Single lacing $2\frac{1}{4} \times \frac{3}{8}$

Problem 8–4. Show that for lacing bars which make an angle of 60° with the axis of the main member the greatest stress to be transmitted by the rivets equals the bar stress, whereas with lacing at 45° the stress equals $\sqrt{2}$ times the bar stress.

(PCB 10) In arranging the eye bars it will be seen that the study has been carried one panel past the center line of the truss. This was done in order to ensure identical conditions at joints L_3 and L_5. In connection with eye bars, attention is directed to the following from pp. 529–530 of "Bridge Engineering," by Dr. J. A. L. Waddell:

In determining the character and size of the various members, care should be taken to see that the pin packing is satisfactory. In so doing, ample clearance must be allowed between the different members attached to each pin. For compound webs, one thirty-second ($\frac{1}{32}$) of an inch should be allowed for each additional plate above two on account of the tendency of such compound plates to build out or thicken. As far as practicable, rivets should not be countersunk, although they may be flattened; and the outstanding legs of angles should be cut just as little as possible, especially in compression members, as such members are materially weakened thereby. In any case projecting rivet heads and angle legs must be taken into account in determining the packing. Not less than one-sixteenth ($\frac{1}{16}$) of an inch should be allowed between adjacent bars in different panels, whereas, on account of painting requirements, at least one-half ($\frac{1}{2}$) of an inch is necessary between bars in

Pin-Connected Truss Railroad Bridge Design Sheet PCB2

Stringer (Continued)

Assume a depth of 48"

Required thickness $= \dfrac{223{,}400}{11{,}000 \times 48} = 0.42"$ [301]

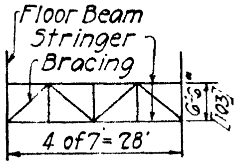
Use $48 \times \frac{7}{16}$

Web $48 \times 7/16$

Assume 6"×6" Flange Angles

$s_c = 18{,}000 - 5\left(\frac{L}{b}\right)^2$ [301]

$= 18{,}000 - 5\left(\frac{84}{12.44}\right)^2 = 17{,}700\ \#/\square"$



Flange $4\text{-}L\ 6 \times 6 \times \frac{15}{16}$

To prevent water pocket, place Ls 48" b-b. [526]

$17{,}700 = \dfrac{1{,}360{,}900 \times 12 \times 24}{I} \quad \left(= \dfrac{Mc}{I}\right)$

Required I = 22,100 in.^{4}

I of $48 \times \frac{7}{16}$ web = 4,032

18,068

Approximate: Neglect $I_{c.g.}$ of Ls to compensate for neglecting rivet holes in web.

$4(\text{Net area} \times 22.16^2 \pm) = 18{,}068$

Net area $= 9.20\ \square"$

$1\text{-}L\ 6 \times 6 \times \frac{15}{16}\ (1\text{-}\frac{7}{8}"\phi \text{ hole out}) = 10.57 - 0.94 = 9.43\ \square"$

Check: I web = 4,030

I, Ls $= 4(33.7 + 10.37 \times \overline{22.16}^2)$ = 20,430

24,460

I of rivet holes $= 2(1 \times 2.31 \times \overline{21.75}^2)$ = 2,190

22,270

$> 22{,}100 \therefore$ OK

End Connection

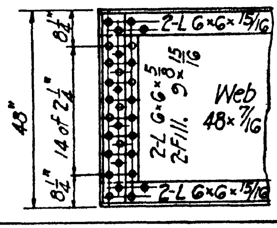
$\frac{7}{8}"\phi$ Rivets, bearing on $\frac{7}{16}"$ metal

$\frac{7}{8} \times \frac{7}{16} \times 27{,}000 = 10{,}320\#$ [301]

$\frac{7}{8}"\phi$ Rivets, double shear

$2 \times 0.601 \times 13{,}500 = 16{,}220\#$

$\dfrac{223{,}400}{10{,}320} = 22 \qquad \dfrac{223{,}400}{16{,}220} = 14$



Rivet Pitch

$m(\text{vertical load per inch}) = \left(45{,}000 \times \frac{1}{3 \times 14}\right) + 100\%\ \text{Impact} = 2150\#$ [203] ties' spacing

End Pitch: $p = \dfrac{10{,}320}{\sqrt{\left(\frac{223{,}400}{44.32}\right)^2 + 2150^2}} = 1.88"$ ←

Similarly, Quarter point pitch = 2.70", Center line pitch = 4.13" ←

the same panel. One-quarter ($\frac{1}{4}$) of an inch, or more, should be provided between a built member and an eye bar or between two built members, this being increased properly for projecting rivet heads.

If 9-in. eye bars are assumed for the bottom chord, the pins at L_2, L_3, L_4 must be $9 \times 0.8 = 7.2 = 7\frac{1}{4}$ in. min. [**440**].

In the handbook (Carnegie, p. 342) it will be seen that the minimum thickness of either a 7-in. or an 8-in. bar for a $7\frac{1}{4}$-in. pin is $1\frac{1}{8}$ in. This will account for the excess area in member U_2L_3.

The counters in panels 2–3 and 3–4 require little area. To provide this in a bar with a standard head would mean the use of a large excess area. Therefore, 5×1 in. bars with a special head have been selected.

Experience teaches that it is impossible to adjust the length of chords, verticals, main diagonals, and counters in a panel so nicely that the diagonals and the counters will be sure to fit. For this reason, it is customary to make the counters adjustable, that is, with a turnbuckle near their mid-point.

(PCB 11) The stresses shown on PCB 8 are not simultaneous stresses. Therefore, in investigating the stresses in a pin, it is necessary to find the stresses in all bars entering the joint for some critical position of the live load. For the pins of this truss, the critical position may be assumed to be that which causes a maximum stress in the main diagonal which attaches to the pin.

In computing the stress, note that conditions are symmetrical about the center line of the pin and that, therefore, the values of loads are only one-half of the totals in the members.

Problem 8–5. Check the use of a $7\frac{1}{4}$-in. pin at joint L_2.

Problem 8–6. Check the use of a $7\frac{1}{4}$-in. pin at joint L_4.

Problem 8–7. Check the use of a $6\frac{1}{2}$-in. pin at joint U_2.

Problem 8–8. Check the use of a $6\frac{1}{2}$-in. pin at joint U_3.

Problem 8–9. Check the use of a $6\frac{1}{2}$-in. pin at joint U_4.

(PCB 12) On this sheet use was made of two values from *later* sheets. In any design problem, use must be made of assumed values which are later checked. If the assumption has been fortunate, no redesign is needed; if not, the design must be revised. Frequently, several sheets of design will be computed at the same time.

(PCB 14) In the computation to determine the size of the cover plate, the gage line on the 4-in. angles has been pushed out as far as it can go. This was done to give the rivets in these lines clearance over the side plates.

Pin-Connected Truss Railroad Bridge   Design Sheet   PCB3

Stringer (Continued)

Web Stiffener

[433] $\frac{48-2\times 6}{7/16} = 82$ ∴ Stiffeners are needed.

At end, $s = \frac{223{,}400}{48 \times 7/16} = 10{,}640$ #/□"

From chart (Fig. 4-10) $d = 53"$ clear distance

Use 2-L $5 \times 3 \times \frac{3}{8}$, spaced (from end) 4'-6", 4'-6", 5'-0"   ← use

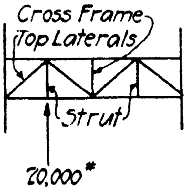
Bracing

Top Laterals [212, 213]

Length $7\sqrt{2} \pm = 10'$

Max. $\frac{L}{r} = 120$ [304]

$\frac{10 \times 12}{120} = 1$ Try 1-L $6 \times 6 \times \frac{3}{8}$, min. $r = 1.19$



Stringer Flange Stress

$= \frac{1{,}360{,}900 \times 12}{44.32} = 369{,}000$#

Shear and Stress in Diagonal:

$369{,}000 \times 2 \times 0.025 = 18{,}450$ [213]

$\left(18450 + \frac{3}{4} \times 20000\right)\sqrt{2} = 47{,}200$# Lateral Stress

Allowable fiber stress in assumed diagonal angle

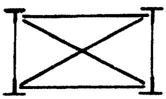
$= 15{,}000 - \frac{1}{4}\left(\frac{120}{1.19}\right)^2 = 12{,}500$#/□" ±

Actual Stress $= \frac{47{,}400}{4.36} = 10{,}850$ #/□" O.K. Use 1-L $6 \times 6 \times \frac{3}{8}$ ← use

Cross Frame [438]

Use one at ℄ of stringer

Each member, 1-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$



Cross Struts

Max. $\frac{L}{r} = 120$ $r_{min.} = \frac{78}{120} = 0.65$

Use 1-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$, min. $r = 0.69$ ← use

Weight

Stringer		Laterals	
1-Web $48 \times 7/16 \times 28'$-0"	= 2000#	4-L $6 \times 6 \times 3/8 \times 10'$-0"	= 596#
4-L $6 \times 6 \times 15/16 \times 28'$-0"	= 3955	4-L $3\frac{1}{2} \times 3\frac{1}{2} \times 3/8 \times 6'$-6"	= 221#
10-L $5 \times 3\frac{1}{2} \times 3/8 \times 3'$-$10\frac{1}{8}$"	= 400	2-L $3\frac{1}{2} \times 3\frac{1}{2} \times 3/8 \times 8'$-0"	= 136
10-Fill. $3\frac{1}{2} \times 15/16 \times 2'$-$11\frac{1}{2}$"	= 331		953#
4-L $6 \times 6 \times 5/8 \times 3'$-$10\frac{1}{8}$"	= 372	$7{,}523 + \frac{953}{2} = 8000$#	
4-Fill. $9 \times 15/16 \times 2'$-$11\frac{1}{2}$"	= 340	$\frac{8{,}000}{28} = 286$ #/'	
500 ± Rivet Heads	= 125	Checks (closely enough)	
	7523	275#/' assumed.	

(PCB 15) In locating centroid, etc., the moments have been figured about an axis through the bottoms of the lower angles. This places the entire area of the member on one side of the axis and eliminates a possibility of numerical error which exists if the axis is so located that some arms must be considered positive, some negative.

Problem 8–10. Show that the section given for U_2U_3 is satisfactory.

(PCB 17) The figures seem to indicate that the area of this member might be reduced by about 2.5 sq in., but this cannot come from the webs or the cover plate (see PCB 14). Also, to decrease the size of the bottom angles would increase the eccentricity and necessitate the use of a deeper web. Therefore, only the top angles could be reduced. But these, through which the heavy cover plate is stressed, are already thinner than the parts which they connect. Good practice suggests that the member be used as designed, in spite of the excess area.

(PCB 19) It will be noted that the top chord bars and the end post have been made of the same general form. This is necessary not only to make simple the matter of connections between top chord members which are practically continuous (the splices being placed beyond the joint in the panel of minimum stress), but also because the change in eccentricity (distance from center of web to centroid of section) must be as little as possible. The pins will be located on a working line which will be the average value of the eccentricities. However, any change of eccentricity from member to member will result in the introduction of secondary stress, a matter treated in Chapter XI of "Structural Theory."

(PCB 21) The pin at L_0 comes out larger than assumed. This would slightly reduce the bearing areas required but would not change the distances between parts enough to reduce the moments to the point where a 9-in. pin would serve. Also, there is an excess of area through the pin hole in bar L_0L_2 so that no change will be required in this member.

(Fig. 8–3) The detail where a floor beam connects to a vertical is shown in Fig. 8–3, which is drawn for joint L_3. A difficulty in this connection arises from the fact that the lower corner of the floor beam must be cut away to provide clearance over the eye bars, pin, and nut. This reduces the floor-beam depth to such an extent that the connection angles and web must be extended above the top-flange angles in order to provide room for sufficient rivets to transmit the floor-beam load.

The reason for certain of the dimensions will be made known. The 1 ft $\frac{1}{4}$ in. dimension from the center of the chord to the bottom of the floor beam is fixed by the half depth of the member L_0L_2. This arrange-

Pin-Connected Truss Railroad Bridge Design Sheet PCB4

Floor Beam

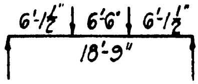
End post will probably have an over-all width 32" or 33". ∴ truss spacing for 16' clearance [105] must be 16'-0" + 2'-9" = 18'-9" = Floor Beam Span [106]

Max. F.B. reaction $= 41{,}200 \times 7.2 \times \frac{1}{2} = 148{,}300^{\#}$

Dead load at stringer connection $= 8000 + (265 \times 28) = 15{,}420^{\#}$

Assume F.B. weighs $250^{\#}/'$

Impact $= \frac{100}{18.75} + (100 - 0.60 \times 18.75) = 94.17\%$ [206]



Moment and Shear

		Shear	Moment
Live		148,300#	× 6.125 = 907,000'#
Impact	94.17%	139,500	94.17% 853,000
Dead		15,420	× 6.125 = 94,500
"	$250 \times \frac{18.75}{2} =$	2,350	$\frac{1}{8} \times 250 \times 18.75^2 =$ 11,000
		305,570	1,865,500

Web — $66 \times \frac{7}{16}$

Assume 66" depth

$$\frac{305{,}570}{11{,}000 \times 66} = 0.42" \qquad \text{Use } 66 \times \frac{7}{16}$$

Flange — $4\text{-}L\ 6 \times 6 \times \frac{13}{16}$

Try 6×6 L, 66" b.b. Unsupported length = distance c.-c. stringers = 6'-6"

$$18{,}000 - 5\left(\frac{78}{12.44}\right)^2 = 17{,}800^{\#}/\square" \text{ allowable stress}$$

Required Moment of Inertia $= \frac{1{,}865{,}500 \times 12 \times 33}{17{,}800} = 41{,}500$

I of web $= 10{,}482$

31,018

Required net area of one L $= \frac{31{,}018}{4 \times 31.2^2} = 7.97\square"$

$1\text{-}L\ 6 \times 6 \times \frac{13}{16}$ (1-1" ϕ hole out) $= 8.28\square"$

End Connection

Must follow design of truss. See Fig. 8-3

Rivet Pitch

End Sections

$$p = \frac{10{,}320 \times 62.4}{305{,}570} = 2.11"$$

Web Stiffeners

$$s = \frac{305{,}570}{66 \times 7/16} = 10{,}600^{\#}/\square"$$

From chart (Fig. 4-10), d = 47"

Use one pair in each end panel

Use

$2\text{-}L\ 5 \times 3\frac{1}{2} \times \frac{3}{8}$

PCB 4

ment permits all bottom chord lateral plates to be at the same elevation. The 1 ft $9\frac{11}{16}$ in. dimension from the center plane of the truss to the cut back portion of the floor beam is based on the maximum width over the eye bars (1 ft $3\frac{13}{16}$ in. at L_3; see sheet PCB 11), the pin projection, $2\frac{3}{8}$ in. (steel handbook), the depth of the pilot nut, 3 in. (a tapered nut, the maximum diameter of which equals the pin diameter; used in inserting the pin through the eye-bar holes of the members at a joint; removed and replaced by a recessed nut: see "Structural Engineers' Handbook," KETCHUM), and $\frac{1}{2}$-in. clearance. The 1 ft 1 in. dimension from center of chord to top of cut back on floor beam is that required to clear eye bar U_2L_3. The $8\frac{1}{2}$-in. dimension between tops of floor beam and stringer is made such that the rail will have 1-in. clearance over the floor beam, even though the ties be dapped $\frac{1}{2}$ in.

The critical factor in the location of the splice was the provision for the stress in the bottom chord angles at the splice. For the splice in the position shown, two (plus) rows of rivets were needed in the splice plates. The development of the bottom flange angles was figured as follows: Their stress was computed on the old assumption that one-eighth of the web area was available to act with the flange. The four rivets to the left of the splice are limited as follows: If the web were thick enough, these rivets might be figured for four shears. As it is, when double shear, 16.24 kips, is reached on the planes between angles and side plates, the corresponding bearing value on the side plates will be 18.55 kips per sq in. Using this value on both side plates and web plate, the allowable load on the rivet will be 23.33 kips. Four rivets at this value are not enough to develop the chord angles. However, if it be assumed that the angles have started to unload to the side plates through the three rivets to the right of the splice, it may be taken for granted that the detail is satisfactory.

Design practice is to investigate the stress due to bending on the line marked *a*. The student should make this computation in which the curved end angles are customarily neglected.

It will be clear that the purpose of the diaphragm placed in member U_3L_3 is to distribute part of the floor-beam load to the other side of the post.

(Fig. 8–4) This figure shows the expansion end of the truss. Whereas the pin at L_0 was designed for the position of loads which produced maximum stress in bar L_0U_1, the greatest end reaction, taking into account the end floor beam, will occur when wheel 2 is at the end of the truss. For this position of the loads, the reaction will be about 852 kips per truss. By trial, a rocker radius of 36 in. was selected. The corresponding allowable bearing pressure [**301**], for d = 72 in., was 25.4 kips

Pin-Connected Truss Railroad Bridge Design Sheet PCB5	
I-web $66 \times \frac{7}{16}$ = 98# 4-L $6 \times 6 \times \frac{13}{16}$ = 124 Rivet Heads = 6± 228 Stiffeners will raise this value, ∴ 250# assumed is close enough.	Floor Beam (Continued) Weight
For a first approximation, assume that increases in loads and stresses offset each other and that truss designed under 1935 spec. for E-72 will weigh about the same as a truss of 1920 spec. for E-60. Constant's formula for E60 (Theory p. 75) $w = 1.12(9 \times 224 + 700) = 3040$ #/l (Total Steel) Floor System: Stringers $7523 \times 16 =$ 121,000# Bracing $953 \times 8 =$ 7,600 Floor Bm. $300 \times 18.75 \times 7 =$ 39,400 Heavy end connection 168,000 $\frac{168,000}{224} = 750$ #/l ∴ Truss and bracing weigh $3040 - 750 = 2,290$ #/l Check by Hudson's rule Moment, Live $= 7,229,000 \times 7.2 \times \frac{1}{2} =$ 26,050,000′# Impact $= \frac{100}{18.75} + \frac{1800}{224-40} + 10 = 25.13\% =$ 6,550,000 Dead (floor) $= \frac{1}{8} \times \frac{750+525}{2} \times \overline{224}^2 =$ 4,000,000 Dead (truss and bracing) $\frac{1}{8} \times \frac{2290}{2} \times \overline{224}^2 =$ 7,180,000 43,780,000′# Assuming 33′ c.-c. chords, maximum bottom chord stress will be $43,780,000 \times \frac{15}{16} \times \frac{1}{33} = 1,244,000$ # $A_1 = \frac{1,244,000}{18,000} = 69.2$ □″ $W = 69.2 \times \frac{50}{3} = 1152$ #/l ∴ Trusses and bracing weigh $1152 \times 2 = 2304$ #/l Checks Constant's 2290 Use 2400 #/l of bridge as assumed weight.	Truss Dead Load Use

per linear inch (based on a yield point of 33 kips per sq in. [**806, 1203**]) and required a bearing length of 34 in. Modern practice is to use concrete piers and abutments for which the allowable bearing pressure is 600 lb per sq in. [**301c**]. The reaction required a bearing area of about 1420 sq in. which was provided in a rolled-steel slab of 52 in. $\times$ $27\frac{1}{2}$ in. = 1430 sq in. Assuming that the pressure is uniformly distributed under the slab and that the load is applied along a line by the rocker, a slab thickness of $4\frac{3}{8}$ in. was required, the entire 52-in. width of slab being considered available to resist bending. The steel handbook shows a minimum standard thickness of 6 in. for a 52-in. slab. However, the $4\frac{1}{2}$-in. slab shown on the drawing should be specified unless time of delivery enters as an element, in which event it might be desirable to take the thicker slab. In the present case, the mill probably would roll a special slab, $27\frac{1}{2}$ in. by $4\frac{1}{2}$ in. The delay in securing this would be likely to be less than that in obtaining the eye bars, and the steel castings for the rockers. Any special material, such as the three items under discussion, should be ordered at the earliest possible moment.

(Fig. 8–5) This figure shows the change which would be made at the fixed end of the truss where the equivalent of the rocker and slab would be provided in a single casting, the distance from pin to bridge seat being maintained as before. It will be appreciated that at each end of the truss there is a considerable space between the end floor beam and the face of the back wall. To provide support for the rail, brackets in line with the stringers are attached to the floor beams. These have the same depth as the stringers and fasten by the stringer connection rivets. The student should sketch such a bracket with a length sufficient to support one tie.

Pin-Connected Truss Railroad Bridge — *Design Sheet PCB 6*

Truss (Continued)

Dead Stresses

Dead floor beam reaction, Track: $\frac{525}{2} \times 28 = 7350^{\#}$

Stringer: $= 7520$

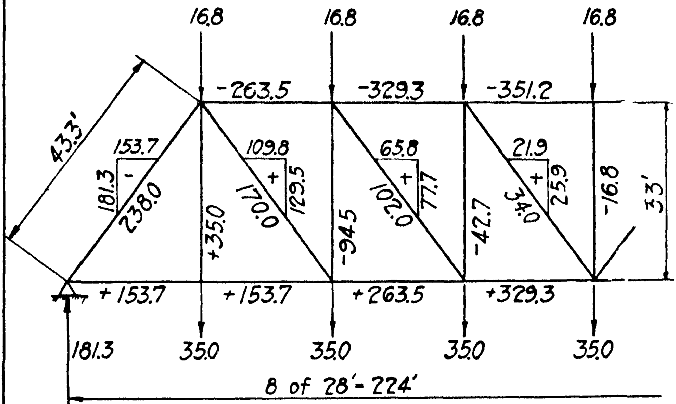
Bracing: $\frac{953}{2} = 500$

Floor Beam $300 \times \frac{18.75}{2} = 2810$

$18{,}180^{\#}$

Truss panel load (top chord) $= \frac{2400}{2 \times 2} \times 28 = 16{,}800^{\#}$

Truss panel load (bot. chord) $= 16{,}800 + 18{,}180 = 35{,}000^{\#}$

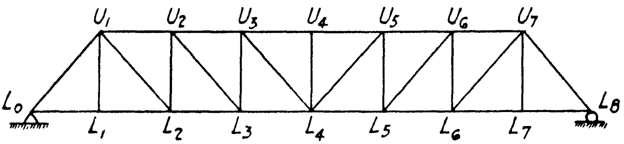


Check $U_3 U_4 = \frac{16.8 + 35.0}{28} \times \frac{1}{8} \times \overline{224}^2 \times \frac{1}{33} = -352$

Pin-Connected Truss Railroad Bridge   Design Sheet   PCB7

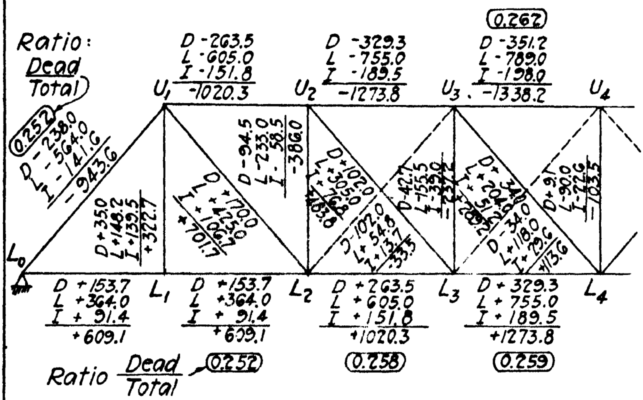
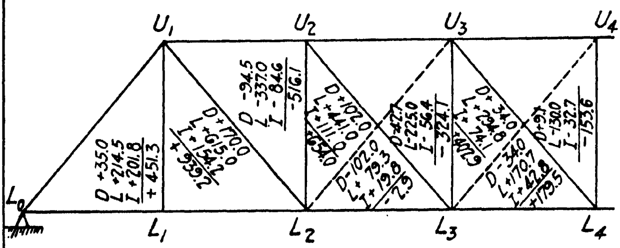
Truss (Continued)

Live Stresses and Impact



Bar	Table, p.p. 384-5	Live Stress	Impact %	Impact
L_0L_1, L_1L_2	$3335 \times \frac{7.2}{2} \times \frac{1}{33}$	364.0	25.1	91.4
L_2L_3, U_1U_2	$5540 \times \frac{7.2}{2} \times \frac{1}{33}$	605.0	25.1	151.8
L_3L_4, U_2U_3	$6914 \times \frac{7.2}{2} \times \frac{1}{33}$	755.0	25.1	189.5
U_3U_4	$7229 \times \frac{7.2}{2} \times \frac{1}{33}$	789.0	25.1	198.0
L_0U_1	$119.2 \times \frac{7.2}{2} \times \frac{43.3}{33}$ Check $364.0 \times 43.3/28 = 563$	564.0	25.1	141.6
U_1L_2	$90.0 \times \frac{7.2}{2} \times \frac{43.3}{33}$	425.0	25.1	106.7
U_2L_3	$64.7 \times \frac{7.2}{2} \times \frac{43.3}{33}$	305.0	25.1	76.8
U_3L_4	$42.2 \times \frac{7.2}{2} \times \frac{43.3}{33}$	204.0	25.1	51.2
U_4L_5	$25.0 \times \frac{7.2}{2} \times \frac{43.3}{33}$	118.0	25.1	29.6
U_5L_6	$11.6 \times \frac{7.2}{2} \times \frac{43.3}{33}$	54.8	25.1	13.7
U_1L_1	$41.2 \times \frac{7.2}{2}$	148.2	94.1*	139.5
U_2L_2	$64.7 \times \frac{7.2}{2}$	233.0	25.1	58.5
U_3L_3	$42.2 \times \frac{7.2}{2}$	155.5	25.1	39.0
U_4L_4	$25.0 \times \frac{7.2}{2}$	90.0	25.1	22.6

* This bar, U_1L_1, is a "floor beam hanger." See [206].

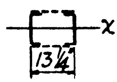
Pin-Connected Truss Railroad Bridge Design Sheet PCB 8	
	Truss Stress Diagram
Ratio, Dead/Live, is least for L_0L_2. To increase unit stress one-third, [305], total stress must be increased $\frac{609.1}{3} = 203.0$ Kips. To produce this, live load must be increased $\frac{203}{364.0+91.4} = 44.6\%$. The corresponding web stresses are here given. 	Design Loads for Web Members at Increased Stresses [305]

Pin-Connected Truss Railroad Bridge Design Sheet PCB9

U_2L_2 $P = -516.1$, Design stress $= \frac{4}{3}\left(15{,}000 - \frac{1}{3}\left(\frac{L}{r}\right)^2\right)$ [301, 305]

Try 2-[15×55, Area $= 2 \times 16.11 = 32.22$ □" $r_x = 5.16$

$\frac{4}{3}\left(15{,}000 - \frac{1}{3}\left(\frac{33 \times 12}{5.16}\right)^2\right) = 17{,}400$ #/□" allowable



$\frac{516{,}100}{32.22} = 16{,}000$ #/□" actual ∴ OK.

For machine riveting, channels must be spaced $5\frac{1}{2}$" clear between toes. Flange width = 3.82"

$3.82" + 5.50 + 3.82 = 13.14"$ Try $13\frac{1}{4}$" b.b.

$I_x = 2 \times 429.0 = 858.0$

$I_y = 2\left(12.1 + 16.11(6.62 - 0.82)^2\right) = 1108$ (>858, ∴ OK)

To keep floor beams alike, U_3L_3 and U_4L_4 will also be spaced $13\frac{1}{4}$" b-b.

U_2L_2 — 2-[15×55 Laced

U_3L_3

$P = -324.1$

Try 2-[15×33.9, $A = 2 \times 9.90 = 19.80$ □", $r_x = 5.62$

$\frac{4}{3}\left[15{,}000 - \frac{1}{3}\left(\frac{396}{5.62}\right)^2\right] = 17{,}800$ #/□" allowable

$\frac{324{,}100}{19.80} = 16{,}370$ #/□" actual ∴ O.K.

U_3L_3 — 2-[15×33.9 Laced

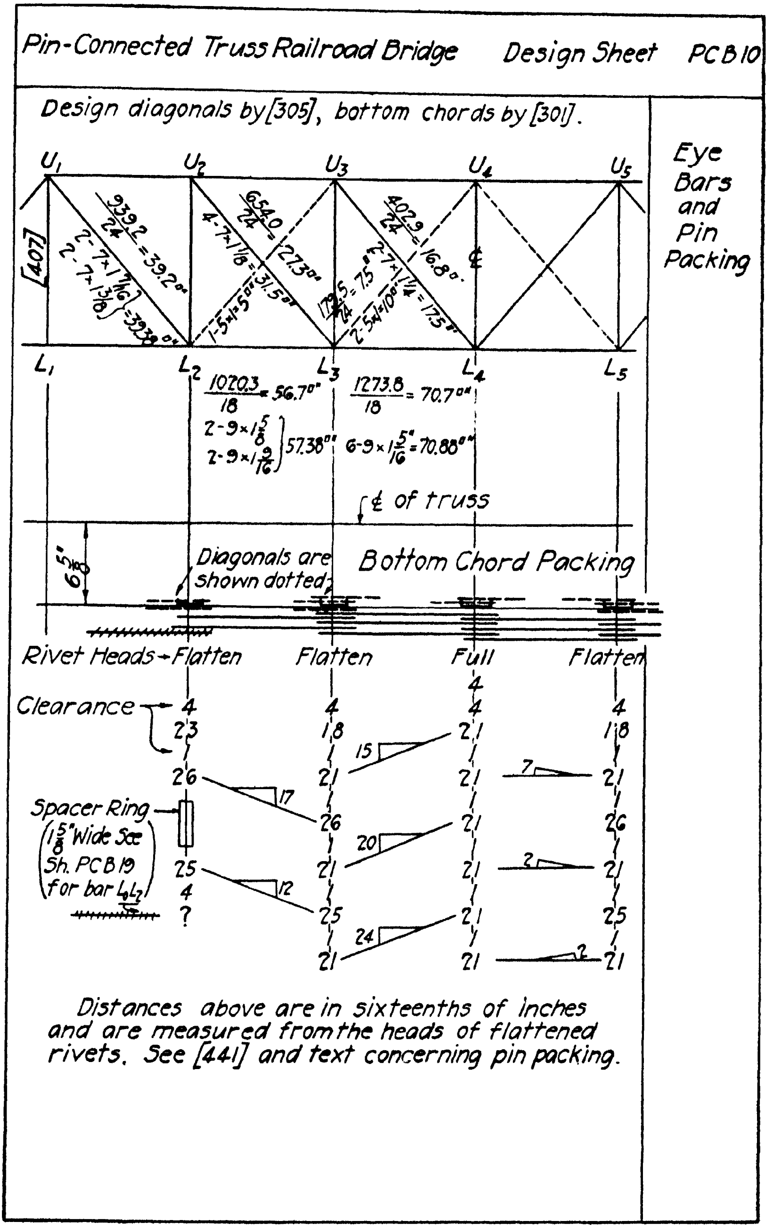
U_4L_4

$P = -153.6$

Considering stress only, 2-[12×25 might be used, but the pin hole at the base of the member would be too large in proportion to the width of the member.

2-[15×33.9 will be used as for U_3L_3.

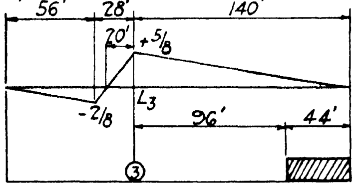
U_4L_4 — 2-[15×33.9 Laced


Pin-Connected Truss Railroad Bridge
Design Sheet
PCB 10
Design diagonals by [305], bottom chords by [301].
Eye Bars and Pin Packing
U1 U2 U3 U4 U5
L1 L2 L3 L4 L5
[407]
939.2/24 = 39.2□"
2-7×1 7/16 2-7×1 3/8 = 39.38□"
654.0/24 = 27.3□"
4-7×1 1/8 = 31.5□"
1-5×1 = 5□"
402.9/24 = 16.8□"
179.5/24 = 7.5□"
2-7×1 1/4 = 17.5□"
2-5×1 = 10□"
1020.3/18 = 56.7□"
2-9×1 5/8 2-9×1 9/16 57.38□"
1273.8/18 = 70.7□"
6-9×1 5/16 = 70.88□"
℄ of truss
6 5/8"
Diagonals are shown dotted
Bottom Chord Packing
Rivet Heads → Flatten
Flatten
Full
Flatten
Clearance
Spacer Ring (1 5/8" Wide See Sh. PCB 19 for bar L0L2)
Distances above are in sixteenths of inches and are measured from the heads of flattened rivets. See [441] and text concerning pin packing.

Pin-Connected Truss Railroad Bridge   Design Sheet   PCB 11

Pin at L_3 $7\frac{1}{4}''$

Pin stress at L_3 will be a maximum when the shear in panel 2-3 is maximum.



Table, App. F, shows maximum shear will occur with wheel (3) at L_3.

$$E\text{-}10,\ S_{2\text{-}3},\quad \frac{8182+142\times 44+44\times\frac{44}{2}}{224}-\frac{115}{28}=68.74-4.11=64.63^{K}$$

Appendix F gives 64.7.

From Sh. PCB 8, $S_{U_2L_3}=483.8$

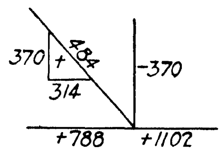
Find corresponding stress in L_2L_3

$$\text{Live}=68.74\times\frac{72}{2}\times\frac{56}{33}=420^{K}$$

$$\text{Impact } (25.1\%)\qquad =105^{K}$$

$$\text{Dead}\qquad =\underline{263}$$

$$788^{K}$$

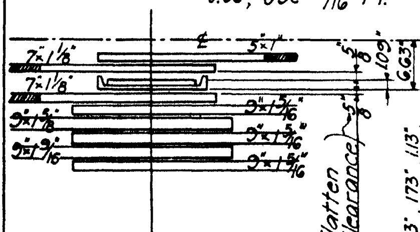


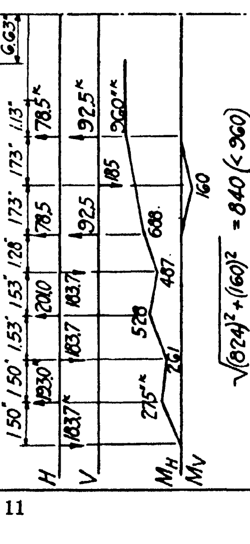
Bearing of U_3L_3 on pin (assumed $7\frac{1}{4}''$ diameter)

$$\frac{370}{2\times 24\times 7.25}=1.06''$$

$\underline{0.40}$ (=[web)

$0.66''$, Use $\frac{11}{16}''$ Pl.





Max. Pin Moment = 960"K

$$S=\frac{M}{S}=\frac{960}{\pi d^3/32}$$

$$d^3=\frac{960\times 32}{27\times\pi}=363$$

$$d=7.13''$$

$\therefore\ 7\frac{1}{4}''\phi$ Pin O.K.

Pin-Connected Truss Railroad Bridge — Design Sheet PCB 12

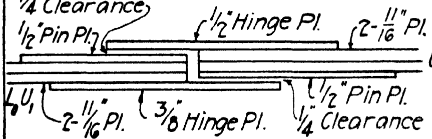
$\underline{U_1U_2}$

$P = -1020.3^k$ Assume a 9" pin

$\frac{1020.3}{2 \times 24 \times 9} = 2.36"$

Made up (See Sh PCB 17)

Web	11/16
Fill	11/16
Hinge Pl.	8/16
Pin Pl.	8/16
	2 3/8"

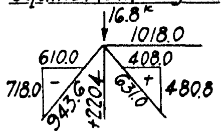


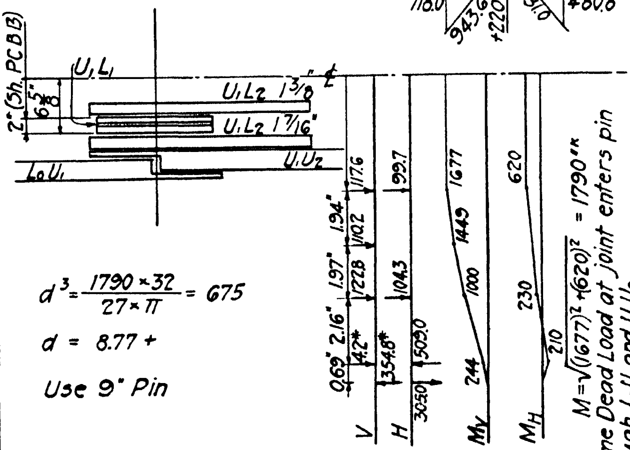
To maintain clearance, material of L_0U_1 must have same thickness as that of U_1U_2 except for hinge plates.

Max. LL in U_1L_1 occurs with ⑬ at L_1. Will be about the same with ④ at L_1, which is loading for max. in L_0U_1. ∴ Max. LL will be assumed to act simultaneously in those members. Vertical component of LL in $L_0U_1 = 119.2 \times \frac{7.2}{2} = 429.5$. Same impact percentage (25.1) will be used for all members.

Vertical in L_0U_1		Stress in U_1L_1
Live	= 429.5	= 148.2
Impact (25.1%)	= 107.5	37.2
Dead	= 181.3	35.0
	718.3	220.4

Other stressess by equilibrium of joint.





$M = \sqrt{(1677)^2 + (620)^2} = 1790''^k$

* Assume Dead Load at joint enters pin through L_0U_1 and U_1U_2.

$$d^3 = \frac{1790 \times 32}{27 \times \pi} = 675$$

$d = 8.77+$

Use 9" Pin

Pin-Connected Truss Railroad Bridge	Design Sheet	PCB 13

Pin at U_1 = 9" (Sh PCB 12)

Bar $U_1 L_1$ $+451.3^K$

$\frac{451.3}{24} = 18.8\,\square''$, required net area

Try 2-[15×40 = 23.40

4 out of flg = 4×1×5/8 = −2.50

4 out of web = 4×1×0.52 = −2.08

Furnished net area = 18.82 □"

Eye-bar head $18\frac{1}{2}$" dia.

Pin 9" "

$\frac{18.5}{2} - \frac{9}{2} = 4.75$" back of pin hole.

[411] $\frac{18.82}{2 \times 4.75} = 1.98''$

Across pin hole, using 16" pl.

16"−9" = 7" net width.

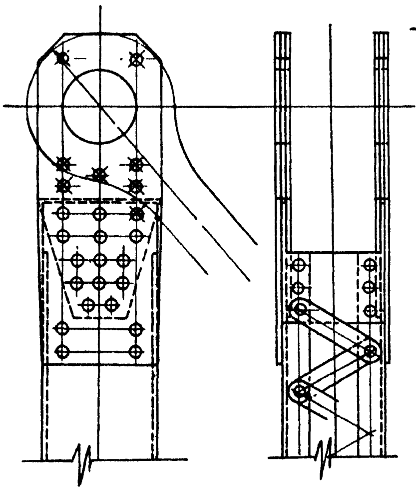
$\frac{18.82 \times 1.4}{2 \times 7} = 1.88$ Reqd. Thickness

Try 1-Web pl. 0.5

2-3/4" Pl. 1.5

Thickness Furn. = 2.0"

Bearing $= \frac{451.3}{2 \times 2 \times 9} = 12.5^K/\square''$ (< 24) ∴ O.K.



Rivets

Bearing on 0.50

$= \frac{7}{8} \times 0.50 \times 27 = 11.8^K$

Double Shear

$= 2 \times 0.601 \times 13.5 = 16.2^K$

Capacity of $\frac{1}{2}$ piece

$\frac{18.82 \times 18}{2} = 169.4^K$

$\frac{169.4}{11.8} = 15$ Rivets in each Channel.

To develop 1/2" Filler Pl.

$15 \times \frac{0.50}{1.88} = 4$

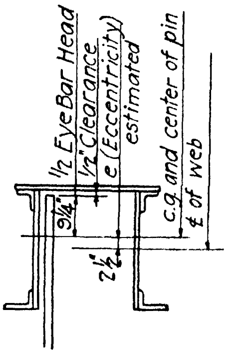
$U_1 L_1$

2-[

15×40

Pin-Connected Truss Railroad Bridge	Design Sheet	PCB14

$U_3 U_4$



Depth

1/2 Eye Bar Head	$9\frac{1}{4}''$
Clearance	$\frac{1}{2}$
Eccentricity	$2\frac{1}{2}$
	$12\frac{1}{4}''$

$12\frac{1}{4}'' \times 2 = 24\frac{1}{2}''$ Try 24" Web

Width (Joint U_1)

U_1L_1	$13\frac{1}{4} + (2 \times \frac{3}{4}) =$	$14\frac{3}{4}''$
Clearance	$2 \times \frac{1}{4} =$	$\frac{1}{2}$
U_1L_2	$2 \times 1\frac{7}{16} =$	$2\frac{7}{8}$
Clearance	$2 \times \frac{1}{4} =$	$\frac{1}{2}$
Hinge Pl. (on U_1U_2)	$2 \times \frac{1}{2} =$	1
		$19\frac{5}{8}''$ clear between webs

Cover Plate

Clear between webs	$19\frac{5}{8}''$
2-Webs $\frac{11}{16}$	$1\frac{3}{8}$
2-L 4×4	8
	$29''$

Use 30" Cover

Distance between gage lines

$19\frac{5}{8} + (2 \times \frac{11}{16}) + (2 \times 2\frac{5}{8}) = 26\frac{1}{4}''$

$\frac{26.25}{40} = 0.66''$ [405]

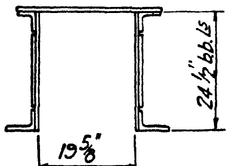
Use $30'' \times \frac{11}{16}''$

Web Thickness [405]

Distance between gage lines $= 24\frac{1}{2} - 2 \times 2\frac{5}{8} = 19\frac{1}{4}''$

$\frac{19.25}{32} = 0.605''$ Web which bears on pins should not be thinner than cover.

$\therefore$ Use Web $24 \times \frac{11}{16}''$



Clear Between webs

U_3U_4, -1338.2

Assume min $\frac{L}{r} = 40$ (a guess to make a start in design)

$15000 - \frac{1}{4}(\overline{40}^2) =$	14,600 #/□"	[301]
Bending =	600 #/□"	[216]
	14,000 #/□"	

Reqd. area, $\frac{1,338,200}{14,000} = 95.6$ □"

Try the section on the following sheet.

Pin-Connected Truss Railroad Bridge Design Sheet PCB15

Section	A = Area	a = Arm	Aa	I_x	arm	Iy	
1-Cov. $30 \times \frac{11}{16}$	20.63	24.85	513	1 12740	– –	1547	U_3U_4
2-Webs $24 \times \frac{11}{16}$	33.00	12.25	404	1584 4950	10.16	1 3410	
2-L $4 \times 4 \times \frac{5}{8}$	9.22	23.27	215	13 5003	11.73	13 1270	
2-L $6 \times 4 \times \frac{11}{16}$	12.80	1.06	14	16 15	12.56	46 2020	
2-Side Pl. $16 \times \frac{5}{8}$	20.00	12.25	245	427 3000	10.81	1 2340	
$e = 14.54 - 12.25$ $= 2.29$	95.65	14.54	1391	27749	I_y =	10,648	
			$-95.65 \times \overline{14.54}^2 =$	20360			
			I_x =	7389			

$r = \sqrt{\frac{7389}{95.65}} = 8.78$

$\frac{L}{r} = \frac{28 \times 12}{8.78} = 38.3$

$15{,}000 - \frac{1}{4}(38.3)^2 = 14{,}630 \text{ \#/□"}$ allowable

Bending

$95.65 \times 3.4 = 325 \text{ \#/'}$

$\frac{3}{4} \times \frac{1}{8} \times 325 \times \overline{28}^2 = 23{,}900 \text{ '\#} \; [216]$

$s = \frac{23{,}900 \times 12 \times 14.54}{7389} = 570 \text{ \#/□"}$

$\frac{1338200}{95.65} = 13{,}990 \text{ \#/□"}$

Bending $\underline{570}$

$14{,}560 \text{ \#/□"}$ actual

O.K.

U_2U_3

Use:

1-Cov. $30 \times \frac{11}{16}$

2-Webs $24 \times \frac{11}{16}$

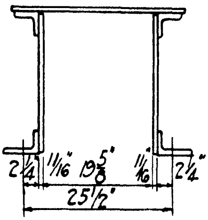
2-L $4 \times 4 \times \frac{5}{8}$

2-L $6 \times 4 \times \frac{11}{16}$

2-Side Pl. $16 \times \frac{1}{2}$

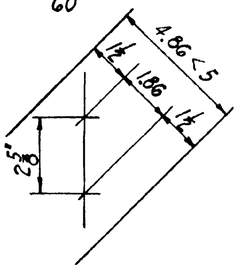
Pin-Connected Truss Railroad Bridge Design Sheet PCB16

Lacing and Stay Plates U_3U_4 U_2U_3



$25\frac{1}{2} > 15$, ∴ double lacing

$\dfrac{25\frac{1}{2} \times \sqrt{2}}{60} = 0.60$ thick



Try $5 \times \frac{5}{8}$

$r = \frac{5}{8}\sqrt{\frac{1}{12}} = 0.18$

$\frac{L}{r} \quad \dfrac{\sqrt{2} \times 25.5}{0.18} = 200$

$U_3U_4 = -1338.2$ [421]

$r_{vertical} = \sqrt{\dfrac{10648}{95.65}} = 10.56 \quad \dfrac{L}{r} = \dfrac{28 \times 12}{10.56} = 31.8$

$V = \dfrac{1338.2}{100}\left(\dfrac{100}{31.8+10} + \dfrac{31.8}{100}\right) = 36.3^K$

Stress in one bar

$= \dfrac{36{,}300}{2 \times 2}\sqrt{2} = 12{,}830^{\#}$

$\dfrac{P}{A} = \dfrac{12830}{5 \times \frac{5}{8}} = 4110\ ^{\#}/\square''$

$\frac{P}{A}$ allowable $= 15{,}000 - \frac{1}{4}(0.7 \times 200)^2$

$= 10{,}100\ ^{\#}/\square''$ O.K.

Connection to main member

$= 2 \times 0.707 \times 12830 = 18250^{\#}$

One rivet good for $8100^{\#}$ (single shear)

∴ 3 rivets needed

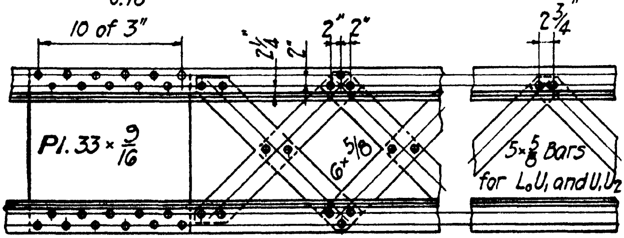
3 Rivet connection requires a 6" bar.

Stay Pl. [420]

$\frac{5}{4} \times 25.5 = 31.8$

$\dfrac{25.5}{50} = 0.51''$

Use $32 \times \frac{9}{16}$ Pl.



Pin-Connected Truss Railroad Bridge Design Sheet PCB17	
	U_1U_2

$U_1U_2 = -1020.3$

One end pinned. ∴ Design as pin-ended column.

Assume $\frac{L}{r} = 40$

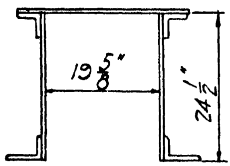
$\frac{P}{A} = 15{,}000 - \frac{1}{3}(40)^2 =$ 14470

Bending 570 ±

13,900

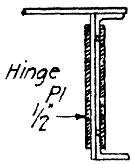
$\frac{1020.3}{13.9} = 73.5\,\square''$

Try section below:



Bearing on 9" Pin at U_1

$\frac{1020.3}{2 \times 24 \times 9} = 2.36''$



Web = $\frac{11}{16}$"

Fill = $\frac{11}{16}$"

Hinge Pl. = $\frac{1}{2}$

Pin Pl. = $\frac{1}{2}$

$2\frac{3}{8}$"

	A = Area	a = arm	Aa	I_x
1-Cov. 30 × $\frac{11}{16}$	20.63	× 24.85	513	1 12740
2-Webs 24 × $\frac{11}{16}$	33.00	× 12.25	404	1584 4950
2-L 4 × 4 × $\frac{5}{8}$	9.22	× 23.27	215	13 5003
2-L 6 × 4 × $\frac{11}{16}$	12.80	× 1.06	14	16 15
$e = 15.00'' - 12.25'' = 2.75$	75.65	× 15.00	1146	24322
	75.65×15^2		=	17030
		I_x	=	7292

$r = \sqrt{\frac{7292}{75.65}} = 9.80$

$\frac{L}{r} = \frac{12 \times 28}{9.80} = 34.3$

$15{,}000 - \frac{1}{3}(34.3)^2 = 14{,}610$#/□" allowable

Bending: $75.65 \times 3.4 = 260$#/′

$\frac{3}{4} \times \frac{1}{8} \times 260 \times \overline{28}^2 = 19{,}100'$#

$S = \frac{19{,}100 \times 12 \times 15.00}{7292} = 470$#/□"

$\frac{1020.3}{75.65} = 13{,}500$#/□"

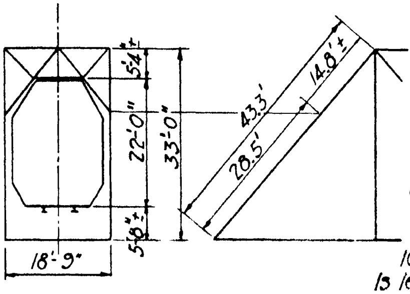
Bending = 470

13,970

< 14610#/□" allowable

Pin-Connected Truss Railroad Bridge — *Design Sheet PCB 18*

End Post L_0U_1



Wind Load [209]

Chord $\frac{25}{12} \times 28 = 58.3^{\#}$

Vert: $\frac{15}{12} \times 16.5 = 20.6$

Diag: $\frac{7}{12} \times 21.7$ } $= 21.7$
Counter $\frac{5}{12} \times 21.7$

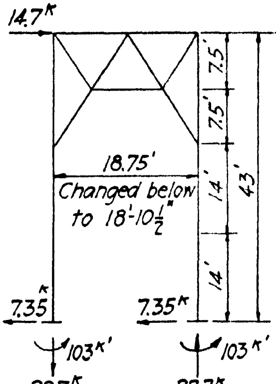
$100.6^{\#'}$

$100.6 \times 30^{\#}/\square = 3020^{\#}$

Is less than $150^{\#}/'$ [209]

∴ Use $150 \times 28 = 4200^{\#}$ panel load

Wind reaction, top of end post $= 3.5 \times 4200^{\#} = 14{,}700^{\#}$



Note: Design of portal members, except end post, will not be for this load. Design will be governed by [210, 217, 304].

Clearance between webs,
$19\frac{5}{8}(U_1U_2) + (2 \times \frac{1}{4})$ Clearance
$+ 2\left(\frac{1}{2}\right)$ Pin Pls. $= 21\frac{1}{8}"$

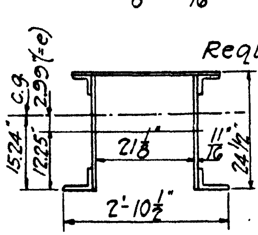
$21\frac{1}{8}" + 2 \times \frac{11}{16}"$(webs) $+ 2 \times 4$ (Top Ls)
$= 30\frac{1}{2}"$

Use $31 \times \frac{11}{16}$ Cover Pl.

Over all width
$21\frac{1}{8} + 2 \times \frac{11}{16} + 2 \times 6"$ (Bottom Ls) $= 34\frac{1}{2}" = 2'\text{-}10\frac{1}{2}"$

Clearance $= 16'\text{-}0"$

Required Truss Spacing $= 18'\text{-}10\frac{1}{2}"$,



∴ The truss spacing will be greater than the 18'-9" first assumed. This affects the design length of the floor beam and the impact (lurch) in the truss. However, the change is negligable.

Try the following section:

Pin-Connected Truss Railroad Bridge — Design Sheet PCB 19

L_0U_1

Section	A = area	a = arm	Aa	I_x	arm	I_y
1-Cov. $31 \times \frac{11}{16}$	21.31	24.85	530	1 13160		1705
2-Webs $24 \times \frac{11}{16}$	33.00	12.25	404	1584 4950	10.91	1 3940
2-L $4 \times 4 \times \frac{5}{8}$	9.22	23.27	215	13 5003	12.48	13 1440
2-L $6 \times 4 \times \frac{11}{16}$	12.80	1.06	14	16 15	13.31	46 2270
$e = 15.24 - 12.25 = 2.99$	76.33	15.24	1163	24742 $-76.33 \times \overline{15.24}^2 = -17730$ 7012		$I_y = 9415$

$r = \sqrt{\frac{7012}{76.33}} = 9.59$, $\frac{L}{r} = \frac{43.3 \times 12}{9.59} = 54.2$

$15{,}000 - \frac{1}{3}(54.2)^2 = 14{,}020\ \#/\square''$ allowable

$W = 76.33 \times 3.4 \times 43.3 = 11{,}230\#$; $M = \frac{1}{8} \times 11{,}230 \times 28 = 39{,}350'\#$

$S. = \frac{39350 \times 12 \times 15.24}{7012} = 1020\ \#/\square''$

$\frac{P}{A} = \frac{943{,}600}{76.33} = 12{,}360$

$13{,}380$ O.K

Member has excess area, but see remarks concerning Sh. PCB 17

Considering Portal Action

From above		$13{,}380\ \#/\square''$
$22.7 / 76.33$	=	300
$\frac{10{,}300 \times 12 \times 17.25}{9415}$	=	$2260'$
		$15{,}940\ \#/\square''$

S allowable [217]
$= 14{,}020 \times 1.25 = 17{,}520\ \#/\square''$
$\therefore$ O.K.

L_0L_2

L_0L_2 $+609.1^k$ [407] $\frac{609.1}{18} = 33.85\square''$;

2-Webs $24 \times \frac{5}{8}$ (5-1" ϕ out) $= 23.75$
4-L $4 \times 4 \times \frac{1}{2}$ (1-1" ϕ out) $= 11.00$
$34.75\square''$

Bearing on pins, assuming 9" pins at L_0
$7\frac{1}{4}$" pins at L_2

At L_0 $\frac{609.1}{2 \times 12 \times 9} = 2.83''$ [301] Bearing on rocker pin

At L_2 $\frac{609.1}{2 \times 24 \times 7\frac{1}{4}} = 1.75''$

Web	5/8"	Web	5/8
Fill	1/2"	Fill	1/2
2-7/8" Pl.	1 3/4"	5/8 Pl	5/8
	2.88"		1.75"

Pin-Connected Truss Railroad Bridge — *Design Sheet PCB 20*

L_0L_2 Continued

Section through pin holes [411]
Test at L_2 (L_0 is greater)

$\frac{34.75 \times 1.4}{2} = 24.30^{\square''}$ each side

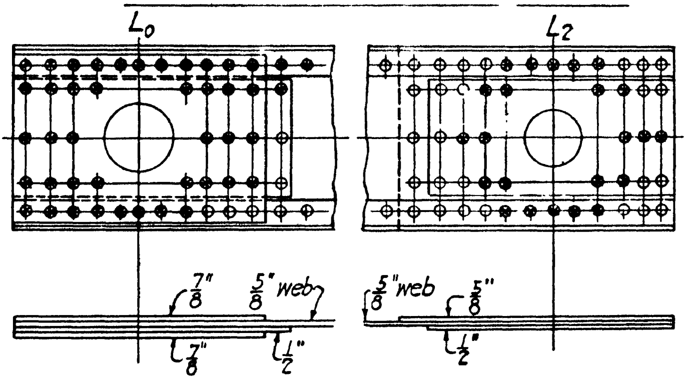
1-Web	$(24-7\frac{1}{4}-2)\frac{5}{8}$	$= 9.22^{\square''}$
2-L	$4\times4\times\frac{1}{2}$ (1-1"ϕ out)	$= 6.50$
1-Fill.	$(16-7\frac{1}{4})\frac{1}{2}$	$= 4.38$
1-Pl.	$(24-7\frac{1}{4}-2)\frac{5}{8}$	$= 9.22$
		$29.32^{\square''}$ OK

Spacing of webs
L_0U_1, $21\frac{1}{8}''$ clear between webs

$21\frac{1}{8}\times\frac{1}{2}+\frac{11}{16}+\frac{11}{16}+1\frac{1}{2}=13\frac{7}{16}''$ (See Below)*

Clearance	=	$\frac{1}{4}$
Pin Pl. (on L_0L_2)	=	$\frac{7}{8}$
		$14\frac{9}{16}''$

$14\frac{9}{16}\times2 = 29\frac{1}{8}''$ Use



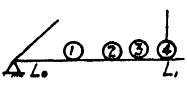
Pin at L_0 $9\frac{1}{4}''$

Bearing on L_0U_1, $\frac{943.6}{2\times12\times9} = 4.37''$ (assumed pin)

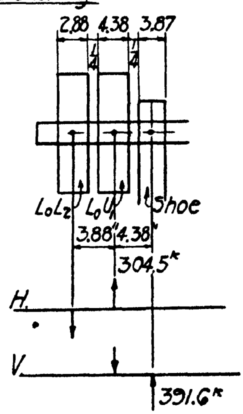
Web	= 0.69	
Fill	= 0.69	
2-$\frac{3}{4}''$Pl.	= 1.50	Inside
2-$\frac{3}{4}''$Pl.	= 1.50	Outside *
	4.38"	

End Floor Beam

Wheel position for max stress in L_0U_1. See Appendix.



Bending



Pin-Connected Truss Railroad Bridge — Design Sheet PCB 21

Pin at L_0 (Continued)

End Floor Beam (Continued)

End F.B. reaction

$$\text{Live} = \frac{72(5+10)+36(18)}{28\times 2} = 30.9^k$$

Impact (25.1%) Sht. PCB 7 =	7.8
Floor and Stringer	7.5
Floor Beam	2.0±
	48.2

Shoe

$$\text{Vertical in } L_0U_1\ \frac{33}{43.3}\times 943.6 = 719.0$$

End F.B. Reaction	48.2
Truss Dead Load	16.0
	783.2

With Wheel ② at end for max. reaction this becomes 852^k

$$\frac{852^k}{2\times 12\times 9\tfrac{1}{4}} = 3.84''$$

Bending (Continued)

See Sketch PCB 20

$M_H = 304.5\times 3.88 = 1180''^k$

$M_V = 391.6\times 4.38 = 1712''^k$

$\sqrt{(1180)^2+(1712)^2} = 2080''^k$

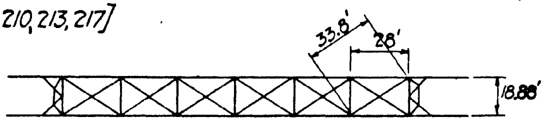
$$d^3 = \frac{2080\times 32}{27\times \pi} = 784$$

$d = 9.22$

Use $9\frac{1}{4}''$ Pin

Top Lateral System

[209, 210, 213, 217]



Sh. PCB 18. Exposed area per joint = 101□

Panel Load 101 × 50 = 5050 # [210]

or 150 × 28 = 4200 # [209]

Panel	Lateral Shear: 30# Wind + Live Load — Wind	[213]	Total	50# Wind without Live Load — Wind	[213]	Total
1-2	15/6 × 4200 = 10500	0.05 × 1020^k = 51,000#	61,500	15/6 × 5050 = 12,600	0.05 × 263^k = 13,200#	25,800#
2-3	10/6 × 4200 = 7,000	0.05 × 1273 = 63,600	70,600	10/6 × 5050 = 8400	0.05 × 329^k = 16,500	24,900
3-4	6/6 × 4200 = 4200	0.05 × 1338 = 66,900	71,100	6/6 × 5050 = 5050	0.05 × 351^k = 17,500	22,700

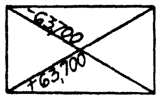
↑ Govern

Pin-Connected Truss Railroad Bridge *Design Sheet PCB 22*

Top Lateral System (Continued)

Designing as double system max. lateral stress will be

$$\frac{71{,}100}{2} \times \frac{33.8}{18.88} = 63{,}700^{\#}$$

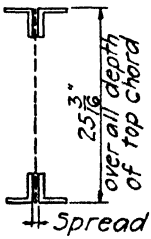


Vertical Leg $3\frac{1}{2}''$ minimum [417]

Unsupported length about vertical axis $= \frac{33.8}{2} = 16.9'$

Min. r, $\frac{16.9 \times 12}{120} = 1.69$ [304]

Try 2-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$



$I_V = 4[2.9 + 2.48(1.39)^2] = 30.8$ – Governs

$I_H = 4[2.9 + 2.48(11.58)^2] = 1341$

Spread $\frac{3}{4}''$ for 2-$\frac{3}{8}''$ Lacing Bars

$r = \sqrt{\frac{30.8}{9.92}} = 1.76 \quad \frac{L}{r} = \frac{16.9 \times 12}{1.76} = 115$

$\frac{P}{A}$ Allowable $= \frac{5}{4}\left[15{,}000 - \frac{1}{4}(115)^2\right] = \frac{5}{4}(11{,}690) = 14{,}600^{\#}/\square''$ [217]

$\frac{P}{A}$ Actual $= \frac{63{,}700}{9.92} = 6420^{\#}/\square''$ O.K.

$\therefore$ Use 4-L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$

Struts

Allowable r about V axis $= \frac{18.88 \times 12}{120} = 1.89$

Use 4-L $4 \times 3\frac{1}{2} \times \frac{3}{8}$ (4" Legs out)

Min. $r = 2.02$ (Steel Handbook)

Bottom Lateral System

<u>Exposed area per panel</u>

Vertical $= \frac{15}{12} \times \frac{33}{2} = 20.6$

Diagonal $= \frac{7}{12} \times \frac{43.3}{2}$

Counter $= \frac{5}{12} \times \frac{43.3}{2}$ } $= 21.7$

Bot Chd. $= \frac{9}{12} \times 28 = 21.0$

Floor $\frac{3}{2} \times \left(\frac{48+8+8}{12}\right) \times 28 = \underline{224.0}$

$287.3^{\square'}$

Panel Load [209]

$287.3^{\square'} \times 30 = 8620$

$300^{\#} \times 28 = \underline{8400}$

$17{,}020^{\#}$

<u>Lurch</u>, $20{,}000^{\#}$ [212]

$\left(17{,}020 \times \frac{10}{8} + 20{,}000 \times \frac{4}{8}\right) \times \frac{33.8}{18.88} = 56.0$

$\left(17{,}020 \times \frac{15}{8} + 20{,}000 \times \frac{5}{8}\right) \times \frac{33.8}{18.88} = 79.4$

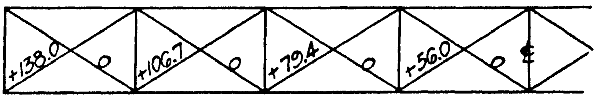
$\left(17{,}020 \times \frac{21}{8} + 20{,}000 \times \frac{6}{8}\right) \times \frac{33.8}{18.88} = 106.7$

$\left(17{,}020 \times \frac{28}{8} + 20{,}000 \times \frac{7}{8}\right) \times \frac{33.8}{18.88} = 138.0$

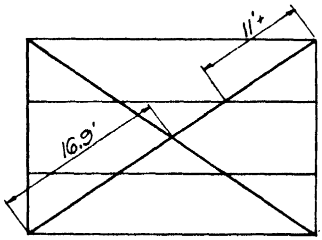
Pin-Connected Truss-Railroad Bridge	Design Sheet PCB 23

Bottom Lateral System (Continued)

Single System:



To reduce bending and unsupported length [304], laterals will be supported by hangers where they pass under stringers. ∴ $L/r < 200$ may be applied about H axis with $L = 11'$ and about V axis with $L = 16.9'$



Try 2-L $6 \times 4 \times \frac{1}{2}$

$r_H = 1.15$

$r_V = 2.76$

$\frac{11 \times 12}{1.15} = 115 (< 200)$

Effective Section [410]

$= 2(4.75 - (1+2)\frac{1}{2}) = 6.50$

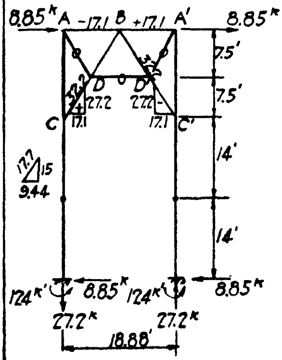
$\frac{138{,}000}{6.50} = 22{,}000\,\#/\square'' \quad (< \frac{5}{4} \times 18{,}000 = 22{,}500)\ \therefore$ O.K.

For all panels except end use 2-L $6 \times 4 \times \frac{3}{8}$

If hanger connection to stringer is stiff (as it will be) system might be figured as double system since L/r max. = 115. Is able ∴ to carry about twice figured stress, but section used is minimum that practice would permit.

Pin-Connected Truss Railroad Bridge — Design Sheet PCB 24

See Sh. PCB 18



Panel Area $101^{\square'}$ (Sh. PCB 18)
Panel Load = $50 \times 101 = 5050^{\#}$
Portal Load = $5050 \times 3.5 = 17.7^k$

Members for this load are too light judging by size of top laterals.
∴ Design will be made for [2/3].

Wind Shear	=	14.7^k
0.05×944	=	47.2
		61.9^k

∴ Stress in AB
$$= -17.1 \times \frac{61.9}{17.7} = -59.8^k$$

Stress in BC'
$$= -32.2 \times \frac{61.9}{17.7} = -112.6^k$$

Portal

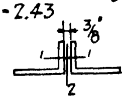
Member BC'

L in plane of end posts = 17.7'

$$\frac{17.7 \times 12}{120} = 1.77$$

Try 2-L $5 \times 3\frac{1}{2} \times \frac{1}{2}$

$r_1 = 1.01$ L = 8.7 governs
$r_2 = 2.43$



$A = 2 \times 4.00 = 8.00$

$$\frac{L}{r} = \frac{8.7 \times 12}{1.01} = 103.5,$$

$$\frac{P}{A} \text{ Allowable} = \frac{5}{4}\left[15{,}000 - \frac{1}{4}(103.5)^2\right] = 15{,}400^{\#}/\square''$$

$$\frac{P}{A} \text{ Actual} = \frac{112{,}600}{8.00} = 14{,}080^{\#}/\square''$$

O.K. Use 2-L $5 \times 3\frac{1}{2} \times \frac{1}{2}$

Member A-A' Has tension over one half its length but for rigidity use same as for BC'.

Members AD, DD', A'D'

Use 2-L $3\frac{1}{2} \times 3 \times \frac{3}{8}$

Long legs vertical.

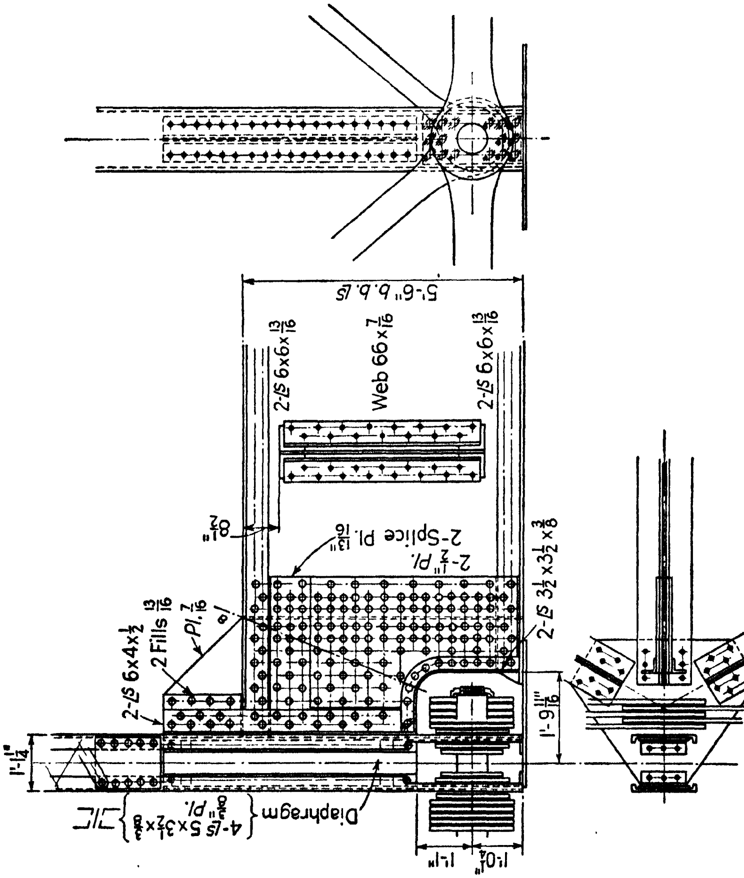
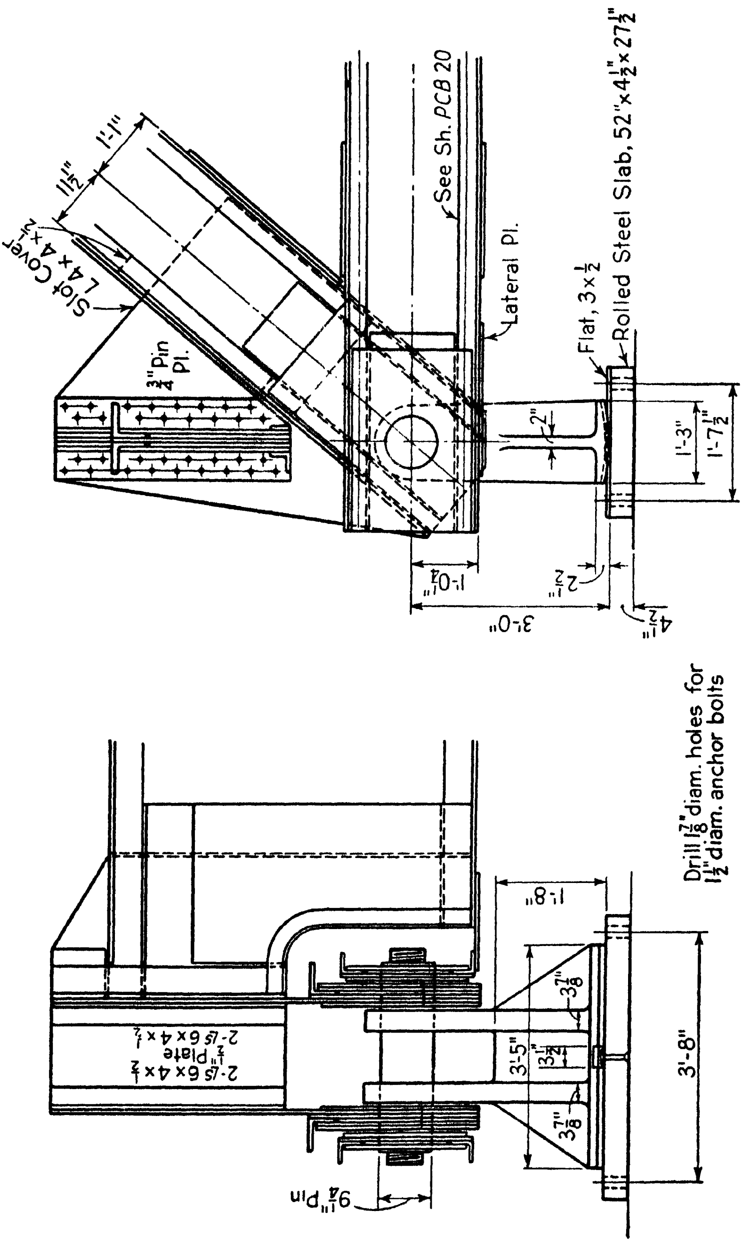

5'-6" b.b. Ls
2-Ls 6x6x13/16
Web 66x7/16
2-Ls 6x6x13/16
8 1/2"
2-Splice Pl. 13/16"
2-1/2" Pl.
2-Ls 3 1/2 x 3 1/2 x 3/8
2-Ls 6x4x1/2
2 Fills 13/16
Pl. 7/16
1'-9 11/16"
1'-1 1/4"
Diaphragm
4-Ls 5x3 1/2x3/8
3/8" Pl.
1'-1"
1'-0 1/4"

FIG. 8–3



Slot Cover
L 4 × 4 × 1/2
1 1/2"
1'-1"
See Sh. PCB 20
Lateral Pl.
3/4" Pin Pl.
Flat, 3 × 1/2
Rolled Steel Slab, 52" × 4 1/2" × 27 1/2"
2"
1'-3"
1'-7 1/2"
1'-0 1/4"
2 1/2"
3'-0"
4 1/2"
Drill 1 7/8" diam. holes for 1 1/2" diam. anchor bolts
2-Ls 6 × 4 × 1/2
1/2" Plate
2-Ls 6 × 4 × 1/2
1'-8"
3 7/8"
3'-5"
3 1/2"
3 7/8"
3'-8"
9 1/4" Pin

FIG. 8-4

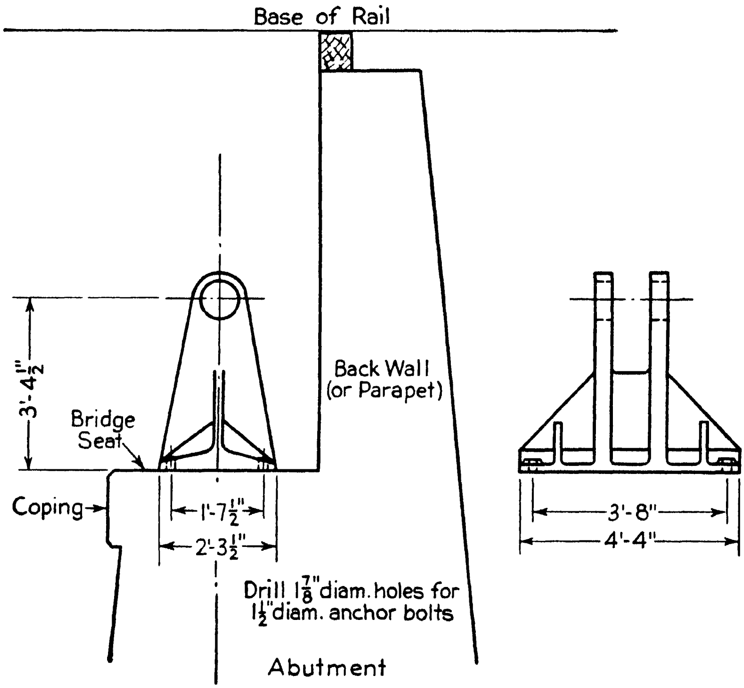

Base of Rail
Back Wall
(or Parapet)
3'-4½"
Bridge
Seat
Coping
1'-7½"
2'-3½"
Drill 1⅞" diam. holes for
1½" diam. anchor bolts
Abutment
3'-8"
4'-4"

FIG. 8–5

CHAPTER IX

MILL BUILDINGS AND ROOF TRUSSES

9–1. The type of building in which the principal structural element is a *bent* composed of a roof truss supported on columns, with or without knee braces, is called a mill building. Fig. 9–1 shows common types of bents. The space between two adjacent bents of a building is known as a *bay*.

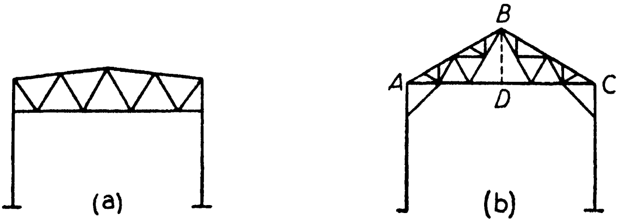


FIG. 9–1

Weights of common covering materials are given in structural handbooks.[1] The roof covering is supported by *purlins*, members which go from truss to truss, and the side covering by *girts*, members which go from column to column.

The *slope* of a truss (or roof) usually gives in inches the vertical rise of the top chord per horizontal foot.

The *pitch*, for a symmetrical truss, is the ratio of rise to span. That is, in Fig. 9–1*b* it is the ratio BD/AC.

There is a definite relation between roof slope and type of roof covering. For example, tar and gravel will run under a hot sun if the slope is great; the wind will carry rain under the lap of slate or corrugated sheeting if the slope is flat. Preferred slopes are:

Tar and gravel	1 in. per ft
Corrugated sheets	6 " " "
Slate	7 " " "

The directions of the manufacturer should be followed if a patented covering material is used.

[1] "Carnegie Pocket Companion," Twenty-fourth Edition, p. 420.

9-2. Corrugated Steel Covering. Corrugated steel sheets are frequently used for roofing and siding on buildings. These are obtainable black (unpainted mill finish) or galvanized. Standard sizes are (stock sizes marked*):

U. S. Gage No.	18, 20*, 24*, 26*, 28*
Width, inches	26* and $27\frac{1}{2}$* (after corrugating)
Length, inches	60*, 66, 72*, 78, 84*, 90, 96*, 102, 108, 114, 120*, 132, 144

Roofing is made from flat sheets 30 in. wide, each having $10\frac{1}{2}$ corrugations $2\frac{5}{8}$ in. deep, one edge of sheet turned up and the other turned down. The sheets after corrugating are $27\frac{1}{2}$ in. wide. They are laid with a side lap of $1\frac{1}{2}$ corrugations and cover approximately 24 in. net width.

Siding is made from flat sheets 28 in. wide, each having 10 corrugations $2\frac{5}{8}$ in. wide by $\frac{1}{2}$ in. deep, with both edges of the sheet turned the same way. The sheets after corrugating are 26 in. wide. They are laid with a side lap of one corrugation and cover approximately 24 in. net width.

Sheets should have an end lap of not less than 6 in. for roofing, and not less than 4 in. for siding.

For siding it is customary to specify two gages lighter than for roofing. Thus: if No. 20 is used for roofing, No. 22 is generally used for siding.

Purlins and girts should be so arranged that sheets span two spaces.

If 96-in. sheets with 6-in. end lap be assumed as average, one $27\frac{1}{2}$-in. sheet will cover an area 2 ft by $7\frac{1}{2}$ ft = 15 sq ft. On this basis the weight of galvanized sheets will be

Gage	Weight, pounds per 100 sq ft covered	Maximum Span between Supports*	
		Roofing	Siding
18	288	5′-9″	5′-10″
20	222	5′-9″	5′-10″
22	187	4′-9″	5′-10″
24	154	3′-9″	4′-10″
26	121	2′-9″	3′-10″
28	104	2′-9″	3′-10″

* "Carnegie Pocket Companion," Twenty-fourth Edition, p. 425.

If the intended use of the building is such that the dripping of condensation would be objectionable, a lining of several thicknesses of asbestos paper, tar paper, and wire netting is placed under the corrugated sheets. The lining weight will not exceed 1.5 lb per sq ft.

9–3. Purlins. Although any rolled or built-up shape might be used as a purlin, in practice the channel is almost always used for this member and is set with its web normal to the sloping top chord of the truss. Consequently, the loads which it is called on to support may not act parallel to a principal axis, and design may have to be made by the method of the S-polygon, p. 21. Generally, however, either of two other conditions is encountered: first, the roofing material and its immediate support have sufficient strength to carry the component of load which acts parallel to the roof, leaving only the normal component for the purlin; or, second, $\frac{5}{8}$- or $\frac{3}{4}$-in. sag rods, spaced 6 to 7 ft c.c., are used. These rods run parallel to the trusses and carry the parallel component of load to the ridge of the roof where double purlins, which may be heavier than the other purlins, are generally used. To prevent objectionable deflection, specifications frequently require that the depth of the purlin be not less than $\frac{1}{32}$ of the distance between trusses. To give rigidity to the roof, purlins are usually made in two-span lengths with alternate purlins splicing at alternate trusses.

9–4. Weights of Steel Roof Trusses. For design, the weight of steel roof trusses of from $\frac{1}{6}$ to $\frac{1}{3}$ pitch and spans up to 80 ft, with the loads applied at panel points and the supports on walls, may be assumed as

$$W = \left[\frac{1}{2}(L - 50)^2 + (L - 20)\left(18 + \frac{8\,w}{100}\right)\right]\left(\frac{16{,}000}{s_t}\right)$$

where W is the weight of one truss in pounds,
L is the span of the truss in feet,
w is the applied vertical load in pounds per foot of top chord, and
s_t is the design stress in tension.

In determining w, the normal wind pressure on one side of the roof may be replaced by a vertical pressure equal to $\frac{2}{3}$ of the normal wind acting over the entire roof surface.

Flat trusses of the Warren type with an end depth of $L/10$ to $L/8$ and a top chord slope of about 1 in. per ft will weigh $8\frac{1}{2}$ per cent more than the above.

Braced Bents. If Fink trusses are used, the weights of *trusses and knee braces* will *exceed* the formula weight by about 15 per cent. If Warren trusses are used with the column taking the place of the end verticals, the weight will be about 5 per cent *less* than the weight of a wall-supported Warren truss; otherwise, knee braces will be required and the weight of these must be added to the truss weight.

If the top chord must be designed for bending because the loads are

not applied at panel points, the above weights will be increased approximately $12\frac{1}{2}$ per cent.

In a mill building where bracing must be placed in the planes of the top and bottom chords of the roof trusses, the weight of bracing may be taken as $3\,L/400$ lb per sq ft of roof surface.

Example 9-1. A roof, the trusses of which are wall supported, has a span of 70 ft and a rise of $17\frac{1}{2}$ ft. The trusses are spaced 16 ft c.c. and are designed for $s_t = 16{,}000$ lb per sq in. The live loads in pounds per square foot of roof surface are: wind on one side $22\frac{1}{2}$, snow 25, ice 10. The design is to be made for dead + snow, or dead + ice + wind, whichever gives larger stresses. The roof covering weighs 4 lb per sq ft of roof surface, and the purlins the same amount. Not all the loads are applied at the panel points. Determine the probable weight of one truss.

Solution.	Covering	4 lb per sq ft	Covering	4 lb per sq ft
	Purlins	4	Purlins	4
	Truss (assumed)	3	Truss	3
	Snow	25	Wind $\frac{2}{3} \times 22\frac{1}{2}$ =	15
			Ice	10
		36		36

In this case the two combinations happen to be equal. Then

$$w = 36 \times 16 = 576 \text{ lb per ft}$$

$$W = \left[\tfrac{1}{2}\,(70 - 50)^2 + (70 - 20)\left(18 + \frac{8 \times 576}{100}\right)\right] 1.00 \times 1.125$$

$$= 3404 \times 1.125 = 3830 \text{ lb}$$

$$\text{Weight per square foot of roof surface} = \frac{3830}{16 \times 70 \times \frac{8}{8}} = 2.99 \text{ lb}$$

The 3 lb per sq ft assumed equals the weight given by the formula. However, extreme accuracy in the assumed weight is unnecessary since the truss weight is only $\frac{3}{36}$ of the load which must be supported and an error of 12 per cent in the assumption will change the truss stresses only 1 per cent.

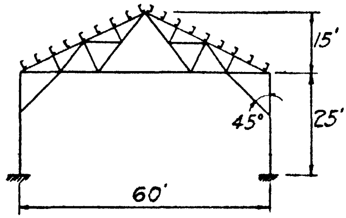
9-5. Design of a Bent. Design sheets MB 1-12 and Plate III which accompany this chapter illustrate the design of a mill-building bent. By comment and question, the articles which follow attempt to explain points which otherwise might be obscure.

9-6. Purlins and Sag Rods. In the design of purlins, it has been necessary to consider the moments due to bending in two directions. (Where does the moment of $wL^2/8$ due to normal loads occur? What is the moment at the same section due to the parallel loads; that is, is the factor $\frac{1}{32}$ exact or approximate?)

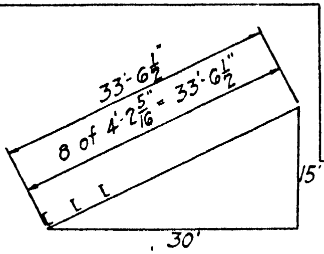
The factor $\frac{8}{7}$ which is used in arriving at the load brought by one pur-

Mill Building — Design Sheet MB1

Data

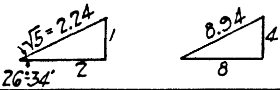


Mill Building 60' × 150'
Bents as shown, spaced 15' c.c.
Specification: A.I.S.C. 1934
Except:
Wind, 30#/□' on vertical surface, and by Duchemin formula on inclined -
Snow, 20#/□' of roof surface.
Ice, 10#/□' of roof surface.
Design for maximum stress from:
Dead + Snow;
Dead + Wind; or
Dead + Ice + Wind



Use corrugated galvanized steel roofing and siding, and anti-condensing roof lining, supported directly on purlins and girts.

Slope



Roofing — #22 Gage Galvanized
Siding — #24 Gage Galvanized

With purlins spaced 4'-2 5/16", #22 gage is lightest that may be used (table, Art. 9-2). Weighs 1.87#/□'.
Following common practice, #24 gage will be used for siding. Max. spacing of girts, 4'-10". Weighs 1.54#/□'

Purlin

Spacing 4'-2 5/16" c.c. (= 4.19) Assume sag rod at center of bay

Wind Pressure $= P \times \frac{2 \times \sin\alpha}{1 + \sin^2\alpha} = 30\,\frac{2 \times 1/\sqrt{5}}{1 + 1/5} = 22.4$ #/□' normal

Specification, (A.I.S.C. 5 f.g.) permits 33% increase in permissible working stress when considering wind alone or wind in combination with other loads. This is the same as reducing combined load one-fourth.

lin to the sag rods is occasioned by the fact that the end shears in the spans of continuous beams are not equal to $wL/2$. (What is the reaction at the first interior support of a four-span continuous beam with uniform load?)

In the case of the ridge purlins, it has been *assumed* that, at the point where the sag rods attach, the moment due to other loads is equal to $\frac{3}{4}$ of the moment for which the other purlins were designed. The moment at the point of support on the intermediate truss exceeds the moment for which the design is made but at the truss the purlin is prevented from buckling, a fact that could have been taken into account in the earlier design.

Problem 9–1. In the end bay, alternate purlins are simple beams. What change will this make in the fiber stresses in both ordinary and ridge purlins? Is a change in size necessary for these simple beams?

9–7. Location of Points of Contraflexure in Columns. The columns of a mill-building bent are attached to the foundation by anchor bolts and thus a certain degree of restraint is introduced which causes moments at the column bases and produces, in general, a point of contraflexure in each column. The degree of restraint and the consequent position of the point of contraflexure depend upon conditions outside of the bent itself. The extremes are: (*a*) the bent is attached to a heavy reinforced-concrete substructure — for example, a wharf — and is, therefore, fully restrained; and (*b*) the bent is attached to light footings built in a new fill, a condition which furnishes so little restraint that the columns may be considered pin ended, or hinged, at their bases. Cases encountered in practice are usually between these extremes. If the building designer has no definite information as to foundation conditions, a safe — but illogical — method is to design the column anchor bolts and connections for (*a*) above, and to design the columns and trusses for (*b*). In the example herewith it has been assumed that the columns are attached to individual footings of sufficient size to justify the assumption that the points of contraflexure are $\frac{1}{3}$ of the distance from the base of the column to the foot of the knee brace. A usual assumption is that the horizontal reactions at the bases of the columns are equal.

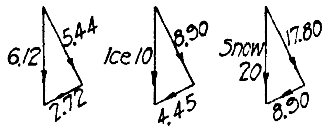
9–8. Selection of Members. The minimum size of angle to use in the truss depends, among other things, on the size of rivets. In light roof trusses $\frac{5}{8}$-in. rivets are sometimes permissible, but the great majority of trusses are built with $\frac{3}{4}$-in. rivets, and this size has been selected in the present example. Earlier specifications require an edge distance of $1\frac{1}{8}$ in. from a $\frac{3}{4}$-in. rivet to a rolled edge ($1\frac{1}{4}$ in. for $\frac{7}{8}$-, $1\frac{1}{2}$ in.

Mill Building — *Design Sheet MB2*

Purlins
8 [11.5

Dead Load

Roofing 1.87 #/□
Lining 1.50
Purlin 2.75 (?)
6.12



Dead + Live Loads lb. per sq. ft.

Normal:
- D + I + W = 5.44 + 8.90 + 22.40 = 36.74 (36.74 × 3/4 = 27.56)
- D + S = 5.44 + 17.80 = 23.24

Parallel:
- D + I + W = 2.72 + 4.45 + 0 = 7.17 (7.17 × 3/4 = 5.38)
- D + S 2.72 + 8.90 = 11.62

$\frac{15' \times 12}{32}$ = 5.6" min depth

Spec. (5c) compression flange not narrower than

$\frac{\text{unsupported length}}{40} = \frac{7.5 \times 12}{40} = 2\frac{1}{4}$" Try 8 [$11\frac{1}{2}$, flg $2\frac{1}{4}$"

$$s_{allowable} = \frac{20{,}000}{1 + \frac{L^2}{2000\, b^2}} = \frac{20{,}000}{1 + \frac{40 \times 40}{2000}} = 11{,}100 \text{ \#/□"}$$

8 [$11\frac{1}{2}$, 30' long, supported 15' c.-c. normal to roof $\left(M_{max.} = \frac{W_n \times \overline{15}^2}{8}\right)$ and 7'-6" parallel to trusses $\left(M_{max.} = \frac{W_p \times 7.5^2}{12}\right)$

8 [$11\frac{1}{2}$ $S_1 = 8.1$, $S_2 = 0.79$

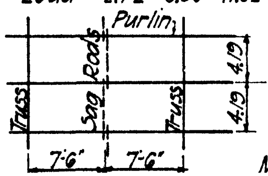
Dead + Ice + Wind	Dead + Snow
$\frac{1}{8} \times \frac{27.56 \times 4.19 \times \overline{15}^2 \times 12}{8.1} = 4800$ #/□"	$\frac{1}{8} \times \frac{23.24 \times 4.19 \times \overline{15}^2 \times 12}{8.1} = 4050$ #/□"
$\frac{1}{12} \times \frac{5.38 \times 4.19 \times \overline{7.5}^2 \times 12}{0.79} = 1600$	$\frac{1}{12} \times \frac{11.62 \times 4.19 \times \overline{7.5}^2 \times 12}{0.79} = 3450$
6400 #/□"	7500 #/□"

No lighter shape meets requirements for flange width.
∴ Use 8 [$11\frac{1}{2}$

$\frac{11.5}{4.19}$ = 2.75 #/□' of roof surface (= assumed).

Sag Rods
5/8"φ
One line per bay

Stress greatest under loading Dead + Snow.
Load = 2.72 + 8.90 = 11.62 #/□'; 11.62 × 4.19 = 48.7 #/' of purlin



Reaction taken by rod = 48.7 × $7\frac{1}{2}$ × $\frac{8}{7}$ = 417#
Max. rod stress at ridge = 417 × 8 = 3340#
$\frac{3340}{18000}$ = 0.185 □" Reqd.
5/8"φ Rod has net area through threads of 0.202□" Use.
Make 4'-6" long Thread 3" with sq nut ea end

for 1-in. rivets). This edge distance, together with the necessary driving clearance, means that the smallest angle leg in which a $\frac{3}{4}$-in. rivet may be driven is $2\frac{1}{2}$ in. The use of $\frac{1}{4}$-in. thick material is commonly allowed in building work [18].[1] Therefore, the minimum size angle for the truss will be $2\frac{1}{2} \times 2 \times \frac{1}{4}$.

Sec. 6 of the specification states that "sections shall preferably be symmetrical." To follow this requirement would necessitate two angles for each member of the truss. However, common practice is to use single angles for members where one *minimum* angle will carry the stress, and this practice has been followed.

The length of compression members has been taken as the distance between panel points.

The angles selected for the columns and for member *CG* are listed by the handbook as *special*. Generally, the only objection to such angles is that they are unlikely to be found in stock. If the fabricator does not have them, and waiting will cause delay, he will call this to the attention of the designer and a substitution may then be made. A similar statement might be made concerning the plain rods selected for anchor bolts on sheet MB 11. If the number of bents to be built is large, a saving will result from the use of $\frac{7}{8}$-in. round rods, upset (Carnegie, p. 345).

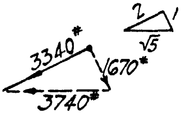
Under wind stress, bottom chord members *AG* and *GG′* and diagonal *GE* are called on to carry a moderate amount of compression. Since these members are primarily tension members, they have been regarded as secondaries in designing for compression and the L/r ratio has been permitted to run up to 200. Considering member *GG′*, strict adherence to the specification, even after considering this as a secondary member, requires the use of two ∠s $3 \times 2\frac{1}{2} \times \frac{1}{4}$, with the 3-in. legs outstanding. If two ∠s $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ were used ($A = 2.38$ sq in., $r = 1.16$ in.), the L/r value would be 233. By the modified Euler formula, $16\ E/(L/r)^2$, the ultimate strength of a column with this ratio is about 8800 lb per sq in. If used here the average stress in the angles would be $1240 / 2.38 = 520$ lb per sq in. The factor of safety would be $8800/520 = 17+$. In practice, the authors would be inclined to choose the lighter angles. The meticulous, if pained by such disregard for a specification, may reach the same result by making the assumption — reasonable under the condition of loading — that more than 50 per cent of the total horizontal reaction is taken by the windward column.

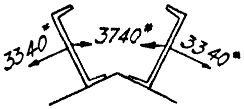
Usually the truss is picked up by the peak for erection, and occasionally the bottom chord is investigated for the compression accompanying this lifting. Generally the section of the member is not increased

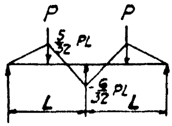
[1] A.I.S.C. Specifications, Appendix B.

Mill Building *Design Sheet MB3*

Ridge Purlins
8 [11.5







$\dfrac{3740}{18{,}000} = 0.208$

$5/8''\phi$ O.K.

Assume 8 [11.5 as above

Loading Dead + Snow

$S = \dfrac{5/32 \times 1670 \times 15 \times 12}{8.1} + 7500 \times \dfrac{3}{4}$

$= 5800 + 5620 = 11{,}420$

$> 11{,}100$ but taken as OK

Girts
7 [9.8
4'-2" c.c.

Use one $5/8''\phi$ sag rod per bay. Dead Load small enough to neglect

6 spaces of 4'-2" = 25'-0" (= col. height)

Wind Load per ft of girt $= 30 \times 4\tfrac{1}{6} = 125^{\#}/'$

Reqd $S = \dfrac{1}{8} \times \dfrac{125 \times \overline{15}^2 \times 12}{11{,}110} = 3.8$

(See above for $\frac{L}{b} = 40$)

Use 7 [9.8, S = 6.0

Flange width = $2\tfrac{1}{8}''$ is less than $2\tfrac{1}{4}''$ reqd. for $L/b = 40$ min. but stress is only

$11{,}100 \times \dfrac{3.8}{6.0} = 7050^{\#}/\square''$

Estimated Weight of Truss and Bracing

$3.2^{\#}/\square'$ of Exposed Surface

Roofing, Lining, Purlin	$6.10^{\#}/\square'$	----------------		$6.10^{\#}/\square'$
Snow	20.00	Ice		10.00
Truss + Br	3.25 (?)	Wind	$2/3 \times 22.4 =$	14.94
	29.35	Truss + Brac		3.25
				34.29

$34.29^{\#}/\square' \times 15' = 515^{\#}/'$ of chord

$W = \left[\tfrac{1}{2}(60-50)^2 + (60-20)\left(18 + \dfrac{8 \times 515}{100}\right)\right] \dfrac{16{,}000}{18{,}000} \times \dfrac{100 + 15 + 12.5}{100}$

$= \left[\tfrac{1}{2} \cdot 100 + 40 \times 59.2\right] \tfrac{8}{9} \times 1.275 \qquad 2750$

Truss per sq. ft. of exposed surface $= \dfrac{2750}{15 \times 33\tfrac{1}{2} \times 2} = 2.74$

Bracing $\qquad \dfrac{3 \times 60}{400} = 0.45$

3.19

say $3.2^{\#}/\square'$

Truss Panel Loads

Dead	1170#
Snow	2520#
Ice	1260#
Wind	2820#

Dead

Roofing, Lining, Purlins $\quad 6.10$

Truss & Br $\quad \underline{3.20}$

$9.30 \times 15 \times 8.38 = 1170^{\#}$

Snow: $20 \times 15 \times 8.38 = 2520^{\#}$

Ice: One Half Snow $= 1260^{\#}$

Wind: $22.4 \times 15 \times 8.38 = 2820^{\#}$

for this condition, but the member is temporarily stiffened, if necessary, by lashing a plank to it.

In spite of the fact that the stress in the top chord changes from panel to panel, the same section has been retained throughout the length since splicing the member would result in a loss of rigidity with but little saving in weight. The top chord bracing is arranged in such a manner that the chord is braced at mid-length. Therefore, about a vertical axis the chord is considered to have an unsupported length of two panels, whereas about the horizontal axis the unsupported length is one panel. Since a purlin is supported at the center of each panel, the chord must be designed for combined direct stress and bending.

9–9. Column Section and Base. Since the column contains a point of contraflexure there is an element of doubt as to the unsupported length that should be used in the beam formula. The use of the entire length is severe but safe.

Several methods are in use for proportioning the anchor bolts at the base of columns. Sometimes the connection is regarded as a reinforced-concrete beam with part of the area between base plate and concrete in compression and the anchor bolts on one side carrying tension. The stretch in the long length of anchor bolt above the base makes the analogy doubtful. A second method of design is to assume that the stress over the area where the base plate is in contact with the concrete may be figured in the manner in which stress is figured in a homogeneous beam. Next, the total tension on this area is computed and the anchor bolts are selected to carry this tension. Usually the bolts are near enough to the resultant of the tension to justify the use of this method, which has been followed in the present example.

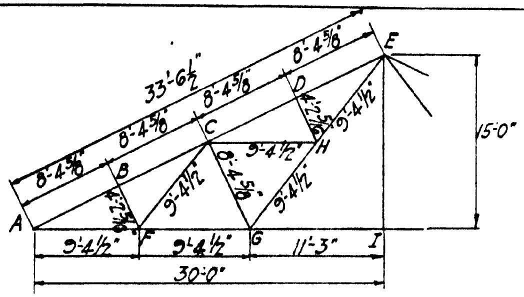
9–10. Drawing of Bent. Plate III shows a general drawing of the bent. In such a drawing, the center lines are laid out and used as the gage lines of the angles. (The center of gravity lines of the angles might be made to coincide with the working lines, thus fulfilling the condition that the gravity lines should meet in a point at a joint. Unfortunately, the gravity lines are so close to the outstanding legs of the angles that rivets cannot be placed on them.)

The gusset plates have been made $\frac{5}{16}$ in. thick although they might be made either thicker or thinner. Their outline is determined by placing the required number of rivets to carry the angle stresses and then drawing the plate around the rivets.

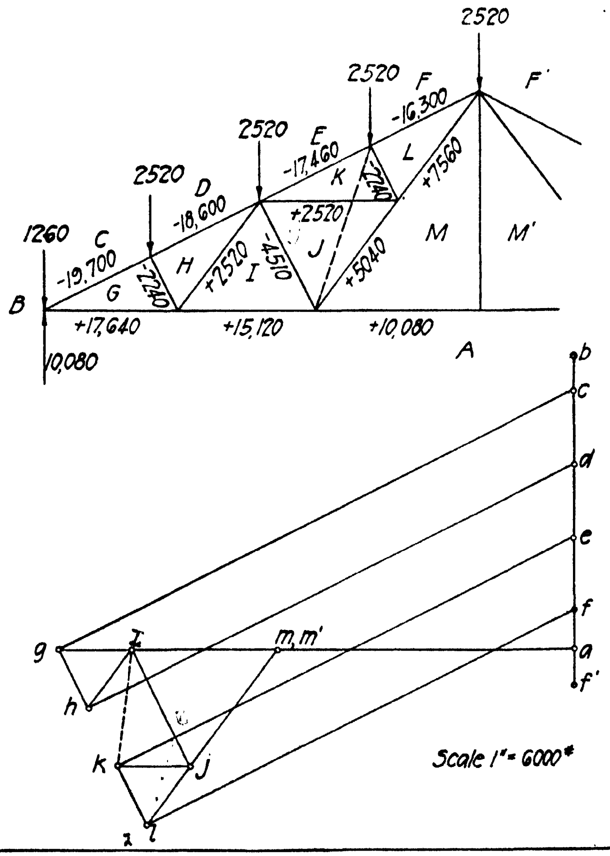
Good practice does not permit the use of less than two rivets in any connection [**15*a***].

Since the top chord has been considered as fixed at panel points in determining the bending moment caused by supporting purlins between

Mill Building — *Design Sheet MB4*



Lengths



Snow Stress

panels, provision must be made at A and E for this moment. This has been done by adding two more rivets than are required for direct stress alone.

For shipment, the truss has been divided into half trusses (AEG), bottom chord (GG'), and hanger (EK). The half trusses will be shipped by placing the top chord (AE) on blocking on the floor of a flat car or gondola. The depth when so placed is less than 10 ft. If this figure were exceeded, it would be necessary to secure permission from the railroad before making shipment. If such permission were not granted, the truss would have to be broken up in some different manner.

It was assumed in determining the necessary number of rivets in field connections that compressed air would be available so that field rivets might be *power driven*. (What difference would it make in the number of rivets if they were *hand driven?*)

The two angles which make up the majority of members must be connected at intervals so that they will act together. This connection is made by rivets which pass through washers of the same thickness as the gusset plates. The spacing of these rivets and washers is fixed by the specification [**14b**].

The computations and drawing give a complete *structural* design and, therefore, present as much as the authors believe proper for this book. Frequently, however, the structural engineer must show door and window details, etc. Such information will be found in "Structural Engineers' Handbook," by KETCHUM. Also the section "Steel Mill Buildings," pp. 44 to 93 in "Steel and Timber Structures," by HOOL and KINNE, may be read with profit.

Problem 9-2. Compute the weight of the truss shown in Fig. 9-2 and compare with the weight assumed on Sheet MB 3.

Problem 9-3. Redesign the bent of this chapter using the type of Fig. 9-1*a*. Give the truss a top chord slope of 1 in. per ft, and use a tar roof on wood sheathing. (Consider the advisability of using rafters as in the truss of Chapter XII.) Make the distance from bottom of column to bottom of truss equal to 22 ft 6 in. Make a general drawing.

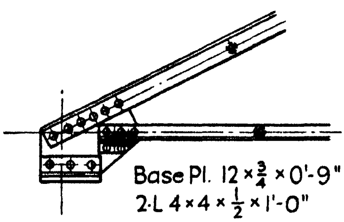


FIG. 9-2

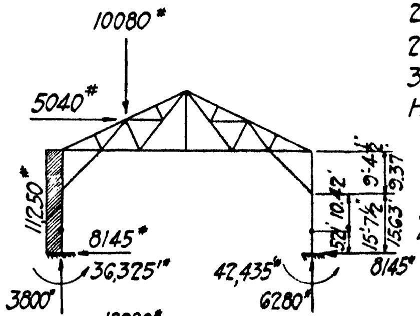
9-11. Wall Bearing Trusses. If the truss previously designed had been wall bearing, the principal difference would have been in the joint at the support. Fig. 9-2 shows a permissible connection for such a point. The center of the bearing angles has been slightly offset to balance the moment in the top chord due to supporting

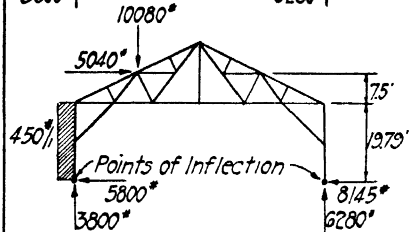
Mill Building — *Design Sheet MB 5*

Dead Stresses / Ice Stresses

Dead and ice stresses may be obtained by direct proportion from snow stresses
See Table, Sheet 7

Wind Stresses and Reactions in Columns and Kneebraces





$22.4 \times 15 \times 33.5 \times \frac{15}{33.5} = 5040^{\#}$

$22.4 \times 15 \times 30 = 10080^{\#}$

$30 \times 15 \times 25 = 11,250$

$H = 11,250 + 5040 = 16,290 \; (\div 2 = 8145)$

$8145 - 450 \times 5.21 = 5800$ (2345)

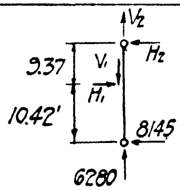
ΣM @ left inflection Pt.

$-60 V_R + 10080 \times 15 + 5040 \times 27.29 + 450 \times \frac{\overline{19.79}^2}{2} = 0$

$60 V_R = 151200 + 137540 + 88,120 = 376,860$

$V_R = 6280$

Right Column



$\Sigma M = 0$ at top

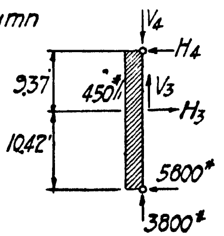
$-9.37 H_1 + 19.79 \times 8145 = 0$

$H_1 = V_1 = 17,200^{\#}$

$V_2 = 17,200 - 6280 = 10,900 \downarrow$

$H_2 = 17,200 - 8,145 = 9,060 \leftarrow$

Left Column



$\Sigma M = 0$ at top

$-9.37 H_3 + 19.79 \times 5800 - 450 \times \frac{\overline{19.79}^2}{2} = 0$

$9.37 H_3 = 114,700 - 88,120$

$H_3 = V_3 = 2,840$

$V_4 = 3,800 + 2,840 = 6,640 \downarrow$

$H_4 = 450 \times 19.79 + 2,840 - 5,800 = 5,940 \leftarrow$

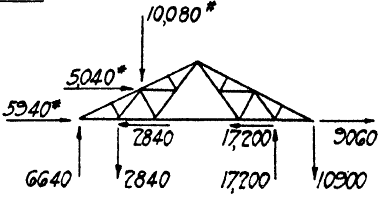
a purlin at mid-panel length. Had there been no bending in the chord, the center of the bearing area would have been placed vertically under the intersection of the top and bottom chords ("Structural Theory," Second Edition, pp. 287–9).

9–12. Industrial Buildings. The past few years have witnessed outstanding developments in industrial buildings, a result of combining new types and new materials. Since their design is an architectural rather than a structural problem it will not be discussed here, but the reader who is interested will find the matter well covered in a series of editorials and articles which appeared in the *Engineering News-Record* during 1936. In some of the structures described, indeterminate elements appear which are beyond the scope of this book.

Mill Building — *Design Sheet MB6*

Wind Stresses and Reactions in Columns and Kneebraces (Continued)

Check



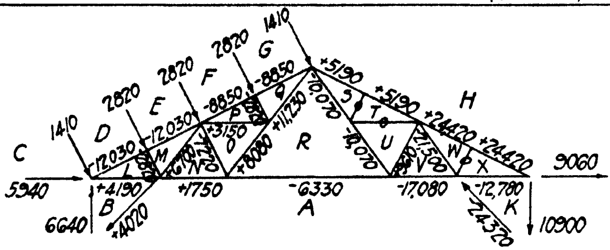
$\Sigma V = 0$

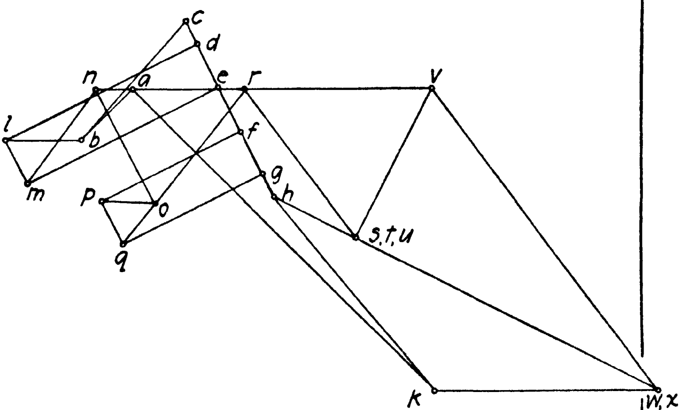
↓	↑
10080	
2840	6640
10920	17200
23,840	23,840

$\Sigma H = 0$

→	←
5040	
5940	2840
9060	17200
20,040	20,040

Wind Stresses

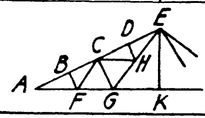




Scale 1" = 10,000#

Mill Building — Design Sheet MB 7

Member	Dead	Snow	Ice	Wind Left	Wind Right	Neglecting Wind: Max Stress	Neglecting Wind: Combination	Including Wind: 3/4 × Max Stress	Including Wind: Combination
AB	-9160	-19,730	-9870	-12,030	+24,420	-28,890	D.S.	-23,300 +11,450	$D.I.W_L$ $D.W_R$
BC	-8640	-18,600	-9300	-12,030	+24,420	-27,240	"	-22,500 +11,840	$D.I.W_L$ $D.W_R$
CD	-8110	-17,460	-8730	-8850	+5,190	-25,570	"	-19,270	$D.I.W_L$
DE	-7590	-16,330	-8170	-8850	+5,190	-23,920	"	-18,460	$D.I.W_L$
AF	+8190	+17,640	+8820	+4190	-12,780	+25,830	"	+15,900 -3,440	$D.I.W_L$ $D.W_R$
FG	+7020	+15,120	+7560	+1,750	-17,080	+22,140	"	+12,250 -7,550	$D.I.W_L$ $D.W_R$
GG′	+4680	+10,080	+5040	-6330	-6,330	+14,760	"	-1,240	D W
BF, DH	-1040	-2,240	-1120	-2820	0	-3,280	"	-3,740	$D.I.W_L$
CG	-2090	-4,510	-2250	-7220	+9610	-6,600	"	-8,670 +5,640	$D.I.W_L$ $D.W_R$
CF	+1170	+2,520	+1260	+6700	+21,500	+3,690	"	+6840 -15,250	$D.I.W_L$ $D.W_R$
CH	+1170	+2,520	+1260	+3150	0	+3,690	"	+4,190	$D.I.W_L$
GH	+2340	+5040	+2520	+8080	-10,700	+7,380	"	+9,700 -6,270	$D.I.W_L$ $D.W_R$
HE	+3510	+7560	+3780	+11,230	-10,700	+11,070	"	+13,890 -5,400	$D.I.W_L$ $D.W_R$
Knee Brace	—	—	—	+4,020	-24,320	—	—	-18,740 +3,020	W_R W_L

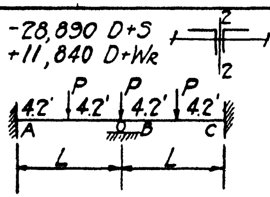


Stresses

Mill Building	Design Sheet MB8
$-3{,}740$, $L = 4'\text{-}2\frac{5}{16}'' = 4.2'$ $\quad \frac{4.2\times 12}{120} = 0.42$ min. r $\quad$ [5b] 1-L $2\frac{1}{2}\times 2\times\frac{1}{4}$, $r = 0.42$, $A = 1.06$, (Min) $\quad \frac{18{,}000}{1+\frac{1}{18000}\left(\frac{L}{r}\right)^2} = 10{,}000$ #/□" allowable $\frac{3740}{1.06} = 3530$ #/□" Actual, O.K.	BF, DH 1-L $2\frac{1}{2}\times 2\times\frac{1}{4}$
$+4190$ 1-L $2\frac{1}{2}\times 2\times\frac{1}{4}$ $\quad A = 1.06-(\frac{1}{4}\times\frac{7}{8}) = 0.84$□" ($\frac{3}{4}$" Rivets) $\frac{4190}{0.84} = 5000$ #/□" $(< 18{,}000)$ $\therefore$ O.K.	CH 1-L $2\frac{1}{2}\times 2\times\frac{1}{4}$
-8670 $+5640$ $\quad L = 8.4'$, $\quad \frac{8.4\times 12}{120} = 0.84$ min. r, 2-L $3\times 2\times\frac{1}{4}$ $\quad r = 0.86$, $A = 2.38$□" f allowable $> 10{,}000+$ $\quad f_{actual} = \frac{8670}{2.38} = 3{,}640$ #/□" $\therefore$ O.K.	CG 2-L $3\times 2\times\frac{1}{4}$
-15250 $+6840$ $\quad L = 9'\text{-}4\frac{1}{2}'' = 9.38$ $\quad \frac{9.38\times 12}{120} = 0.94$ min r, 2-L $3\times 2\frac{1}{2}\times\frac{1}{4}$ $r = 0.95$, $A = 2.62$□" $f_{allowable} > 10{,}000$ #/□" $\quad f_{actual} = \frac{15250}{2.62}$ $= 5800$ #/□" O.K.	CF 2-L $3\times 2\frac{1}{2}\times\frac{1}{4}$
$+25{,}830$ $\quad L_1 = 9.38'$ $\quad \frac{9.38\times 12}{200} = 0.56$ min r_1 $-7{,}550$ $\quad L_2 = 18.75'$ $\quad 1.12$ min. r_2 2-L $2\frac{1}{2}\times 2\frac{1}{2}\times\frac{1}{4}$, $A = 2.38$□" $r_1 = 0.77$ $r_2 = 1.16$ Compression $f_{allowable} > 5600$#/□", $f_{actual} = \frac{7550}{2.38} = 3{,}180$#/□" Tension, $A = 2.38-2(\frac{1}{4}\times\frac{7}{8}) = 1.94$□" $\frac{25830}{1.94} = 13{,}300$#/□" $(<18{,}000)$	AF, FG 2-L $2\frac{1}{2}\times 2\frac{1}{2}\times\frac{1}{4}$
$+14{,}760$ $-1{,}240$ $\quad L = 22.5'$ $\quad \frac{22.5\times 12}{200} = 1.35$ min. r Use 2-L $3\times 2\frac{1}{2}\times\frac{1}{4}$ $\quad$ 3" legs outstanding	G-G' 2-L $3\times 2\frac{1}{2}\times\frac{1}{4}$

Mill Building — *Design Sheet MB9*

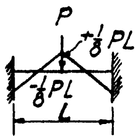
Stress and computation	Member
$+13{,}890$ $-6{,}270$ Length same as for AF, FG and stresses less. ∴ L/r governs. Use as above.	GH, HE 2-L $2\frac{1}{2}\times2\frac{1}{2}\times\frac{1}{4}$
0	Center Hanger 1-L $2\frac{1}{2}\times2\times\frac{1}{4}$
$-24{,}320$ $+4{,}020$ Assume $12\frac{1}{2}''$ column and figure length of knee brace to face of column. $L=(9'\text{-}4\frac{1}{2}''-6\frac{1}{4})\sqrt{2}=12.52'$ $\frac{12.52\times12}{120}=1.25$ min. r 2-L $4\times3\times\frac{1}{4}$, Area $=3.38^{\square''}$ $r_{min.}=1.27$ $f_{allowable}>10{,}000^{\#}/^{\square''}$; $f_{actual}=\frac{24{,}320}{3.38}=7{,}200^{\#}/^{\square''}$	Knee Braces 2-L $4\times3\times\frac{1}{4}$ JL
$-28{,}890$ D+S $+11{,}840$ D+W$_R$ (see below)	AE 2-L $4\times4\times\frac{1}{4}$



$L_1=8.4'$ $\frac{8.4\times12}{120}=0.84$ r_1 min.

$L_2=16.8'$ 1.68 r_2 min

Try 2-L $4\times4\times\frac{1}{4}$ $\begin{cases}A=3.88^{\square''}\\ r_1=1.25\\ r_2=1.77\end{cases}$



Axis 2-2 Bending may be neglected since at B, mid-length, bending stress in fibres distant from 2-2 is tension.

$f_{allowable}>10{,}000^{\#}/^{\square''}+$

$f_{actual}=\frac{28{,}890}{3.88}=7{,}440^{\#}/^{\square''}$ O.K.

Axis 1-1

Dead + Snow

Normal Load (MB2) $=5.44+17.80=23.24^{\#}/^{\square'}$

$\times4.2'\times15'=1{,}465^{\#}$

$M=\frac{1}{8}\times1465\times8.4\times12=18{,}450''^{\#}$

At midlength of AB, top fibres in compression due to bending ∴ $y=1.09''$

$f=\frac{28890}{3.88}+\frac{18{,}450\times1.09}{6.0}=7{,}450+3{,}320=10{,}770^{\#}/^{\square''}$

Dead + Ice + Wind

$(5.44+8.90+22.40)\,3/4$

$=27.56^{\#}/^{\square'}\times4.2\times15'=1735^{\#}$

$M=\frac{1}{8}\times1735\times8.4\times12=21{,}900$

$f=\frac{23{,}300}{3.88}+\frac{21{,}900\times1.09}{6.0}$

$=6000+3950=9{,}950^{\#}/^{\square''}$

$L/r=\frac{8.4\times12}{1.25}=81$

$f_{allowable}=13{,}200^{\#}/^{\square''}$ O.K.

Mill Building | Design Sheet MB10

Bending stress is so much greater than direct that latter might almost be neglected. Will therefor design as a beam, with flange supported at top and bottom. This is perhaps a severe assumption since column contains a point of contraflexure. Assume 6" outstanding L legs and a 12"×5/16" web.

$$\frac{L}{b} = \frac{25\times 12}{12.31} = 24.4$$

$$s_{c\,allowable} = \frac{20{,}000}{1+\frac{(L/b)^2}{2000}} = \frac{20000}{1+\frac{595}{2000}} = 15{,}400\ \#/\square''$$

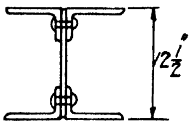
Investigate leeward column at foot of kneebrace under dead + ice + wind.

Direct Load, $d+i+w = 4680 + 5040 + 6{,}280 = 16{,}000$

$3/4 \times 16{,}000 = 12{,}000$

Moment, wind $= 8{,}145 \times 10.42' \times 3/4 = \quad 63{,}700'\#$

Spec., Sec. 7a directs design by "I of net section"



Try 1-Web $12\times 5/16$
4-L $6\times 3\frac{1}{2}\times 5/16$

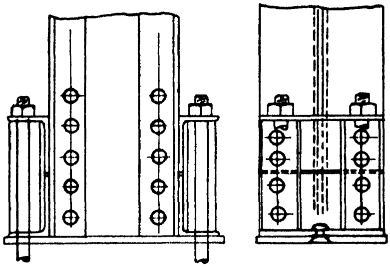
	Area	I
1-Web $12\times 5/16$ $= 3.75 - 2\times\frac{7}{8}\times\frac{5}{16} =$	$3.21^{\square''}$	45.0
4-Ls $6\times 3\frac{1}{2}\times\frac{5}{16}$ $= 4(2.87 - 1\times\frac{7}{8}\times\frac{5}{16}) =$	10.40	$4(2.9 + 2.87\times 5.49^2) = 357.6$
	13.61	402.6
		$-2(\frac{7}{8}\times\frac{15}{16}\times 4.25^2) = 29.6$
		373.0

$$s = \frac{P}{A} + \frac{Mc}{I} = \frac{12{,}000}{13.61} + \frac{63{,}700\times 12\times 6.25}{373}$$
$$= 880 + 12{,}800 = 13{,}680 (< 15{,}400) \therefore O.K.$$

Columns
1-Web $12\times\frac{5}{16}$
4-L $6\times 3\frac{1}{2}\times\frac{5}{16}$

Mill Building | *Design Sheet MB11*

Column Base



Base Pl

$(12\frac{1}{2}+8=20\frac{1}{2})+\frac{1}{2}=21''$

say $21''\times 12\frac{1}{2}''=262\frac{1}{2}\,\square''$

At column base,
Leeward Col

$D+I+W=12{,}000$ (Sh.10)

Moment: (Sh. 5)

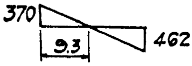
$=\frac{3}{4}\times 42{,}430=31820^{\#}$

Stress on Concrete

$$=\frac{12{,}000}{262}\pm\frac{31820\times 12}{\frac{12\frac{1}{2}\times 21^2}{6}}$$

$=46\pm 416$ = 370 tension
462 compression
O.K.

1- Base Pl $21\times\frac{3}{4}\times 1'\text{-}0\frac{1}{2}''$
4-L $6\times 4\times\frac{3}{8}\times 1'\text{-}0\frac{1}{2}''$
4-L $3\frac{1}{2}\times 3\frac{1}{2}\times\frac{3}{8}\times 11\frac{3}{4}''$

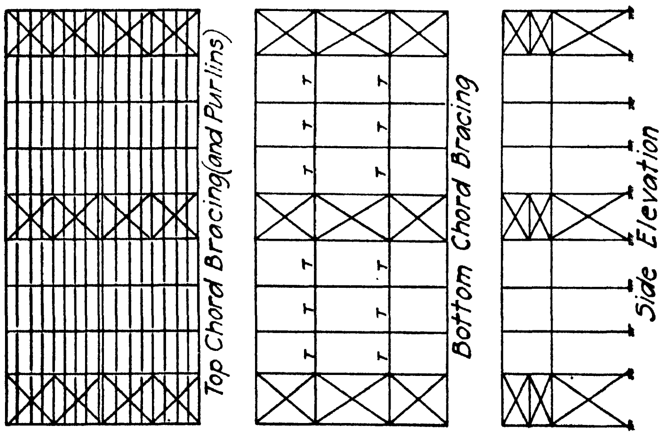


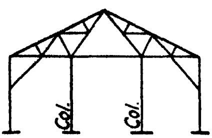
$T=\frac{1}{2}\times 370\times 9.3\times 12.5$
$=21{,}500$

$21{,}500/18000=1.20$

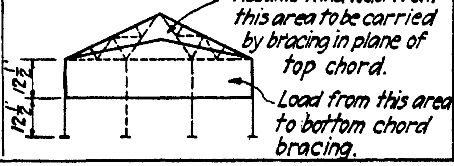
Use 2-$1\frac{1}{8}''\phi$ Bolts ($1.38\,\square''$ net)

Bracing



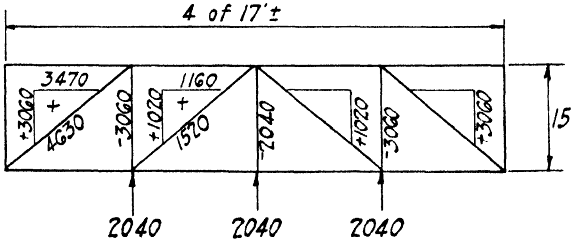


End Elevation



Mill Building — *Design Sheet MB12*

Top Chord Bracing
$\frac{3}{4}$" ϕ Rods (Not Upset)



Average depth of exposed area = 4'±
$4 \times 30^{\#} = 120^{\#}/'$
$120 \times 17' = 2040^{\#}$
Assume all carried by rods in end bay

$\frac{4630}{24000} = 0.193\,\square''$ reqd

Use $\frac{3}{4}$" ϕ. Net area under threads $= 0.30\,\square''$

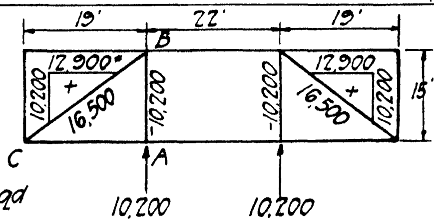
Purlin Compression

$\frac{3060}{3.36} = 910^{\#}/\square''$

O.K. as purlin stress (sh. 2) is much less than allowable

Bottom Chord Bracing
Diagonals 1-L $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$
Struts 2-L $3 \times 2\frac{1}{2} \times \frac{1}{4}$
Ties 1-L $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$

Average depth 17'±
$17 \times 30 = 510^{\#}$
$510^{\#} \times 20'\pm = 10,200^{\#}$



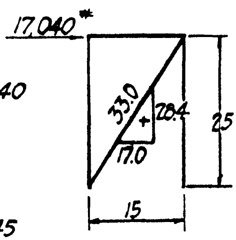
BC.
$\frac{16,500}{24,000} = 0.69\,\square''$ reqd
Use 1-L $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$
Net Area $= 1.19 - \left(\frac{7}{8} \times \frac{1}{4}\right) = 0.97\,\square''$

AB
Min $\frac{L}{r} = 200$ $\quad \frac{15 \times 12}{200} = 0.9$
Use 2-L $3 \times 2\frac{1}{2} \times \frac{1}{4}$ $\quad r = 0.95$

Ties ("T" in Sketch Sh 11)
Use 1-L $2\frac{1}{2}" \times 2\frac{1}{2}" \times \frac{1}{4}"$

Side Bracing
1-L $3 \times 3 \times \frac{5}{16}$

Takes reaction from top and bottom chord bracing plus wind load at column top.
$10,200 + 3,060 + (14\pm \times 9 \times 30) = 17,040$
$\qquad\qquad\qquad\quad 3780$



$\frac{33,000}{24,000} = 1.38$
Use 1-L $3 \times 3 \times \frac{5}{16}$
Net Area $= 1.78 - \left(\frac{7}{8} \times \frac{5}{16}\right) = 1.45$

CHAPTER X

OFFICE BUILDING FRAME

10–1. The frame of an office building is a very simple structure, fundamentally composed of vertical columns which support beams (or girders) at the roof and floor levels. Detailed study of such a structure would have little to teach the student who has already mastered the preceding chapters of this book. It may not be amiss to remark that there is an essential difference between a bridge, for which the designer usually makes his own decision as to type, and a structure of the kind treated in this chapter. In this case, another, the architect, has made plans and located column centers before the engineer sees the job. The latter, therefore, has as his problem the fitting of a skeleton into the scheme, and must be governed by the limitations and clearances set by the architect. The same general statement might be made concerning the part taken by the structural engineer in power, or manufacturing, plant design where the framing must be arranged to support boilers, retorts, engines, turbines, and machines selected and located by others.

10–2. Columns. In the earlier days of high-building construction, the columns were built up of a web and flange angles, without, or more commonly with, cover plates (Fig. 10–1*a*, *b*). At the present time,

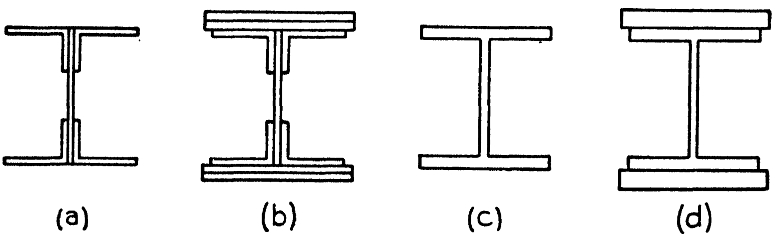


FIG. 10–1

rolled wide-flange sections are available and are generally used (Fig. 10–1*c*, *d*). These columns are supported laterally at such frequent intervals (that is, their L/r ratio is so low) that usually they fall into the class of short columns ($L/r < 60$, say) and are designed for a flat allowable unit stress without the use of a column formula.

Fig. 10–2, a portion of a *column schedule*, shows the manner in which the column designer presents his results to the drawing room. As is

usual, the columns have been designed in two-story lengths with the splices located 2 ft 6 in. above the floor lines. These lengths of columns are spoken of as " tiers " and are called first tier, second tier, etc., counting up from the bottom. The figures recorded in triangles are the design loads; their magnitudes in the upper stories are due to the fact that the

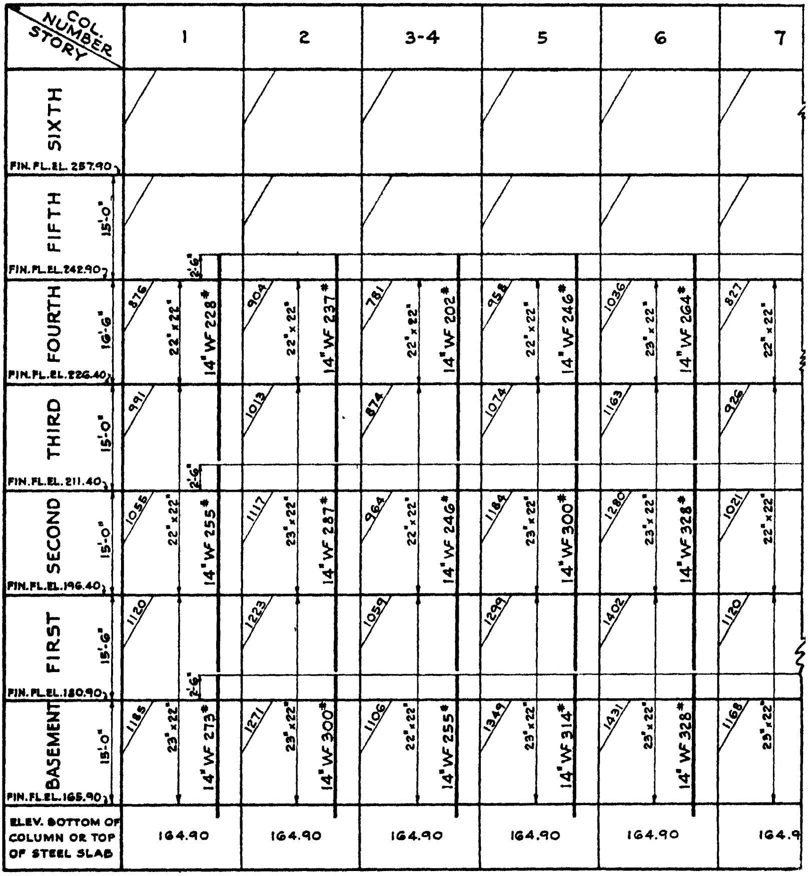


Fig. 10–2

building of Fig. 10–2 was designed to be extended in the future to twelve stories. The additional figures — 23 in. × 22 in., for example — denote the size of the concrete encasement, the first dimension being in the direction of the web.

The newcomer in the steel company drawing room will probably spend some time preparing shop drawings for beams and columns. Plates IV and V are such drawings from which the shop will fabricate columns.

It will be noted that Plate IV shows two columns which are to be milled (that is, machined or finished) at each end (M 2 E = mill two ends). Also, it is shown which derrick will erect each column (Derr. C, etc.). The sizes of the main material are noted in a way which will be clear to those familiar with the system. The student with the aid of his handbook can learn the meaning of the symbols B 14e $\times$ 219, B = $15\frac{7}{8}$, etc. Note in particular that the nominal and actual depth of a wide-flange section are generally not the same. Furthermore, Col. A7 is detailed and it is noted that R7 is " opposite hand " (that is, A7 is a *right*, R7 a *left*). (Neglecting the difference in length, just what will be the difference between these two?) A rereading of Art. 5–6 will probably clear up any questions concerning the system of assembly marks used. The variation shown here from that described before arises from the fact that on a large job it is customary to prepare " standards " — that is, a sheet will be prepared for splice plates, as an example, which will list and mark all the plates which will be required on the contract.

Plate V also shows the details of more than one column. In this case the columns are especially heavy. Note, in particular the weight of Col. 12.

10–3. Beams. The beams in an office building are usually assumed to carry uniform loads and may generally be selected directly from a steel handbook. Irregular and concentrated loading will demand special attention and may call for the use of a built-up girder. Here again the rolled beams now obtainable are serving in many situations where earlier practice, of necessity, would have fabricated a member. Fig. 10–3 shows a portion of a floor (or framing) plan and illustrates the manner in which the information comes to the detailer. The column numbers are shown in circles; the beam marks are the heavy numbers shown on those members. This particular drawing, a portion of an American Bridge Co. typical drawing, would also serve in the field as an erection plan. On this sheet, the symbol " CB " has been used in place of " WF."

Fig. 10–4 shows a floor plan as received by the steel company from the architect. It will be seen that the sheet lacks beam erection marks, these usually being added by the fabricator. Also, on this particular drawing, the distances between column centers have not been given, being known from the plans for lower floors, but it is more usual to repeat them. The symbol " B " has been used instead of " WF." The encircled numbers in the body of the plan give information about the concrete floor slab. For example, all panels numbered " 6 " are like the one of that number for which complete information is given on the drawing. The other numbers are given originally on the part which is not copied.

Typical shop, or detail, drawings for beams are shown in Figs. 10–5

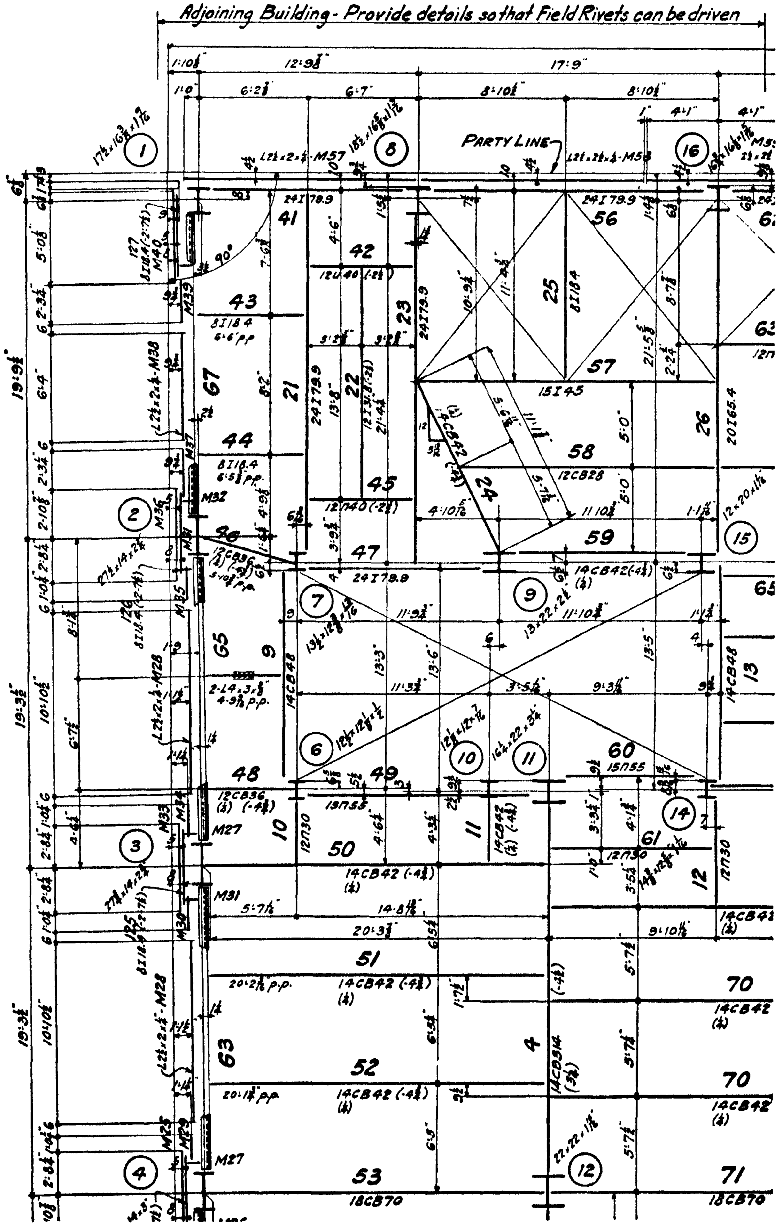

Adjoining Building - Provide details so that Field Rivets can be driven
PARTY LINE
24I79.9
15I45
12CB28
14CB42
18CB70
14CB314

FIG. 10–3

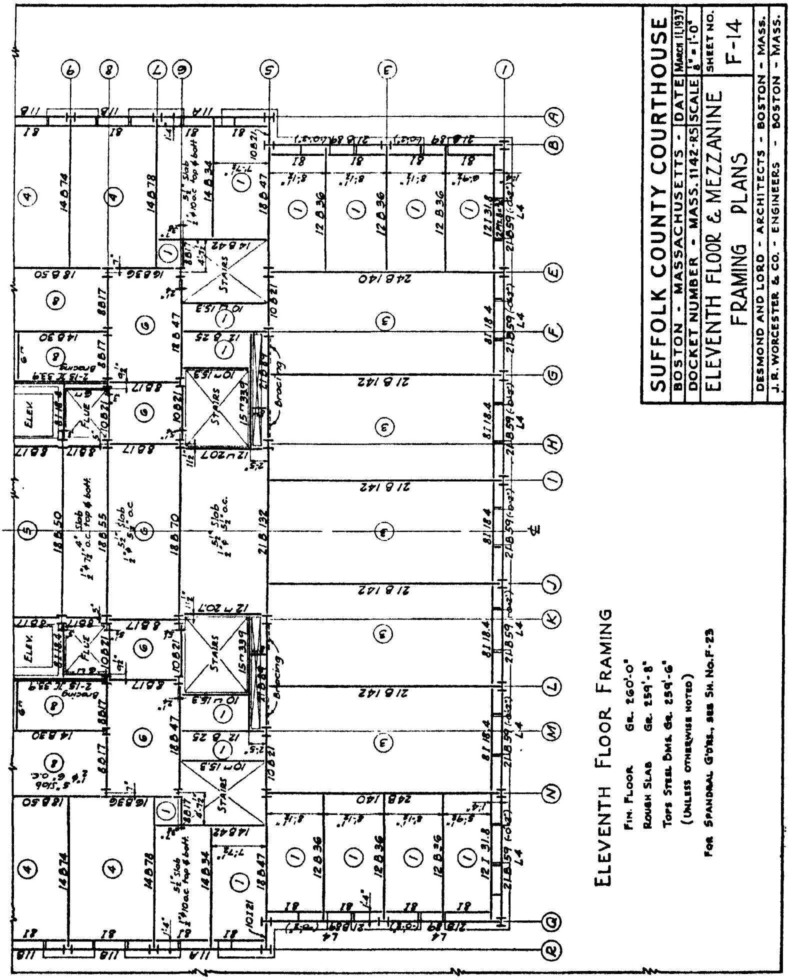

SUFFOLK COUNTY COURTHOUSE
BOSTON - MASSACHUSETTS - DATE MARCH 11, 1937
DOCKET NUMBER - MASS. 1142-RS SCALE 1/8" = 1'-0"
ELEVENTH FLOOR & MEZZANINE FRAMING PLANS
SHEET NO. F-14
DESMOND AND LORD - ARCHITECTS - BOSTON - MASS.
J. R. WORCESTER & CO. - ENGINEERS - BOSTON - MASS.
ELEVENTH FLOOR FRAMING
FIN. FLOOR GR. 260'-0"
ROUGH SLAB GR. 259'-8"
TOPS STEEL BMS. GR. 259'-6"
(UNLESS OTHERWISE NOTED)
FOR SPANDRAL G'DRS., SEE SH. NO. F-23

Fig. 10–4

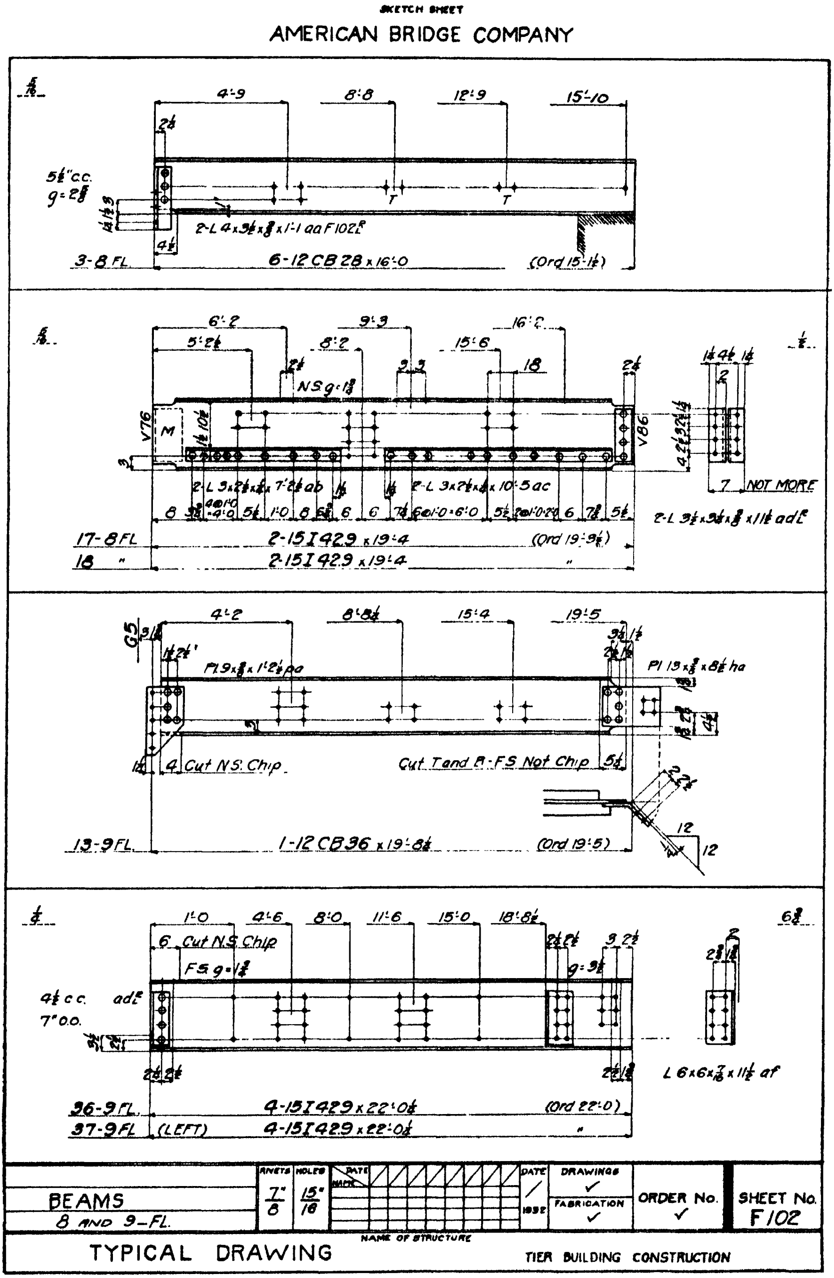

SKETCH SHEET
AMERICAN BRIDGE COMPANY
3-8 FL.
6-12 CB 28 x 16'-0
(Ord 15'-1½)
17-8 FL
18 "
2-15 I 42.9 x 19'-4
(Ord 19'-3½)
NOT MORE
13-9 FL.
1-12 CB 36 x 19'-8½
(Ord 19'-5)
Cut N.S. Chip
Cut T and B - F.S. Not Chip
36-9 FL.
37-9 FL (LEFT)
4-15 I 42.9 x 22'-0½
(Ord 22'-0)
Cut N.S. Chip
BEAMS
8 AND 9-FL.
RIVETS
HOLES
DATE
DRAWINGS
FABRICATION
ORDER No.
SHEET No.
F102
NAME OF STRUCTURE
TYPICAL DRAWING
TIER BUILDING CONSTRUCTION

FIG. 10-5

and 10–6. Printed sheets frequently are used for this purpose, all beams being shown the same size regardless of differences in depth and length. Consequently, scale has no meaning on such drawings.

10–4. Beam Connections. The great majority of beam connections are made with the aid of standard connection angles, to be found listed in every steel handbook.[1] Practice is to use these connections for any loads less than the maximum for which they are good, and this means for all except abnormally short spans carrying excessively heavy loads. Reference to a handbook will show that the open holes in the outstanding legs of connection angles are generally spaced $5\frac{1}{2}$ in. c.c. in spite of the fact that this necessitates a number of different standard angles for any one depth of beam, due to variations in beam web thicknesses. The advantage of the system arises from the fact that the detailer of one beam does not have to hunt up the detailed drawing of a connecting beam to reach an understanding as to the spacing of holes in a common connection. These considerations will explain why additional dimensions are not needed in locating the open holes in the webs of beams.

Difficulty arises in providing for spandrel beams (those beams at floor level which support exterior walls) owing to the fact that they are usually off the line joining the column centers and must be carried by brackets or other special supports. Design of such connections applies the principles of stress computation in eccentric rivet groups, already explained in Chapter III, but facility and ease in making such connections comes only after much drafting-room experience.

10–5. Wind-Bracing Connections. The remarks of the preceding paragraph are also applicable to the matter of end connections to take care of the moments (often of great magnitude) in the ends of beams which help form the bents that resist wind pressure. (Chapter IX of "Structural Theory" considers the determination of these moments.) Fig. 10–7 illustrates several of these connections. Of those shown, Fig. 10–7*c*, although structurally less effective than the others, is a favorite with the architect as it minimizes the interference with clearances. This same detail is illustrated on Plates IV and V with the difference that, for ease in erection, the connection material is shop riveted to the columns.

10–6. Grillage. The unit stress in the lowest section of a column is far too great to be transferred directly to its support, which is, in most instances, concrete. Some means must be taken to spread the load over a considerable area, and for this purpose a *grillage* consisting usually of a steel slab (or billet) — sometimes resting on one or more layers of rolled

[1] Bethlehem, 1934, pp. 203–7; Carnegie, 1934, p. 349; A.I.S.C., 1937, pp. 156–8, 242–9.

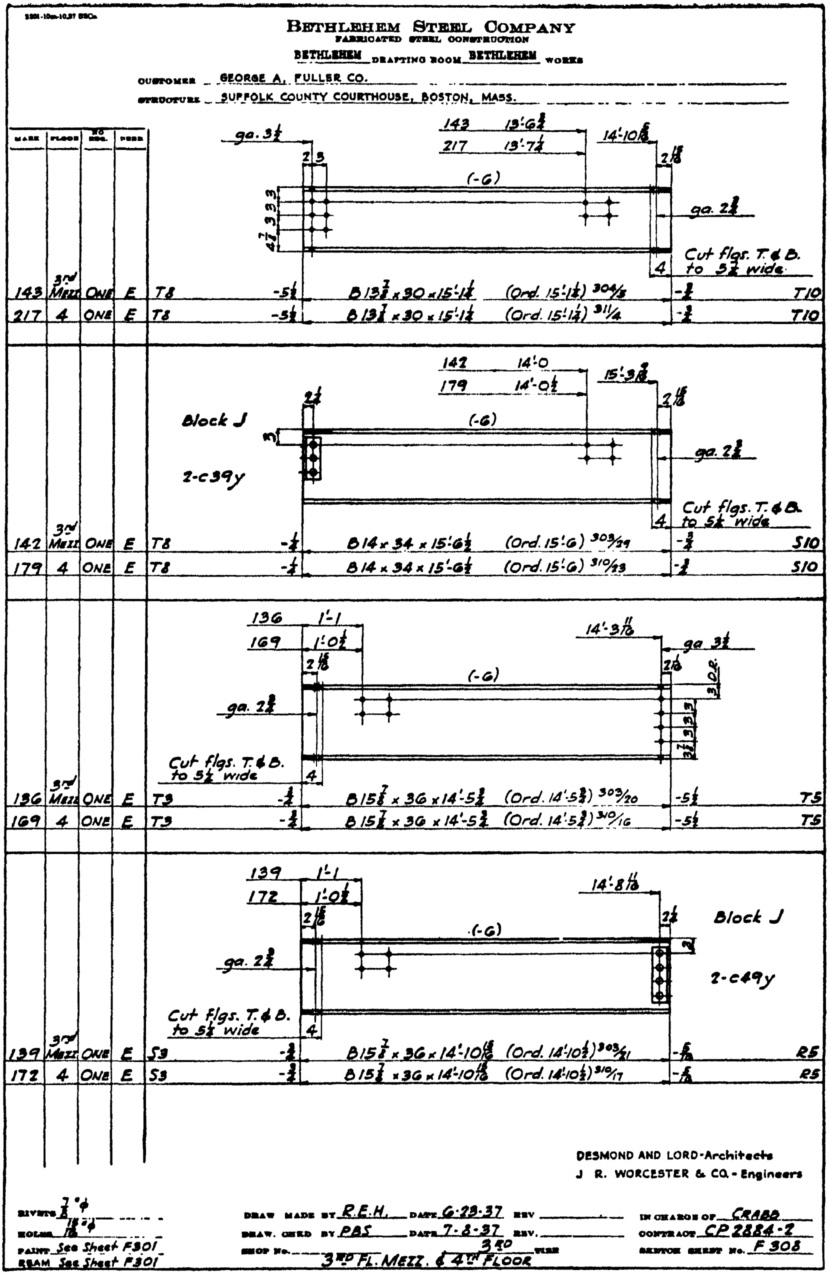


FIG. 10–6. BEAM DETAILS

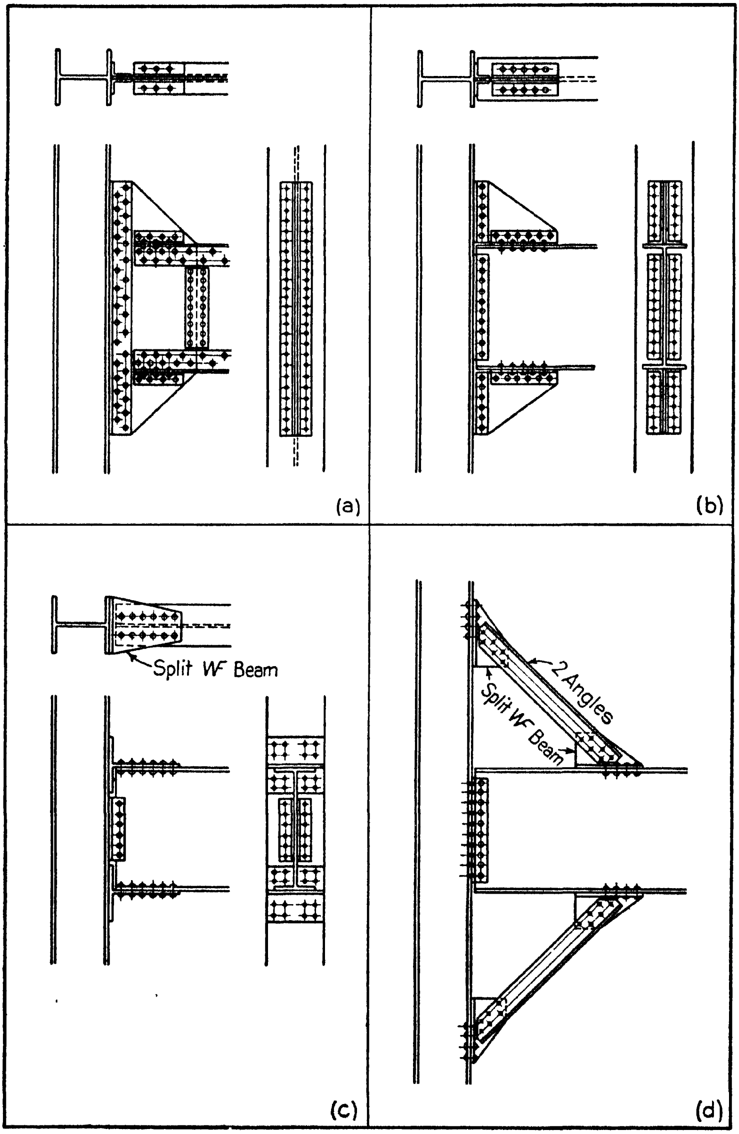


FIG. 10-7. WIND CONNECTIONS

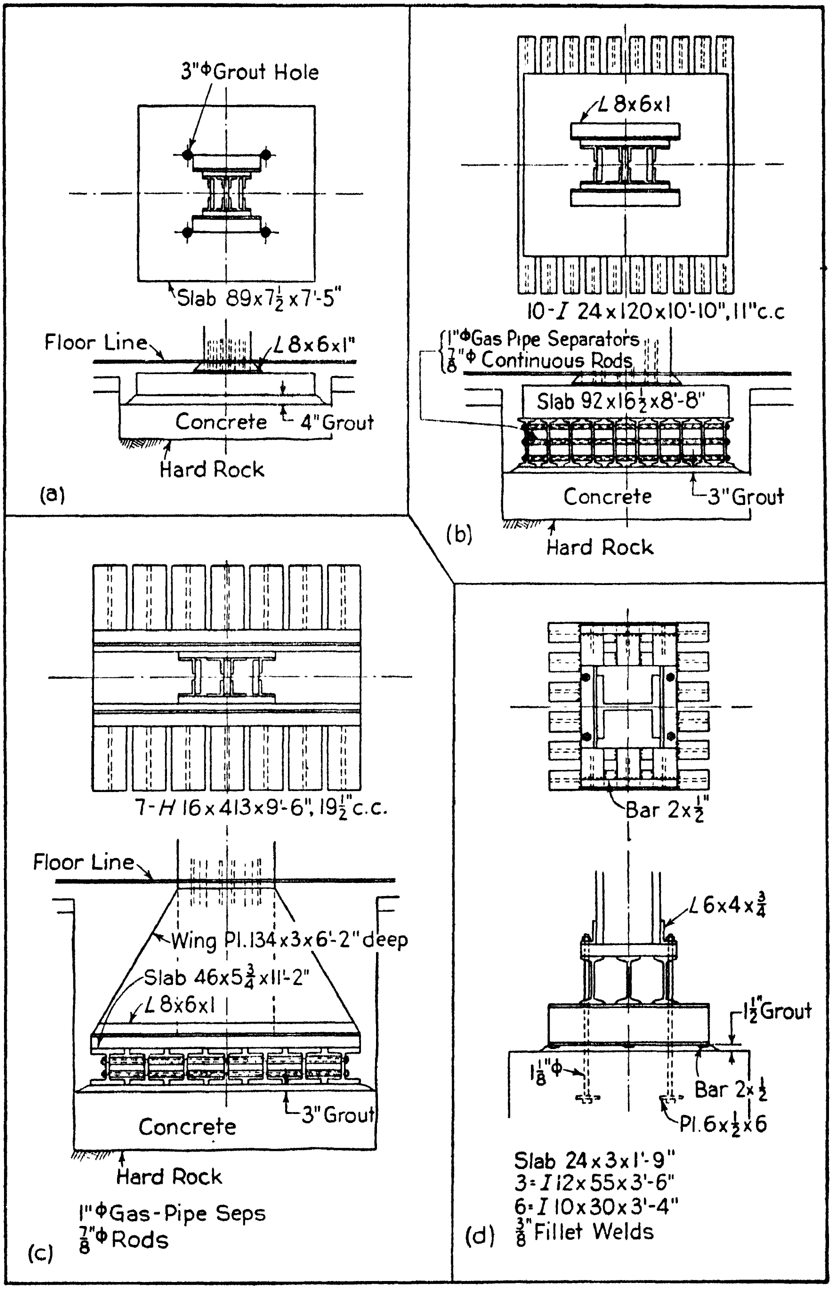


FIG. 10–8. GRILLAGE DETAILS

beams — is used. Fig. 10–8 shows typical examples. Parts *a*, *b*, and *c* of Fig. 10–8 are copied from a design of 1929. The use of gas-pipe separators which space the beams but do not interfere with the free flow of the encasing concrete is to be noted. These, as the name denotes, are simply pieces of pipe cut to proper length. Through them and the holes punched in the webs of the beams, long rods, threaded for a nut at each end, are placed. The sets of separators shown in the figure are in common vertical planes; the sets are spaced about 5 ft c.c. (beams less than 7 ft long, 2 sets; 7 to 12 ft, 3 sets; 12 to 17 ft, 4 sets; etc.).

Fig. 10–8*d* is from a design of 1937. Noteworthy are the absence of separators and the introduction of welding, because of which the grillage is shown here. However, it should be said that for a grillage as light as this many engineers — perhaps a majority — would select a single slab as shown in Fig. 10–8*a*.

Plate VI shows a grillage plan which will also serve as an erection drawing.

The design of slabs and grillage beams is empirical, as the following problems will indicate.

Problem 10–1. List the assumptions made in the design of a slab on p. 109 of "Steel Construction" (1937), the handbook of the American Institute of Steel Construction.

Problem 10–2. List the assumptions used in the design of a grillage on pp. 274–7 of the Bethlehem Steel Co.'s "Manual of Steel Construction" (1934).

Problem 10–3. List the assumptions used in the design of grillage foundations on pp. 311–4 of the Carnegie "Pocket Companion" (1934).

10–7. Ordering Material. As soon as it can be arranged after a contract comes into the office, "mill orders" are written for all the main material and an estimate is made of what will be required for details, generally taken from stock. The rolling mills furnish rolling schedules (i.e., a certain weight of beam will be rolled on a certain day), and an effort is made to order from the next scheduled rolling in order that there may be no delay. Before the shop drawings are made, this ordering is done on numbered sheets, each item on a sheet in turn being numbered. As the work is detailed, the draftsman must identify his piece in the order list and place on the drawing the order sheet and item number. These will be seen on Plate IV in the right-hand column and on the sheets of beam details at the right ends of the lines giving the beam lengths.

Additional information concerning drafting-room and fabrication practice will be found in:

" Detailing and Fabricating Structural Steel," F. W. Dencer, McGraw-Hill Book Co., 1924.

" Structural Drafting and the Design of Details," C. T. Bishop, John Wiley & Sons, 1922.

CHAPTER XI

STRUCTURAL WELDING

11–1. Since the World War, the joining of structural members in trusses and frames by welding has become increasingly prevalent. Large savings of weight are possible with this method of connection because the joints are more compact than with riveting and no increase of tension member section is required to allow for holes. Welded connections may be made stiff enough to resist end moments in continuous construction, and when this is done further weight saving is made. Many buildings in this country, some of considerable height and size, have been built with welded connections. In bridge work, welding has been restricted largely to repair and strengthening operations, for which the method is ideally adapted, only a few unimportant bridges having been erected with welded joints. The silence of welding in contrast with the noisy clatter of riveters favors the new process for many locations.

This chapter deals only with the calculation of welded joints. For the technique of the process, the reader is referred to these valuable treatises:

"Arc Welded Steel Frame Structures," by Gilbert D. Fish, McGraw-Hill Book Co., New York, 1933.

"Procedure Handbook of Arc Welding Design and Practise," Lincoln Electric Co., Cleveland, Ohio, Fifth Edition, 1938.

Welding practice in this country is established by the American Welding Society. All design in this chapter conforms to the codes of this society (henceforth referred to as A.W.S.), which are reprinted in part in Appendix E (p. 359).

11–2. Fusion Welding. Except for the use of resistance welding[1] in the manufacture of floor joists and of various light assemblies such as stairs, structural welding is by the fusion process which is defined by the A.W.S. code as "the process of joining metal parts in the molten (or molten and vapor) state, without the application of mechanical pressure or blows." At present, the method in most extensive use for welding structures employs the electric arc with metal electrode; the gas welding process is more expensive and is less common. The heat of the arc fuses

[1] In resistance welding, the parts are brought together and heavy pressure applied at the spot where intense localized heat is developed by the passage of an electric current. The metal fuses at the point of contact with the consequent union of the two pieces.

the metal of the parts to be joined along their line of contact and also melts the electrode, resulting in the deposition of additional metal. Until recently, bare-wire electrodes were most used, employing direct current, but today practically all structural welding is with electrodes heavily coated with a flux which provides a gaseous shield to the arc, excluding the air and its embrittling effect upon the deposited weld metal. Coated electrodes are used with either direct or alternating current and with a higher voltage than bare wire. Welds made with coated electrodes are stronger, more ductile, and more uniform than those made with bare wire, and the codes recognize this by permitting a 20 per cent higher stress. Increasingly it is becoming the tendency to require such high strength and ductility of the filler (that is, weld) metal that only covered electrodes will suffice.[1]

Welding is an exacting art, and it is often difficult to secure competent operators. Since no field methods for testing a weld to determine its quality completely are available, it is necessary that the welders on any job prove their competency by qualification tests and that a sufficient number of inspectors be employed "to observe with adequate continuity the actual manipulation of each operator." Also, on any construction job there should be qualification tests of the welding process "to prove that the material, the equipment, the kind and sizes of filler metal used, the kind and amount of current, the shape of groove if any, and the method of actual laying and building up the welds are all satisfactory and will produce a welded joint of the prescribed strength and quality." These matters are dealt with in the A.W.S. Code for Qualification of Welding Process and Operators, the essential portions of which are printed as an appendix to the A.W.S. bridge specification but not reprinted in this volume.

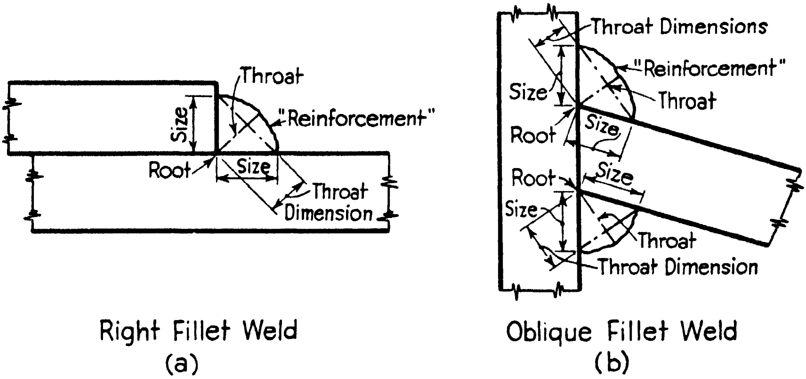
Weld metal in place is cast steel, usually low in carbon content, and has approximately the qualities here given:

	Filler Metal from Electrodes	
	Bare	Covered
Ultimate tensile strength, pounds per square inch	40,000 55,000	65,000 75,000
Elongation — free bend test	5% to 10%	20% to 30%

The shearing strength of welds is discussed in the next article.

[1] See A.W.S. specification for bridges [**401**]; compare table at end of this article. Bold-face numerals in square brackets in this chapter refer to articles in the A.W.S. specification, a portion of which is printed as Appendix E of this volume.

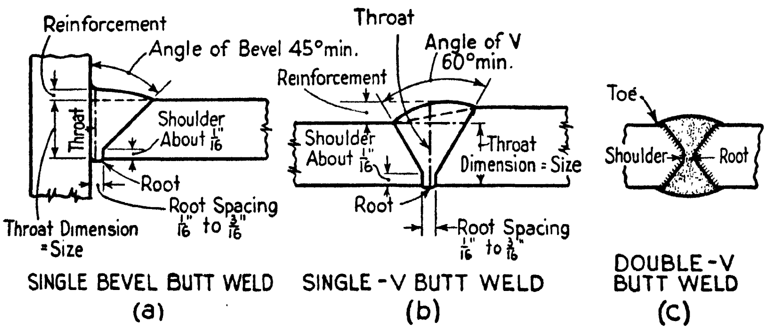
11-3. Structural Welds. Two kinds of welds are used for structural connections: *fillet welds* and *butt welds*. A fillet weld (Fig. 11-1) is a metal bead of approximately triangular cross section laid along the inter-



The reinforcements shown are highly undesirable as they tend to cause heavy stress concentrations.

FIG. 11-1

section angle of two surfaces of the pieces joined. A butt weld (Fig. 11-2) is a mass of weld metal connecting two pieces and laid in extension of one or both of them [**217, 218**].



The reinforcement shown in (*a*) reduces the abrupt transition of section and so reduces somewhat the resulting stress concentration. The reinforcements of (*b*) and (*c*), however, reduce the possibility of easy transfer of stress and where there is impact or stress repetition these protuberances should be avoided.

FIG. 11-2

In order to keep joints compact, the perimeter of a piece available for fillet welding is often increased by one or more slots or by punched holes [**219**].

The "reinforcement" of a weld, that is the excess of metal outside the geometrical outline of a triangular fillet or beyond the thickness of the pieces connected by a butt weld (Figs. 11–1, 11–2), should be kept small as irregularities of this sort lead to large stress concentrations which are

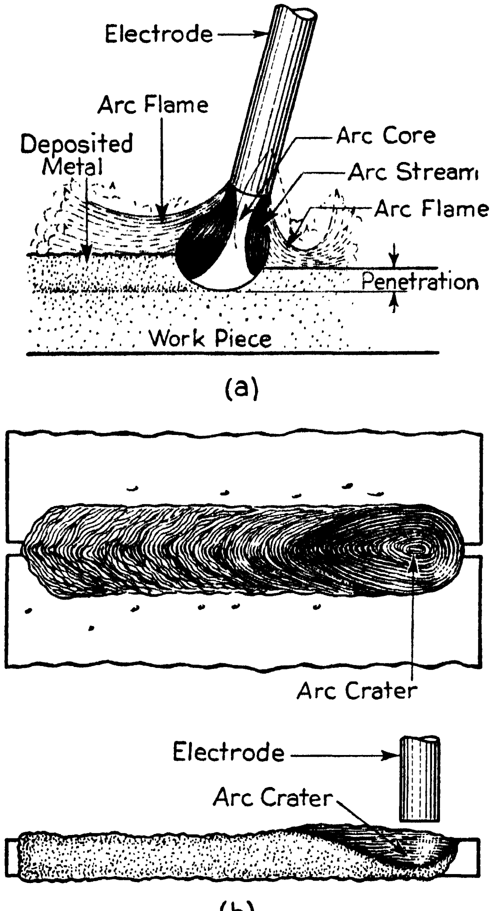


FIG. 11-3

likely to be serious if there is repetition or reversal of stress. "Reinforcement" is here a misnomer.

The depth of fusion (penetration) of the metal of the pieces to be joined is commonly $\frac{1}{16}$ in. or more (Fig. 11–3a). This melted metal unites with that from the electrode to form the connecting bead. As the electrode advances, there is a pool of molten steel directly beneath it

which quickly cools to form a crater on the withdrawal of the wire and the breaking of the arc (Fig. 11-3*b*). In computing the strength of a weld, the length is taken exclusive of the crater, an allowance of $\frac{1}{4}$ in. or more being made for it and for rounded ends.

Welds are most easily and economically made in a horizontal position, a fact which should be kept in mind in locating welds and planning fabrication procedures. Overhead welding is possible but slow and difficult.

11-4. Symbols. In Fig. 11-4 are shown the A.W.S. symbols (1934) for the representation of welds on drawings which have been in general use in this country but are now being replaced by those shown in Fig. 11-5, "a shorthand system whereby a tremendous volume of information may be accurately indicated with a few lines and a minimum amount of numerical data," much more compact and efficient than the system replaced. The beginner will find the older symbols easier to read, however, and both sets will be used in this chapter.

Inspection of Fig. 11-5 shows that the term "butt weld" is replaced by "groove weld" with five different symbols to represent the shape to which the abutting edges are to be cut. A new symbol is added to permit indication of the laying of a bead to build up thickness: both the "bead" and "flush" symbols are used to indicate on which side the welding for a groove weld is to start, near or far, the "bead" indicating that the weld is to be left with a slight reinforcement, the "flush" indicating that the limits of the welded pieces are not to be exceeded. In order to be conversant with this system, the student must go through the table carefully, noting where the numeral is to be placed to indicate size of weld (Figs. 11-1, 11-2), root opening and bevel angles for a groove weld, length and spacing of intermittent welds, and the significance and designation of "near" and "far side." The tail of the indicating arrow may be omitted when no references to specifications occur.

"Whenever fillet welds in a structure are predominantly of one size, a general note to this effect on the face of the drawings may be employed, such as 'All fillet welds shall be of such and such size unless otherwise noted,' thus permitting time saving by omitting the size notation wherever the prevailing size is intended."

11-5. Stresses in Welds. The exact analysis of the unit stresses in fillet welds under various conditions of loading is a very complicated matter and is not attempted. As in riveted joints, conventional simplifications are employed which have been justified by test results. The forces acting on a fillet weld are assumed to develop an actual resistance in shear on the throat section (Fig. 11-1) and the strength of the weld, F, is taken as equal to the longitudinal shearing strength of the fillet which

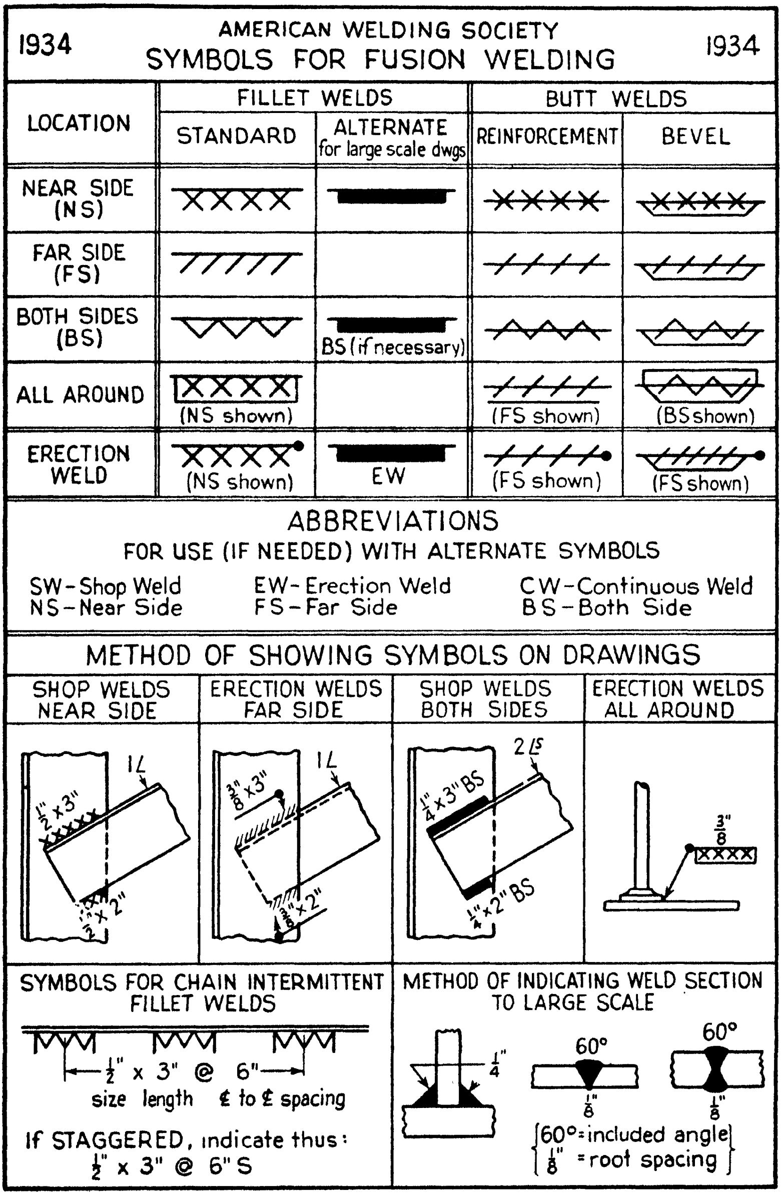
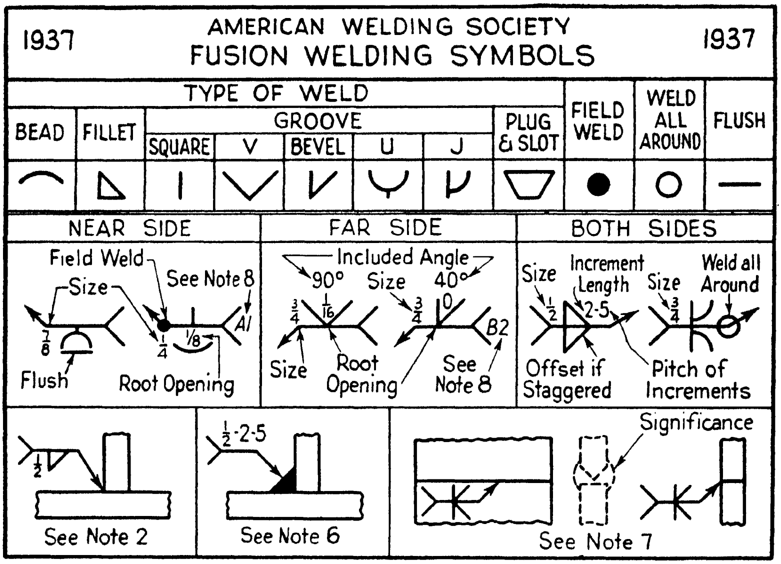

1934
AMERICAN WELDING SOCIETY
SYMBOLS FOR FUSION WELDING
1934
LOCATION
FILLET WELDS
STANDARD
ALTERNATE for large scale dwgs
BUTT WELDS
REINFORCEMENT
BEVEL
NEAR SIDE (NS)
FAR SIDE (FS)
BOTH SIDES (BS)
BS (if necessary)
ALL AROUND
(NS shown)
(FS shown)
(BS shown)
ERECTION WELD
(NS shown)
EW
(FS shown)
(FS shown)
ABBREVIATIONS
FOR USE (IF NEEDED) WITH ALTERNATE SYMBOLS
SW-Shop Weld
NS-Near Side
EW-Erection Weld
FS-Far Side
CW-Continuous Weld
BS-Both Side
METHOD OF SHOWING SYMBOLS ON DRAWINGS
SHOP WELDS NEAR SIDE
ERECTION WELDS FAR SIDE
SHOP WELDS BOTH SIDES
ERECTION WELDS ALL AROUND
1∠
½"x3"
½"x2"
1∠
⅜"x3"
⅜"x2"
2∠s
¼"x3" BS
¼"x2" BS
⅜"
SYMBOLS FOR CHAIN INTERMITTENT FILLET WELDS
½" x 3" @ 6"
size length ℄ to ℄ spacing
If STAGGERED, indicate thus:
½" x 3" @ 6" S
METHOD OF INDICATING WELD SECTION TO LARGE SCALE
¼"
60°
⅛"
60°
⅛"
60° = included angle
⅛" = root spacing

FIG. 11–4



1. In plan or elevation, near, far and both sides locations refer to nearest member parallel to plane of drawing and not to others farther behind. 2. In section or end views only, when weld is not drawn the side to which arrow points is considered near side. 3. Welds on both sides are of same size unless otherwise shown. 4. Symbols govern to break in continuity of structure or to extent of hatching or dimension lines. 5. All welds are continuous and of user's standard proportions and all except V- and bevel-grooved welds are closed (welded edges in contact) unless otherwise shown. 6. When welds are drawn in section or end views, obvious information is not given by symbol. 7. In joints in which one member only is to be grooved arrows point to that member. 8. Tail of arrow used for specification reference.

Note. All dimensions are in inches.

FIG. 11–5

equals the area of the longitudinal throat section multiplied by the allowable unit shearing stress, s,

$$P = s \times \frac{n}{8} \times \frac{1}{\sqrt{2}} \times L$$

$$= \frac{s}{11.3} \times n \times L \qquad 11\text{–}1$$

where $n/8$ is the size of the weld (Fig. 11–1) and L is the length. Using the value of s prescribed by the A.W.S. specification for welding in building construction the allowable load on a fillet weld would become $1000 \times n \times L$, or 1000 lb for each eighth of an inch of weld size for

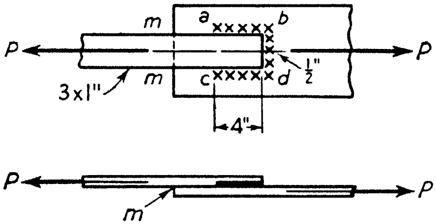
each inch of length for bare wire electrodes, 1200 lb for coated electrodes. The corresponding specification for bridges permits only 850 lb per eighth of inch of size per inch of weld [**204**].

Stresses in butt welds are figured conventionally as in plates.

When the load on a weld is applied eccentrically, the resulting unit stress is usually computed as a combination effect of direct stress and bending, using the common beam theory, assuming straight-line stress distribution.

Example 11-1. Compute the permissible value of P as limited by the weld shown. Use $s = 11{,}300$ lb per sq in.

Solution. The $\frac{1}{2}$-in. fillet weld shown can carry 4000 lb per in. of length, a total of 44,000 lb.



Ex. 11-1

The arrangement shown in the foregoing example is considered objectionable by some because it combines side welds (ab, cd, parallel to line of stress) and an end weld (bd, perpendicular to line of stress). Tests have shown end welds to be 35 per cent stronger[1] than side welds but somewhat less predictable in behavior, which accounts for equal value being given to both welds. The objection to the combination of side and end welds is that there is bound to be a very unequal division of the load between the two on account of the variation of distortion in the length of the connection. Tests have shown a tendency of the end welds to fail first, indicating that they carry the major part of the load until failure brings the side welds into play.

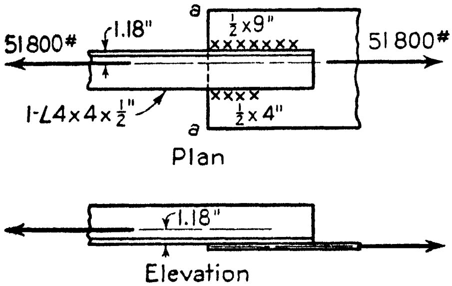
The side elevation of this joint illustrates another consideration to be kept in mind. Evidently, the action of the pull shown is to bend the joint, bringing the two forces P into a common line of action. This prying effect on the weld makes the actual maximum stress considerably greater than the average. Where eccentricity is marked, its stress effect should be computed and allowed for. This joint would be much stronger if two $\frac{1}{2}$-in. plates were used, one on each side of the larger plate. Symmetrical joints are to be preferred wherever possible. The joint would be greatly strengthened against the prying action of eccentricity by a weld along the line m–m, and if a connection of this sort is permitted, this should be insisted on.

[1] Report of the Structural Steel Welding Committee of the American Bureau of Welding (published by A.W.S.), 1931. See report on Series 2500–2800, p. 67f: Art. 7–6–2, p. 103.

Example 11–2. Is the welded connection shown adequate?

Note. The use of a single angle connection is not good practice for the reasons presented in the paragraph above. Two angles should be used, one on each side of the gusset plate or connecting piece, with separators at intervals to prevent the bowing together of the two under pull.

Solution. Taking moments about the upper weld in the plan view, the stress to be carried by the lower weld is $51{,}800 \times 1.18/4 = 15{,}300$ lb, which at 4000 lb per in. requires nearly 4 in. of $\frac{1}{2}$-in. fillet weld. Similarly, the length required at the top is $(51{,}800 \times 2.82/4) \div 4000 = 9.15$, say 9 in. Were the load applied at the center of the 4-in. leg, there would be required a total of 51,800/4000 = 13 in. of weld. Note that the total length is the same as before; in the unsymmetrical case the distribution of weld is inversely proportional to the lever arms. But these figures which apparently justify the connection do not tell the whole story.

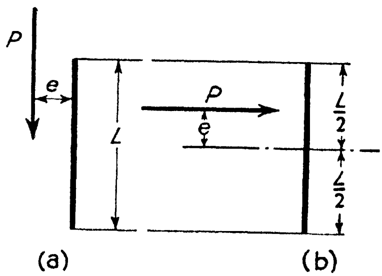


Ex. 11–2

The elevation shows that there is a moment of approximately $51{,}800 \times 1.18$ in-lb tending to tear the weld from the plate along the line *a–a*. The true magnitude of the consequent bending stresses in the welds is not known. The common beam theory does not apply since the distortion of the welds will not be planar under the combined action of pull and moment. (What is the effect of heavy shear upon the distortions of a prismatic beam?) If the action is like that of a beam with abrupt change of section, the unit stresses may easily be double and more those given by the common beam formula. Mr. Fish has proposed an analysis[1] which assumes prying action as about a fulcrum at the end of the weld.

Problem 11–1. Compute the allowable eccentric load, P, for the two cases shown. Length of weld = L: leg (size) = $D = n/8$ in.: allowable shear on throat section = 11,300 lb per sq in.

Suggestion. The resulting formulas were used in constructing Plates E1–2–3, Appendix E, pp. 368-370.



Prob. 11–1

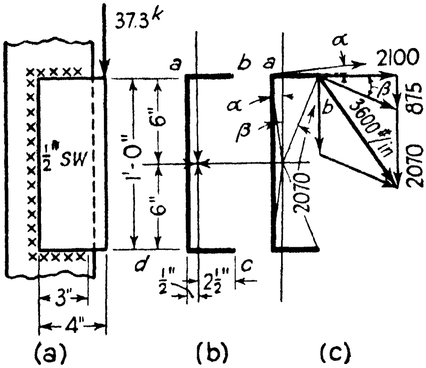
The engineer is accustomed to work in terms of the allowable load per inch of length of weld, and, accordingly, it would be well for the student to solve this problem in this way, as well as directly in terms of unit stress. For example, in the one

[1] "Arc-Welded Steel Structures" by Gilbert D. Fish, McGraw-Hill Book Co., New York, footnote on p. 210.

case the unit stress due to direct stress is $\dfrac{P}{LD/\sqrt{2}}$, and in the other method of treatment the load per inch on the weld due to direct stress is P/L. The corresponding terms for maximum bending effect are: $\dfrac{6\,Pe\,\sqrt{2}}{DL^2}$ and $\dfrac{6\,Pe}{L^2}$.

Example 11–3. The $\frac{1}{2}$-in. plate shown is attached to a building column by a $\frac{1}{2}$-in. shop weld and carries an eccentric load of 37.300 lb. Compute the maximum stress in the weld.

Solution. The centroid of the weld is located $2 \times 3 \times \frac{3}{2}/18 = 0.5$ in. from the 12-in. length. Consequently, the eccentricity of the load is 3.5 in. The effect of the eccentric load is then equal to that of 37.3 kips applied at the centroid and a moment of 37.3×3.5 in-kip. The effect of the central load is $37.3/18 =$ 2070 lb per in. Inspection of the figure (*c*) shows that the combination of the vertical direct effect with the bending effect, normal to the line from point to centroid, produces a greater resultant stress at *b* and *c* than at *a* and *d*. Before computing the bending effect, it is necessary to find the polar moment of inertia for the weld, the sum of the moments of inertia about the *X* and *Y* axes. This equals 373 in.^{3} ($\frac{1}{12} \times 12^3 + 2 \times 3 \times 6^2 + 12(\frac{1}{2})^2 + 2 \times \frac{1}{12} \times 3^3 + 2 \times 3 \times 1^2$). To the vertical stress due to shear only, 2070 lb per in., must be added the vertical component due to bending, which for *b* and *c* (letting *R* equal the lever arm from centroid to point) equals



Ex. 11–3

$$\frac{(37{,}300 \times 3.5)R}{373} \times \frac{2.5}{R} = 875 \text{ lb per in.}$$

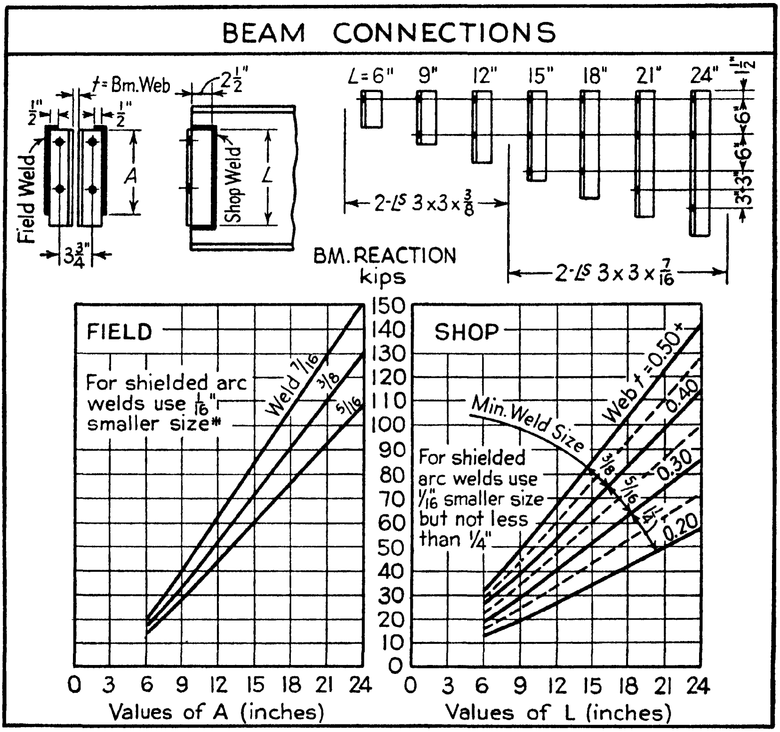
giving a total vertical stress component of 2945 lb per in. The horizontal component due to bending equals 2100 lb per in. The resultant equals 3600 lb per in., well within the 4000 lb per in. limit for the $\frac{1}{2}$-in. weld.[1]

11–6. Beam Connections without Continuity. Structural-steel handbooks give detailed information concerning standard riveted connection angles for the rolled shapes which they list, but as yet no such information is easily available for welded connections. In the *Journal* of the American Welding Society for August, 1933, H. M. Priest presents diagrams for the two common types of beam supports used with welding,

[1] See "Eccentric Welded Connections," by Odd Albert, *Journal* of the A.W.S., April, 1931.

connection angles and *seat angles.* These diagrams are reproduced (with slight change) as Figs. 11-6 and 11-7. In the Appendix, pp. 372-373, are given Plates E5-6, similar diagrams for use with coated electrodes.

The connection angles in Fig. 11-6 are shown punched for erection bolts. It is possible to use this sort of connection without the bolts, but



* This note represents common practice. Actually with covered electrodes it is proper to increase unit stresses 25% or more over those recommended for bare wire electrodes.

FIG. 11-6

then a seat angle should be provided for erection. There is an objection to using this arrangement without the bolts which illustrates considerations which must continually be kept in mind in welded work and which are less often thought of in riveted work. As a beam deflects under load, the ends rotate slightly parallel to the web so that the tops of the connections are pulled away from the supports, as shown in exaggerated fashion in Fig. 11-8. With rivets the flexibility of the

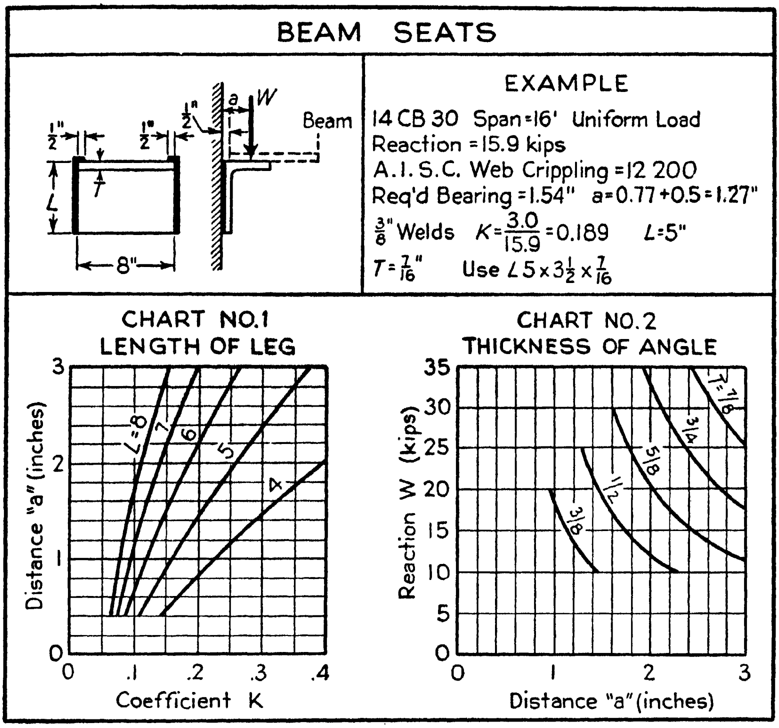


Fig. 11–7

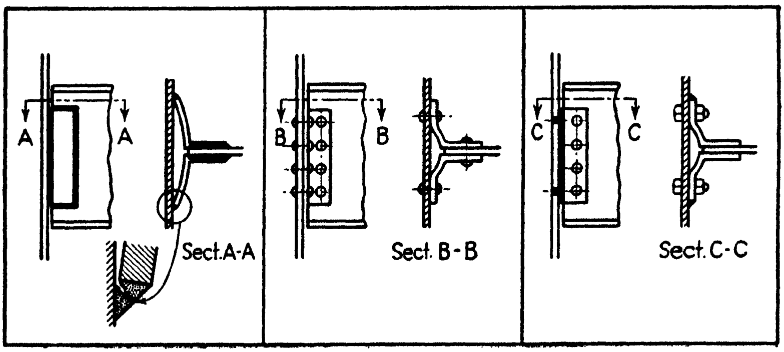


Fig. 11–8

angles is enough to permit the motion without damage, while with welds tension, which is undesirable, is thrown into the roots of the welds. If erection bolts are used and left in place in the finished structure, this pull at the tops of the welds is minimized or largely prevented. In Fig. 11–6 the field welds connecting the outstanding legs of the hitch angles to the support are shown carried up and over the tops of the angles for $\frac{1}{2}$ in. as reinforcement against this possible tension at the roots of the

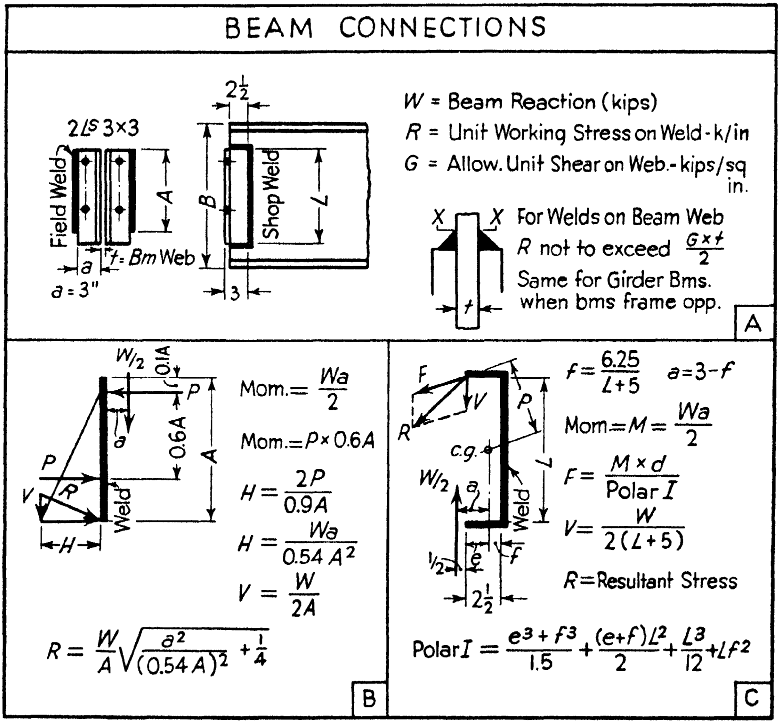


FIG. 11–9

vertical welds at the top. This reinforcement was not included in the strength calculations.

All steel frames must be plumbed and squared before final riveting or welding is done, and the prime purpose of bolted connections is to permit necessary adjustments. In light work, hooks and clamps are sometimes permissible instead of bolts.

The values plotted in Fig. 11–6 may be checked by Mr. Priest's formulas given in Fig. 11–9, which are largely explained by Prob. 11–1 and

Ex. 11–3 of the preceding article. The limitation on the weld stress per inch of weld, R, in Fig. 11–9A, in terms of the allowable unit web shear, G, is explained by the necessary relation: end reaction $= GBt = 2R(L + 5)$, where B is the depth of the beam; and $R = Gt/2$ when $B = L + 5$. The limits of size of shop weld in Fig. 11–6 were worked out for $G = 12{,}000$ lb per sq in.

Some designers calculate the field welds on the assumption of shear without bending, but the eccentricity of the load should be taken into account as here, Fig. 11–9B. Evidently the hitch angle legs against the support (compare Fig. 11–8) tend to pull together at the top and push apart at the bottom, with resultant bending of the field welds in the plane of the supporting surface. The opposition of the beam web in between the two hitch angles prevents this distortion from being linear and causes the neutral axis to lie higher than mid-length of weld. Fig. 11–9B shows the variation of bending stress assumed by Mr. Priest which does not differ greatly from the assumption made in constructing Plate E5, p. 372.

Hitch angle sizes vary with the standards of different designers: about the smallest used are 2×2 for light loads. One rule to ensure the provision of sufficient flexibility makes the width of the outstanding legs against the support at least eight times the weld size. When flexibility is of small importance, the angles may be replaced by narrow bars laid flat against the beam web, 2 in. in width or wider. It is customary to make weld size and angle or bar thickness alike.

The problems to be considered in the design of *seat angles* are outlined in Fig. 11–10 from Mr. Priest's paper, part A of which shows the usual method of welding angle to column, preferably with the welding carried over the top a short distance which is not counted on for strength. Usually beams are cut 1 in. shorter than the clear span, which gives the $\frac{1}{2}$-in. clearance shown. The location of the resultant reaction on the angle is uncertain. Some designers take the length of bearing, A in Fig. 11–10B, as the length required to develop the reaction at the usual allowable unit bearing stress, with the width of bearing taken as equal to the beam web thickness plus the flange thickness, Fig. 11–10B. Mr. Priest advocates a more refined analysis which takes account of beam deflection. This action is shown in D of the figure; as the beam bends it tends to ride on the outer edge of the seat, which results in the bending of the horizontal angle leg. The point of application of the resultant reaction between beam and seat angle at this stage depends on the thickness of the angle; the thicker the angle, the farther out the reaction must act to accomplish the bending. This bending is accompanied by yield-point stress in the angle. In E of the figure, the reaction

W is shown acting at a distance from the face of the angle equal to $B + \frac{1}{8}$ in., where B is taken as 50 per cent greater than the leverage required by load W to produce yield-point stress in the angle. If W acts at this point, yielding will be continuous under constant load. Accordingly, this marks the outer limit to which it is possible for the point of application of the reaction to pass. The one-eighth inch marks the assumed point of maximum bending in the fillet of the angle.

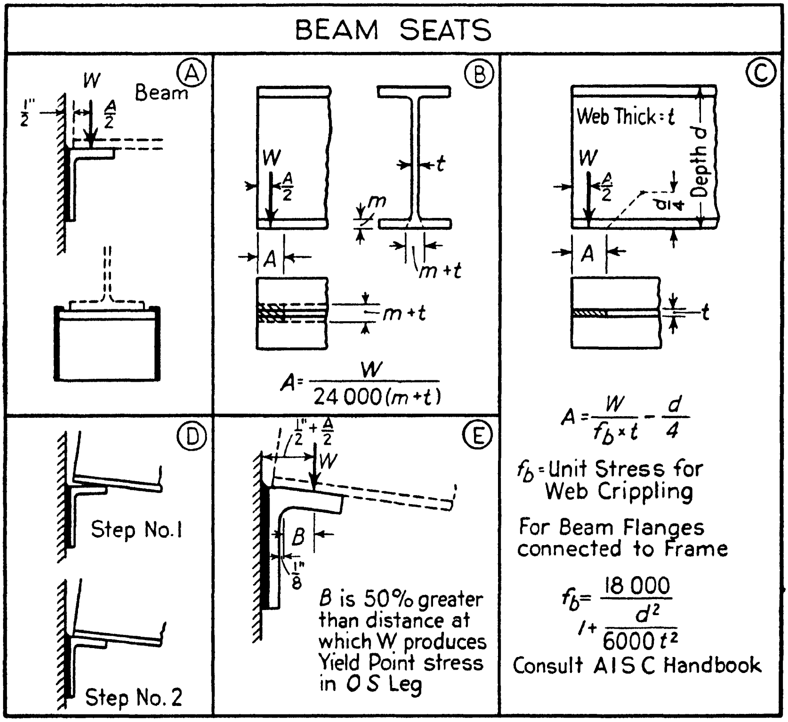


FIG. 11–10

Another limitation on the position of the resultant reaction is set by the requirement that the bearing shall be long enough to ensure a sufficient length of web over the support to act as a column, transmitting the reaction from beam to seat. From a study of test results the American Institute of Steel Construction (henceforth A.I.S.C.) recommends that this length be taken as the bearing plus one-quarter of the beam depth ($A + d/4$) (see Art 1–10 and Fig. 11–10C). The reaction then must equal $R = f_b t(A + d/4)$, where f_b is given by Rankine's column

formula

$$f_b = \frac{18{,}000}{1 + d^2/6000\,t^2}$$

a value independent of the length of web assumed. This value is given directly in some handbooks: in others, the reaction is given for $A = 3.5$ in. which enables quick computation of f_b. From this the length of bearing, A, required for any reaction can be computed.

The thickness of the seat angle is found by equating the two values of the bearing A just outlined: $\frac{1}{2} + A/2 = t + \frac{1}{8} + B$, Fig. 11–10$E$. Fig. 11–7 has been prepared to facilitate seat angle design, assuming a length of 8 in. as sufficient to provide space for field welding beams to seat. Chart 2 of Fig. 11–7 avoids the necessity of figuring the arm, $\frac{2}{3}B$, which would enable the given reaction to develop the yield-point strength of the outstanding leg of the seat angle. With the reaction and the length $a = \frac{1}{2} + A/2$ known (A being given by $A + d/4 = W/f_bt$), Chart 2 gives the seat angle thickness.

The length of the vertical leg of the seat angle must be that which will provide sufficient weld to resist the combined shear and bending. Chart 1 of Fig. 11–7 enables this to be done quickly, $a = \frac{1}{2} + A/2$ being again one of the required coordinates, the other being a coefficient equal to the allowable weld stress in pounds per inch divided by the reaction.

The practice of a large engineering organization is given by Plate E6, p. 373, and differs radically from that of Mr. Priest. The assumptions are made that after the beam deflection has bent the outstanding angle leg the length of bearing of the beam on the angle equals that required by the web-crippling formula; and that the maximum stress in the angle is 18,000 lb per sq in. at a critical section distant from the column face a distance equal to the seat angle thickness plus $\frac{3}{8}$ in., that is, at the assumed point of beginning of the fillet. This method gives heavier angles than required by Fig. 11–7.

Professor Inge LYSE[1] and Mr. N. G. SCHREINER[1] recommend, as the result of an extensive series of tests at Lehigh University, that seat angle thickness be determined with respect to bending strength and that the welds connecting angle to column be proportioned to carry the vertical shear. For the details of the bending computation the reader is referred to the original paper.

The requirement for bearing as limited by web crippling assumes that the beam is supported at the top at the end. This may be accomplished satisfactorily by a light angle connection as shown on Plate E6, p. 373. The beam should be fastened positively to the seat as shown on

[1] "An Investigation of Welded Seat Angle Connections" in the *Journal* of the American Welding Society, Feb., 1935.

the same plate. Note that these welds are at the edge of the horizontal angle leg, the correct position whenever a beam is welded to a horizontal support. Query: if these welds are at the end of the beam, is there any prying action bringing stress to the roots of the welds?

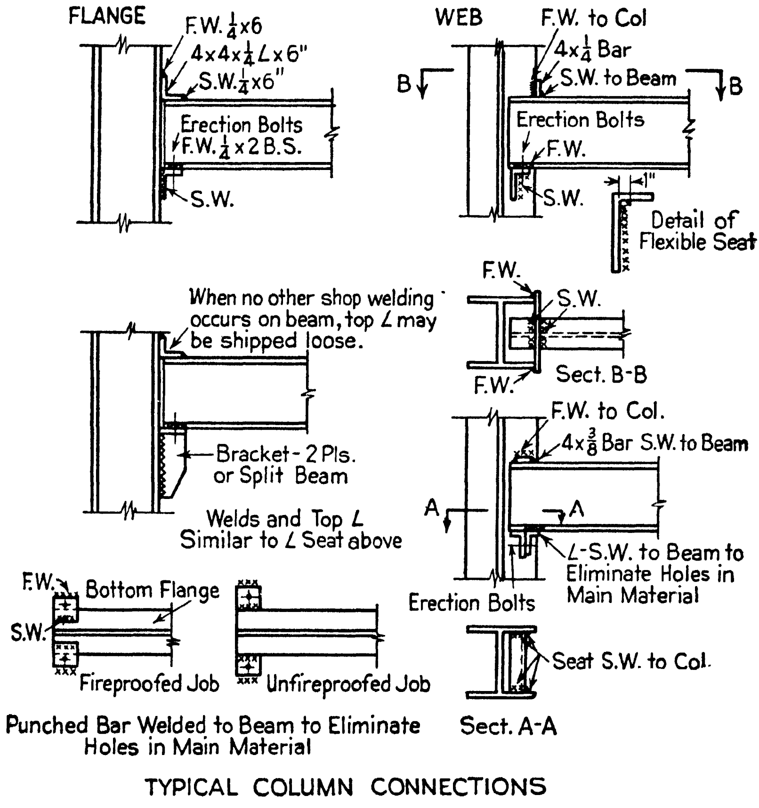
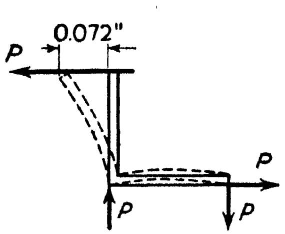


FIG. 11-11



PROB. 11-3

Problem 11-2. Compute the horizontal movement at the end of the top of an 18 WF 50, simply supported, loaded with a center load producing a maximum unit stress of 18,000 lb per sq in., span 20 ft. *Ans.* 0.072 in.

Problem 11-3. Compute the maximum unit bending stress in a 4 × 4 × $\frac{1}{4}$ top connection angle for the beam of the previous problem, arranged as on Plate E6. What proportion of the working strength of the weld is developed?

Suggestion. Assume the force system on the angle and the deflection as here shown. Neglect the angle thickness and the fillet in figuring moment arms, which are thus taken as 4 in.

Ans. 25,000 lb per sq in. approximately.
About one-quarter.

Problem 11-4. Check Chart 1, Fig. 11-7, by computing the value of K for $L = 5$ in. and $a = 2$ in.

Check Chart 2, Fig. 11-7, by computing the distance a for $W = 20$ kips and $T' = \frac{5}{8}$ in.

Suggestion. Chart 1. At 11,300 lb per sq in. a weld can safely develop a maximum intensity of pull per inch of length of 8 D kips, where D is the leg length (size). The maximum stress in this case is at the top of the weld and is the resultant of the longitudinal shear and the transverse pull due to the eccentricity of the load. The linear intensity of this last is $3\,Wa/L^2$, assuming zero stress due to bending at mid-length. (Remember that two welds share the load.) Setting $8\,D$ equal to this resultant, transferring R to the left-hand side, gives

$$\frac{8\,D}{W} = K = \frac{\sqrt{1 + 36\,a^2/L^2}}{2\,L}$$

Chart 2. Assume the yield point of steel at 33,000 lb per sq in. and compute the moment of resistance of the angle leg, neglecting the fillet. From this B (Fig. 11-10E) is computed, and then $a = \frac{1}{2} + A/2 = T + \frac{1}{8} + B$.

Problem 11-5. A 14 WF 87 carries a total uniformly distributed load of 66 kips on a span of 25 ft, framing into a column at each end. Design the end connection using (a) connection angles and (b) seat angles. Pick the connections from Figs. 11-6 and 11-7 and then verify results by computation without using the diagrams.

When the beam runs into the web instead of into the flange of the supporting column, it may be rested on a seat angle supported between the column flanges, welded thereto by vertical welds which should not be allowed to extend horizontally under the outstanding angle leg for more than an inch. Such an angle has little torsional resistance, and this lack of stiffness is counted on to permit the rotation of the end of the beam as it deflects. The stress analysis of such a connection is quite involved. The angle is stressed in torsion and also acts as a more or less completely restrained beam between column flanges. The end welds are stressed in shear by the vertical load, in shear by torsion due to the twisting of the angles, and in tension by the pull of the angle at the ends due to fixation. It is not customary to attempt this analysis for reactions of less than perhaps 40 kips but instead simply to provide enough length of weld to carry the vertical shear, using a long thin weld and an angle no thicker than the weld size, plus a turning up of the weld for an inch under the outstanding angle leg. The width of this horizontal leg must be sufficient to provide adequate bearing for the beam.

The length of the vertical leg and the angle thickness should be checked in a manner similar to that for a seat on a column flange. The support of the top of the beam in this case is made by a horizontal bar, say $4 \times \frac{1}{4}$ in., extending from flange to flange of column and welded to the top of the beam also.

For heavy reactions stiffened seat angles or brackets are used. Their design is sufficiently indicated by Figs. 11–12[1] and 11–13[1] and in variation

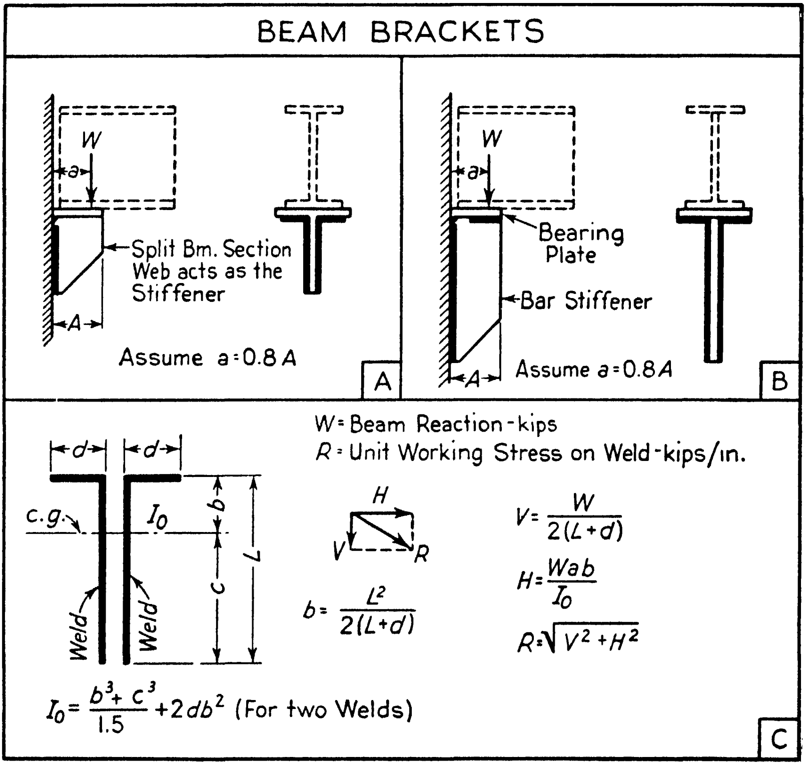


FIG. 11–12

by Plate E7, p. 374. The change from an unstiffened to a stiffened seat angle in riveted work is made when the beam reaction is about equal to the value of the four $\frac{7}{8}$-in. rivets which can be placed in the usual connection. Many designers place the same limit on the use of unstiffened seats which are welded to the column although there seems to be no particular reason for this since a greater reaction can easily be provided for.

[1] H. M. PRIEST in *Journal*, A.W.S., August, 1933.

Occasionally a beam is supported by means of fillet welds connecting its web directly to the support. Usually this is not at all an advisable procedure for it requires the beam to be cut much more accurately to length than is common or economical and also it brings excessively heavy secondary stresses into the beam web unless top and bottom connections are used in addition to relieve the web of all stress and strain

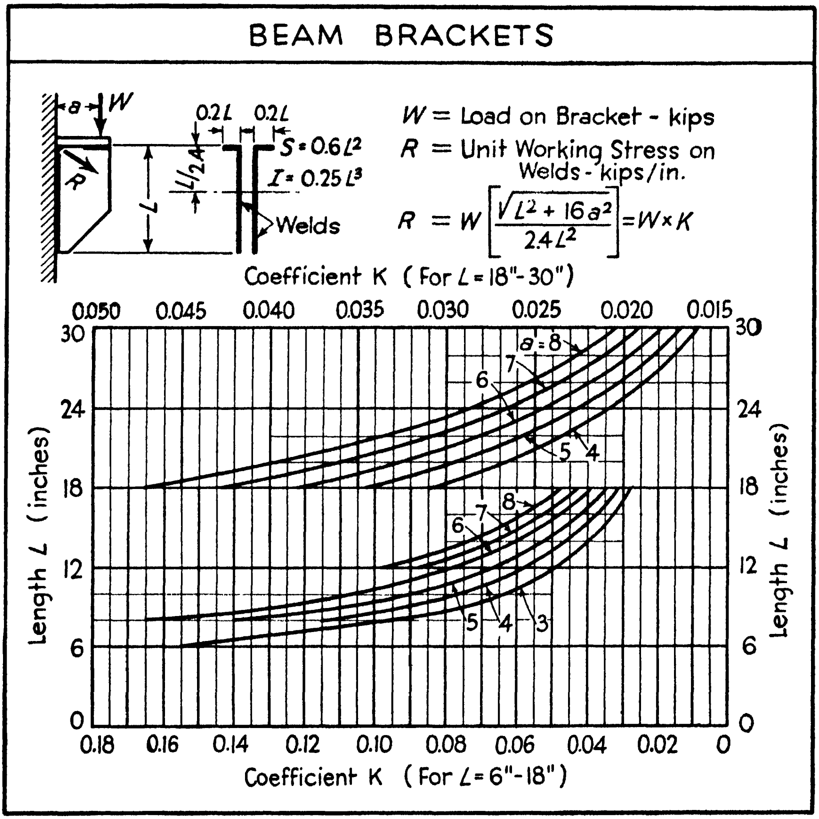


FIG. 11–13

incident to end rotation or fixation. Another objection is that oversize welds are likely to be required. The size of the weld in this case is taken as the width of the leg in contact with the beam web; the leg of the weld in contact with the support must be made wider than this by the amount of the gap between the end of the beam and the face of the support, except that a clearance as small as $\frac{1}{16}$ in. is disregarded.

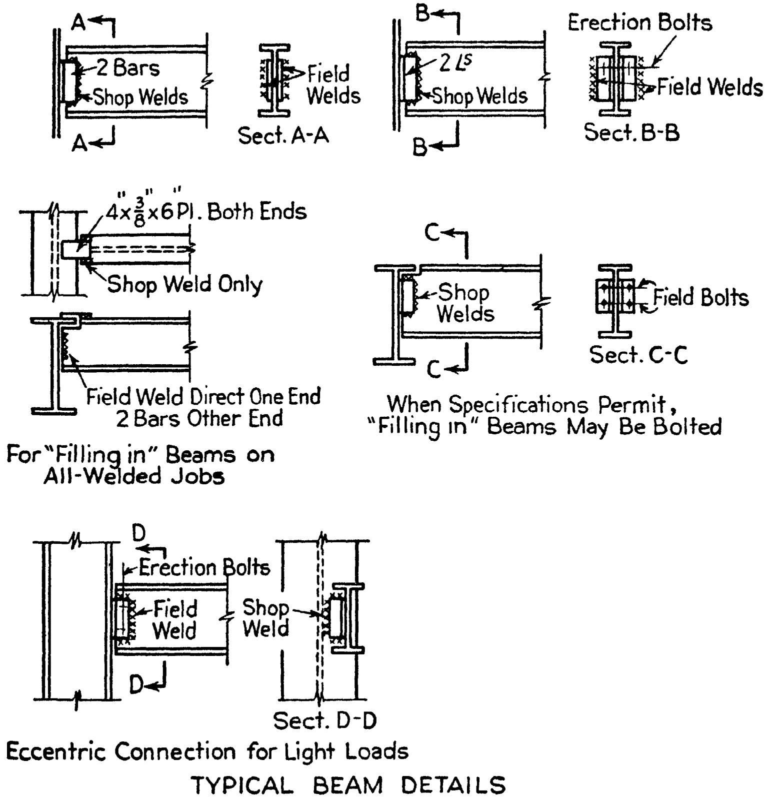


Fig. 11–14

11–7. Beam Connections with Continuity. A simply supported beam is one where there is complete freedom for the ends to rotate with the deflection of the beam, a phenomenon considered in the previous article. When this end rotation is restrained by the construction, a smaller beam is required to carry a given load on a given span than when simply supported. In riveted construction some restraint is inevitably imposed by the end connections, but this is always ignored except in specially designed rigid supports. There is sufficient slipping of plates on rivets and flexibility of angle connections so that the usual connection offers relatively little restraint. Welded connections tend to greater stiffness, and considerable economic gain results from utilizing this fact and employing smaller sections. It is necessary, however, to employ a type of support or connection which will develop the stiffness assumed in the

design. More research is necessary before practice is standardized in this matter.

Continuity of beam construction over a series of supports provides restraint and consequent decrease of positive bending moments if the loaded spans are always flanked by other loaded spans: the greater these adjacent loadings, dead or live plus dead, the greater the restraint at the ends of the intermediate span. When continuous construction consists of a series of beams and supporting columns with rigid joints, we have a rigid frame, inherently a stiffer and more economical structure than the frame made up of independent members.

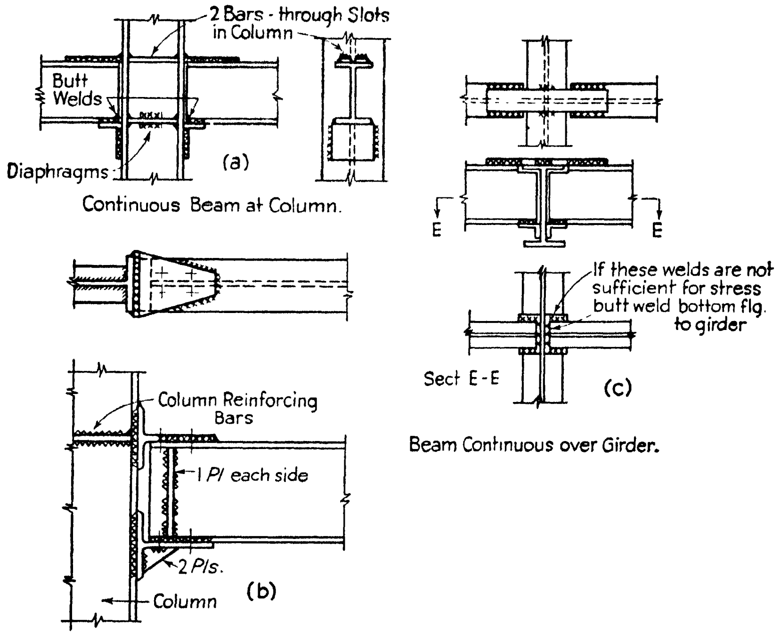


FIG. 11-15

In Fig. 11-15 are shown three methods of securing continuity of beam action, representative of the many variations in practice due to different ideas of designers in a field not yet standardized and to differences of size and position relation for the beams and supporting members. The fundamental principles to be kept in mind are principally two. Heavy stress concentration at any point in a connecting weld is to be avoided, and care must be taken that adequate provision is made for the complete transfer of stress across any joint. The connection shown at (*b*) in Fig. 11-15 most clearly illustrates the application of these principles.

Here we have the wind-bracing connection so common with riveted work transferred directly to welded construction. Were the column reinforcing bars omitted, the tension at the top of the beam due to negative bending moment would be resisted only by the web of the column, evidently an indirect path since the tension at the connection is spread over the width of the column flange. This pull on the flange will tend to deflect outward the relatively flexible flange edges, resulting in heavy and unsafe stress concentration at the middle of the weld. The reinforcing bars prevent flange deflection and thus ensure uniform stress distribution over the weld connecting the split I-beam support to the column, and at the same time they provide a direct stress path across the column.

The connection of Fig. 11–15*b* is more simply made without the split beam supports, welding the top flange directly to the column flange by a bevel groove weld (single V butt weld, Fig. 11–2*a*), and adding a small auxiliary plate on top of the beam flange to permit of a larger groove weld if needed. In this case the only function of the weld at the bottom of the beam is to keep the lower flange butted solidly against the column to ensure good bearing. However, if this type of connection is used at one end of the beam it cannot be used at the other since the beam cannot be expected to be the exact length of the clear distance between column faces. (Practice is to order beams 1 in. shorter than the clear distance, and the usual mill tolerance on cutting is plus or minus $\frac{3}{8}$ in.) Here the connection between top flange and column face may be made by a tie plate with a groove weld, and that between bottom flange and column by a square groove weld, securing additional stress path for the compression, if necessary, by welding the flange to the seat angle and by means of auxiliary plates. If difficulty is encountered in securing sufficient length of fillet weld between plate and beam, a slot weld may be indicated. Different shapes of tie plates are used, the trapezoidal form being common, with wide parallel base equal to the width of column flange.

If rigid beam-to-column connections are desired to make the frame stiff against lateral loads, it should be kept in mind that there will be tension at the bottom of some beams, with reversal if the transverse force is reversed. Adequate stress analysis provides for contingencies of this sort.

Weld-connected wind bracing is much more economical than bracing with rivets and often is more compact.

11–8. Plate Girders. The direct connection of parts possible with welding makes unnecessary the many angles required in riveted plate girders and permits large reductions in weight and in number of pieces.

Welding makes possible the assembly of a rather wide variety of pieces to form a plate girder, a flexibility which is very advantageous. Besides the conventional assemblage of plates, the combinations shown in Fig. 11-16 have proved effective.

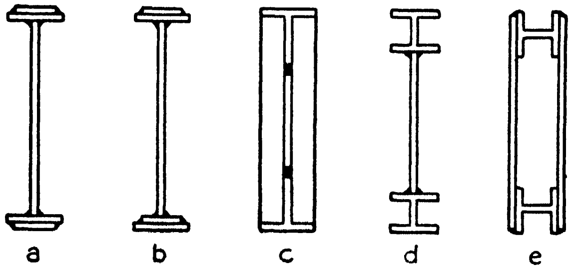
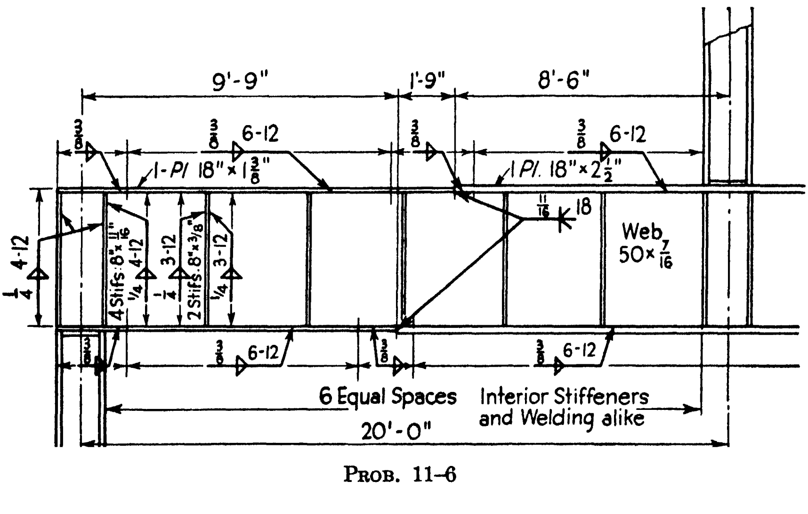


FIG. 11-16

Web Splice. Changes in web thickness are very easily made with welding, since the splice may be had by groove welding both plates to a transverse bar placed between them and extending outwards on both sides to form a stiffener.



PROB. 11-6

Problem 11-6. The girder here shown, designed by the Lincoln Electric Co. ("Procedure Handbook" (1938), Figs. 603, 605, pp. 548, 551), carries a center load of 350 kips and a uniformly distributed load of 700 lb per ft over its whole length. Check the details of design listed below and revise if necessary, using the A.I.S.C. and A.W.S. Specifications for Buildings (pages 310 and 359).

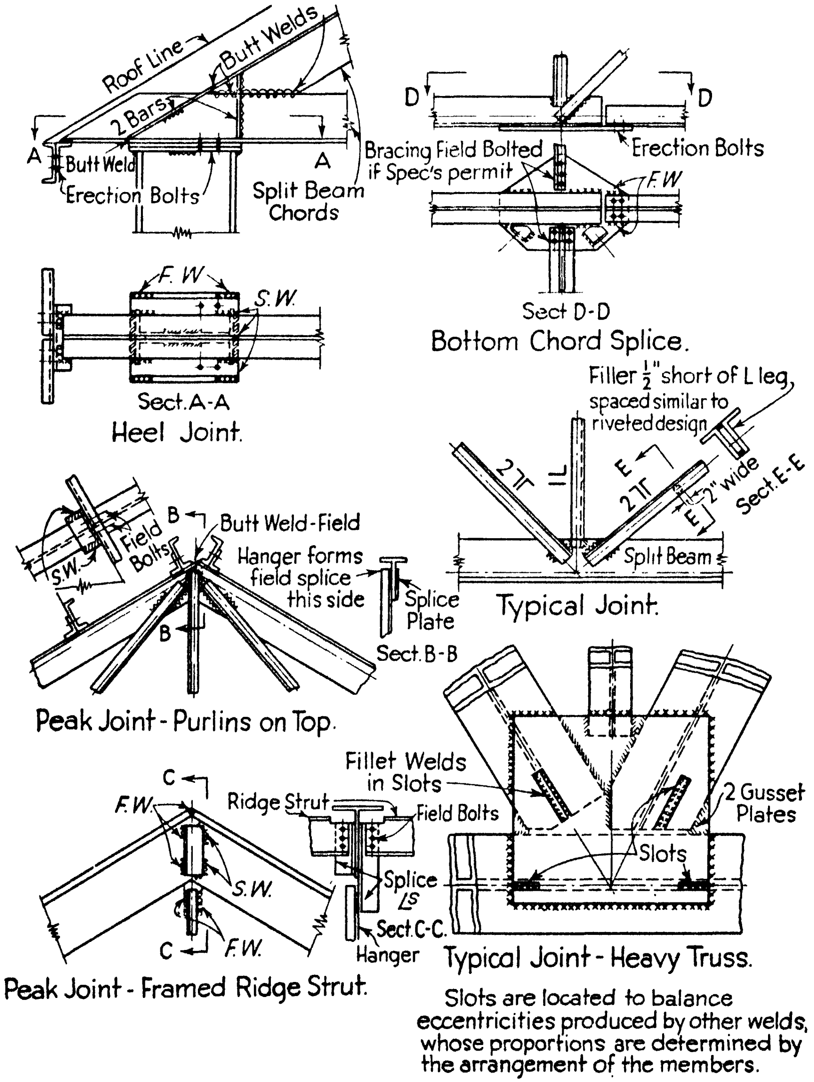
(*a*) Web thickness and interior stiffener spacing.

Query: could a thinner web be used with the stiffener spacing shown, which is about 39 in.?

(*b*) Chord plate thicknesses, using $I/c = M/s$. Assume top chord completely prevented from buckling. When checking the short unsymmetrical section, 1 ft 9 in. long, consider whether the neutral axis actually can move from the mid-point of girder as required by theory.

(*c*) Welding between web and chord plates.

(*d*) Load and reaction stiffeners and welding.



TRUSS DETAILS

FIG. 11-17

Problem 11–7. Design a riveted girder to replace that of Prob. 11–6 and compare the number of pieces and the weights of the two designs.

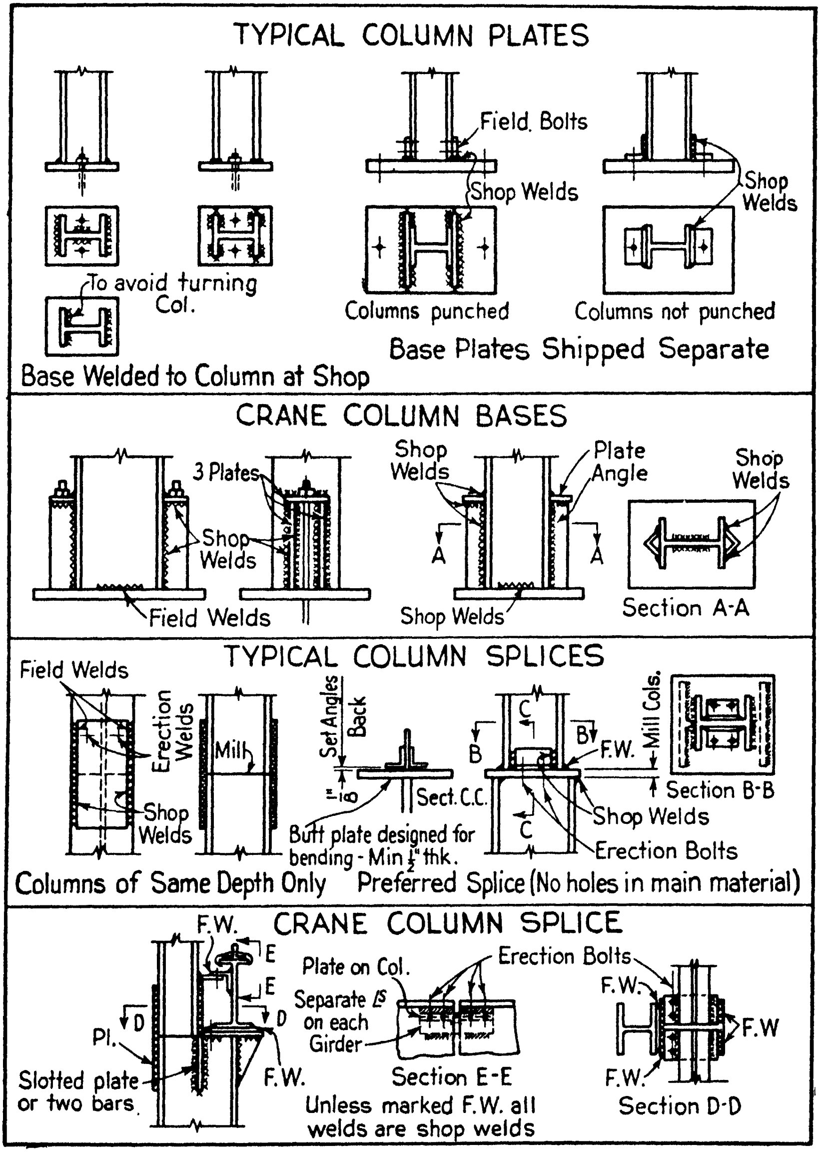
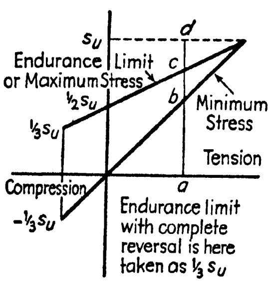


Fig. 11–18

11–9. Welded Trusses. By means of welding, considerable savings in weights of trusses are possible, since tension members may be smaller,

no allowance being needed for rivet holes, and gusset plates may be omitted in many situations where the direct connection of members is possible. Typical truss joints of the Bethlehem Steel Co. are shown in Fig. 11-17.

11–10. Repeated and Reversed Stress. A structural piece will fail under the repeated application of a load considerably less in magnitude than that required to cause rupture on a single application. The more nearly the stress approaches the static ultimate the fewer will be the applications necessary to cause failure. The most severe manner of applying a load, apart from application suddenly or with shock, is in such a way as to cause reversal of stress from tension to compression and back, continuously through a series of cycles. Experiment has shown that structural materials possess an *endurance* or *fatigue limit*, the maximum unit stress which may be repeated indefinitely without failure. For structural steel the endurance limit with complete reversal of bending stress is approximately one-half of the static ultimate strength in tension, when determined with smoothly polished specimens of uniform or nearly uniform section. The endurance limit is lower when there is surface roughness or abrupt change of section.



Goodman Diagram

FIG. 11–19

For complete reversal of shearing stress in torsion the endurance limit of steel is about one-half of that in reverse bending.

A general idea of the variation of endurance limit with variation of the range of stress may be had by inspection of the Goodman diagram (1903), Fig. 11–19, which assigns a somewhat smaller value to the endurance limit with complete reversal ($\frac{1}{3}\ s_{\text{ultimate}}$) than is given by modern carefully machined specimens. The diagram may be taken to represent working stresses by dividing all ordinates by a constant, the factor of safety. The equation for the endurance limit for any ratio of minimum and maximum stresses is

$$s_{\text{en lim}} = ab + (bc = \tfrac{1}{2}\ bd)$$

$$= s_{\text{min.}} + \tfrac{1}{2}\ (s_u - s_{\text{min.}})$$

$$= \tfrac{1}{2}\ s_{\text{min.}} + \tfrac{1}{2}\ s_u \qquad 11\text{–}2$$

In terms of the endurance limit for complete reversal, s_r, here taken as $\frac{1}{3}\ s_u$, this becomes

$$s_{el} = \tfrac{1}{2}\ s_{\text{min.}} + \tfrac{3}{2}\ s_r$$

Rewriting as $2\ s_{\text{max.}} - s_{\text{min.}} = 3\ s_r$, dividing by $s_{\text{max.}}$ and solving for its value, gives

$$s_{el} = s_{\text{max.}} = \frac{3\ s_r}{2 - \text{min./max.}} \tag{11-3}$$

$$= \frac{\frac{1}{2}\ s_u}{1 - \text{min./2 max.}} \tag{11-4}$$

called the Goodman or Goodman-Johnson formula. Of several similar formulas, notable are the Launhardt formula

$$s_{\text{max.}} = \tfrac{1}{2}\ s_u\ (1 + \text{min./max.}) \tag{11-5}$$

for stresses of the same kind, and the Weyrauch formula for reversed stress

$$s_{\text{max.}} = \tfrac{1}{2}\ s_u\ (1 - \text{min./2 max.}) \tag{11-6}$$

By dividing any of these formulas by a factor of safety there results a formula giving the reduced working stress to use with many repetitions of live load.

The A.R.E.A. rule [**215**] may be expressed

$$A = \text{area required} = \frac{1}{s}(\text{max.} + \text{min./2})$$

$$= \frac{\text{max./max.}}{\dfrac{s}{\text{max.} + \text{min./2}}}$$

$$= \frac{\text{max.}}{s'}$$

where

$$s' = \frac{s}{1 + \text{min./2 max.}} \tag{11-7}$$

s being the usual working stress and s' the reduced stress to use with reversal for the determination of area required to carry the maximum stress. The reduced working stress to use with the minimum stress is two-thirds of *s-usual* [**215**]. As written, the ratio *min./2 max.* of Eq. 11–7 employs the arithmetical values of the stresses which are of opposite kind. It is preferable, however, to use these values with opposite signs which results in the Goodman formula

$$s' = \frac{s_{\text{usual}}}{1 - \text{min./2 max.}} \tag{11-4'}$$

The number of repetitions of stress for structural parts is not sufficient to require any lowering of working stress unless there is reversal as well as

repetition, it being necessary in any case to keep the total stress (dead, live, impact, lateral, and secondary) within the elastic limit.

Problem 11–8. Plot Eq. 11–2 to 11–7 with the ratio $s_{working}/s_{usual}$ as ordinates and minimum/maximum as abscissas.

Fatigue Strength of Welded Joints. The endurance limit for welded joints is below that of structural steel. For butt joints the fatigue limit may be taken at about 16,000 lb per sq in. for bare wire and 30,000 lb per sq in. for covered wire, about 0.60 and 0.90, respectively, of the limit for the base metal.[1] Transverse fillet welds made with coated electrodes have an endurance limit of about 16,000 lb per sq in., longitudinal fillets a limit about 15 per cent less. The abrupt change of section at a fillet weld produces inevitable concentrations of stress with resultant lowering of fatigue resistance.

Consideration of the above brief factual summary makes it plain that with welded connections repetition and reversal of stress have an important effect and recent specifications in this country and abroad have given large emphasis to these stresses. The A.W.S. Bridge Specifications (1936), Appendix E, permit a design stress in butt welds of 13,500 lb per sq in. for stress repeated from zero to a maximum and two-thirds of that value for complete stress reversal; the corresponding figure for fillet welds is 7,200 lb per sq in., coated wire being specified in both cases.

Inspection of Tables 1 and 2 [**202, 203, 204**] show that the A.W.S. specification employs the Goodman formula, Eq. 11–4′.

[1] Fatigue Strength of Welded Joints, W. Spraragen and G. E. Claussen, *Journal of the American Welding Society*, October, 1936.

CHAPTER XII

TIMBER ROOF TRUSS

12–1. Timber, one of man's first building materials, has been used since before the days of written history. Despite the present predominance of steel and concrete, timber is of great interest to the modern engineer because large quantities of it are now used in forms for concrete, for falsework and scaffolding, and for such structures as trestles, wharves, caissons, quays, and cofferdams. Design in any material is affected by the physical properties of that material and by the details and connections to which it is adapted. This is especially true of timber, and it is for the purpose of illustrating its peculiar properties that the present brief chapter is written. Most important to note, probably, is the variation in the allowable fiber stresses of various kinds. This is explained by the structure of wood, essentially a mass of hollow tubes, strong in end tension and compression, weak in transverse compression, and weakly resistant to longitudinal shear. The rules of Appendix G should be carefully noted.

12–2. Specifications. As added data for the design problem which is to follow, a section of a timber specification that includes tables of allowable working stresses is reproduced in Appendix G, herewith.

12–3. Design Problem. It is desired to design a 50-ft roof truss for the conditions stated on the first design sheet, TRT 1, p. 293. The type selected is the "English," in which certain members are vertical as shown. If, as is sometimes done, the members shown inclined are placed perpendicular to the top chord and the other web members are sloped to meet this position, the result is known as a "Belgian" truss.

The *sheathing* (sometimes called *sheeting*) is continuous over a number of rafters. Such a member, under continuous uniform load, is, in most of its spans, similar to a beam fixed at the ends. A coefficient of $\frac{1}{12}$ is, therefore, justified.

The *rafters* — members which support the sheathing and run parallel to the trusses — will probably be continuous over two panels. In such a case, the maximum moment of $\frac{1}{8}\,wL^2$ occurs over the middle support. The shear adjacent to this support is $\frac{5}{8}\,wL$.

The *purlins* — roof members which run from truss to truss — will also be continuous over two spans. As noted in the drawing, Plate VII, adjacent purlins will not be spliced over the same truss. This serves better to tie the roof together as a whole.

As is common practice, the design is made for *equivalent vertical loads,* instead of for combined dead load, snow or ice load, and inclined wind load. The design sheets of Chapter IX illustrate the method of combining stresses and may be followed if desired.

In timber, better than in any other material, the fact is demonstrated that the *design of members* is not completed until the details are worked out. The areas selected for the chords appear excessive at first sight. As the succeeding parts of the problem are developed, it is seen that these members are the smallest that may be used.

The *top chord,* a compression member, carries its highest stress midway between panel points because of its buckling tendency. For this reason the *splice* should be located near a panel point.

Usual practice is to locate the *bottom-chord splice* in the third panel, thereby using for the chord three sticks of approximately equal length. In that case, the half splice shown would be immediately repeated. In the present instance, it was found impossible to splice the size selected in the third panel, and the splice was moved to the fourth. As the splice was developed, it became apparent that the middle piece of the chord would be very short and the two splices would almost overlap. The unusual expedient was adopted, therefore, of omitting any center piece and allowing the plates to carry the stress at the middle of the truss.

Handbooks and catalogs give what are listed as standard *washers.* The use of these, however, results in excessive bearing pressures. As an example, consider a "standard" cast-iron ogee washer. This washer is $3\frac{1}{4}$ in. in diameter and has a $\frac{7}{8}$-in.-diameter hole. Its bearing area is $\frac{\pi}{4}\left[(3\frac{1}{4})^2 - (\frac{7}{8})^2\right] = 7.7$ sq in. The handbook states that a $\frac{3}{4}$-in.-diameter bolt or rod, with which this washer is intended to be used, has an area under the threads of 0.30 sq in. If stressed to 16,000 lb per sq in. the resulting total stress in the rod will be 4800 lb. The bearing pressure under the washer will be $4800/7.7 = 625$ lb per sq in., a value higher than the allowable on any kind of timber. The situation as regards malleable-iron washers is even worse. These, which have the same outside diameter as cast-iron washers, are of ribbed construction and have a bearing area equal to about 60 per cent of that of a cast washer. These examples will make plain why square plate washers of steel were used at most joints. The thickness of each plate washer was determined by the common method of determining the required thickness in a reinforced-concrete footing. The thicknesses are greater than required by a frequently quoted rule which says that the plate thickness should equal one-half of the rod diameter. Placing a plate washer at joints U_1, U_2, and U_3 would have necessitated a deep cut in the chord. For this

reason, a simple cast-iron washer, of a kind that can be obtained readily from any foundry, was used.

To determine the required sizes of *notches*, a truss diagram was constructed, to the largest scale that would go on a page, giving the correct angle between members at a joint. A larger scale was then used in laying out the members. For convenience of workmanship, a 90° notch angle was adopted. Also, 1 in. was taken as the minimum notch depth. This depth was found satisfactory at two joints. At the others the required depth was found by trial. In these computations, bar stresses were resolved graphically into components normal to notch surfaces, and allowable unit stresses for these surfaces were read from a transparent diagram of the type illustrated on design sheet TRT 6.

In some of the books listed in Art. 12–6 the reader will find castings, side plates, etc., used at the *end joint*. An all-timber joint was used here because it suited better the purpose of the problem.

The *truss weight* was figured on the basis of 48 lb per cu ft. This is on the side of safety, for, while it approximates the weight of the material when green, 40 lb would be a better value for air-dried lumber.

Plate VII is a drawing of the truss as designed.

12–4. Destruction; Preservation. Under certain conditions, timber will suffer destruction by rot, termites, and marine borers. However, this damage may be retarded or prevented by means of preservatives, the most common of which are creosote and zinc chloride. An elementary textbook is not the proper place for an extended discussion of these matters, but the designer or builder who has anything more than a casual contact with timber should speedily inform himself concerning them.

12–5. Connectors. The engineer who is working with timber now has at his disposal connectors which permit the use of details that it would be impossible to make otherwise. Axel H. OXHOLM, Director, National Committee on Wood Utilization, states: "Modern connectors consist of various types of rings, plates, and disks inserted in the faces of two wood members to be joined, with a single bolt to hold the pieces together. The connectors act as the load-transmitting agency. Extensive European experience, substantiated by the comprehensive test program, has established certain clear-cut advantages for this new construction system. The strength of the ordinary bolted joint is increased several fold; in some cases where the wood is stressed perpendicular to the grain the increase may amount to as much as 12 times or even more. Next, the rigidity of structures framed with modern connectors is approximately doubled. In other words, the slip or play in the joints is only about half of that for the usual type of bolted joint. Another advantage, following as a natural corollary of the first two, is the

reduction which may be accomplished in the sizes of timbers and in the quantities of hardware. Finally, the connectors facilitate the shop fabrication of structural timbers under methods similar to those in common practice with structural steel. All of these features result in important construction economies." [1]

To date there have been many applications of connectors, but almost all of them have been European. The readily available sources of information on the subject are the publication from which the above quotation was taken, and "Modern Connectors for Timber Construction," [2] Fig. 83 of which is reproduced here as Fig. 12-1. This figure shows "suggestive diagrams of joints assembled with modern connectors." [3]

12-6. Bibliography. The reader will recall that the purpose of this chapter, which comprises what the authors consider the minimum amount the student should know about timber, was to provide an introduction to working in that material. Therefore, only such instruction was given as was needed for the one design problem. To treat the subject of timber exhaustively would require a large volume. The worker who finds himself confronted with many structures in the material should consult the following:

"American Civil Engineers' Handbook," MERRIMAN-WIGGIN, John Wiley & Sons, New York. Much information on timber is scattered throughout the volume.

"Architects' and Builders' Handbook," PARKER-KIDDER-NOLAN, Wiley. Same comment as above.

"Wood Construction," principles, practice, design. A project of the National Committee on Wood Utilization. McGraw-Hill Book Co., New York.

Proceedings and "Manual," American Railway Engineering Assn., Chicago.

"Timber Design and Construction," JACOBY and DAVIS, Wiley.

"Structural Members and Connections," HOOL and KINNE, McGraw-Hill. Sec. 4, Design of Wooden Members. Sec. 5, Splices and Connections for Wooden Members.

"Steel and Timber Structures," HOOL and KINNE, McGraw-Hill.

"Manual," Southern Pine Assn., New Orleans.

Current periodicals: *Engineering News-Record, Railway Age, Wood Preserving News*, etc.

[1] From introduction to "Structural Applications of Modern Connectors," U. S. Department of Commerce, Washington, D. C.

[2] Superintendent of Documents, Washington, D. C., 15 cents. "This report was prepared jointly by engineers of the National Committee on Wood Utilization and the Forest Products Laboratory. It demonstrates the use of more than 60 types of metal connectors which strengthen wood joints from 4 to 8 times. Also gives the results of tests of these connectors with American woods and explains their application to American construction. This bulletin is of particular value and interest to construction engineers, architects, and builders."

[3] Connectors are available through the Timber Engineering Co., 1337 Connecticut Avenue, Washington, D. C., a company organized by lumbermen to acquire patent rights and make the connectors commercially available at nominal cost.

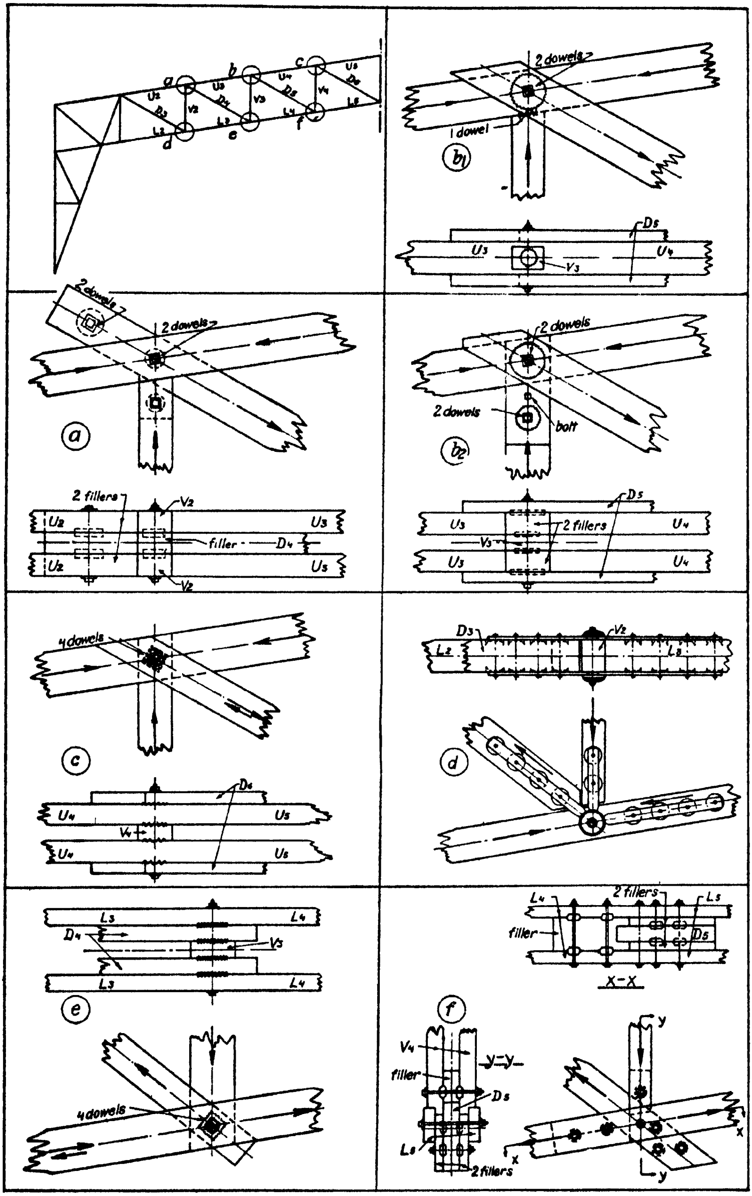

2 dowels
1 dowel
2 fillers
filler
bolt
4 dowels
X-X
Y-Y
2 fillers

FIG. 12-1

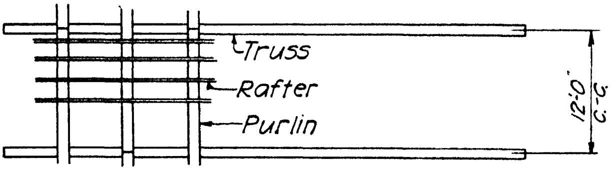
Problem 12–1. Design a wooden English roof truss of the span, etc., assigned, assuming a location in the central states (see Appendix, Art. F–2). Prepare a drawing similar to Plate VII.

Part No.	Span, ft	Panels	Pitch (or Slope)	Spacing c.c. Trusses, ft
a	48	6	(30°)	12
b	48	6	$\frac{1}{4}$	12
c	48	6	$\frac{1}{5}$	12
d	51	6	(30°)	12
e	51	6	$\frac{1}{4}$	12
f	51	6	$\frac{1}{5}$	12
g	54	8	(30°)	12
h	54	8	$\frac{1}{4}$	12
i	54	8	$\frac{1}{5}$	12
j	57	8	(30°)	12
k	57	8	$\frac{1}{4}$	12
l	57	8	$\frac{1}{5}$	12
m	57	8	(30°)	14
n	57	8	$\frac{1}{4}$	14
o	57	8	$\frac{1}{5}$	14
p	60	8	(30°)	14
q	60	8	$\frac{1}{4}$	14
r	60	8	$\frac{1}{5}$	14

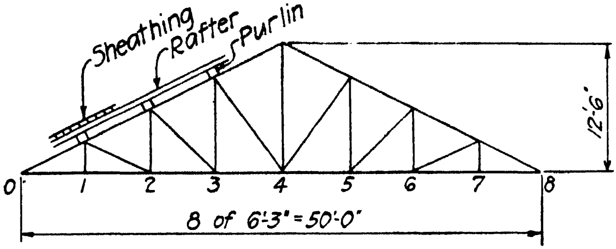
Problem 12–2. Similar to Problem 12–1, except that the joints shall be made with the aid of "modern connectors."

Timber Roof Truss — Design Sheet - TRT 1

Wooden English Roof Truss for New England States



Plan



Elevation

Materials

Roofing - Slate.
Sheathing and Rafters - 1100 lb. f Eastern Hemlock.
Purlins and Truss Lower Chord - 1800 lb. f Dense Longleaf or Shortleaf Southern Pine.
Truss Upper Chord and Diagonals - 1300 lb. c. Dense Longleaf or Shortleaf Southern Pine.

Working Stresses

Timber. As stated above in stress-grades and as modified in specification printed as Appendix G, herewith.
Steel. 16,000 lb. per sq. in.
Brickwork. Bearing on brickwork in lime mortar, 8 tons per sq. ft. (= 111 lb. per sq. in.)

Loads

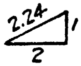
Truss, snow, and combined wind and snow from Appendix F, herewith.
Wind. 30 lb. per sq. ft. on vertical surfaces, reduced by Duchemin's formula for sheathing, rafters and purlins.

Timber Roof Truss — Design Sheet - TRT 2

Sheathing

1" Hemlock
1100 lb. f.

Investigate 1" sheathing ($\frac{7}{8}$" std. thickness) on 2' span (max. allowable with slate because of deflection)



Sheathing	4#/□'
Slate	10
Ice	10
	24 vertical

$$24 \text{ vertical} \times \frac{2}{2.24} = 21.4\#/\square' \text{ normal}$$

$$\text{Wind, } P\frac{2\sin\alpha}{1+\sin^2\alpha} = 30\,\frac{2(1/2.24)}{1+1/5} = \underline{22.4}$$

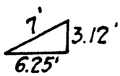
$$43.8$$

$$S = \frac{6M}{bh^2} = \frac{\frac{1}{12}\times 43.8\times 2^2\times 12}{\frac{1}{6}\times 12\times(7/8)^2} = 115\#/\square''\ (<1100,\ \therefore O.K.).$$

Rafters

2×6
Hemlock
1100 lb. f.

Assume 2" hemlock (2" Nominal, $1\frac{5}{8}$" Actual)
Find required depth.



Rafter	2#/□' ±
	44 (sheathing, etc.)
	46×2 = 92#/' of rafter)

Rafter continuous over 2 spans.

$$M = \frac{1}{8}\times 92\times 7^2\times 12 = 6760''\#$$

$$S = \frac{6M}{bh^2} = 1100 = \frac{6760}{\frac{1}{6}\times 1\frac{5}{8}\times h^2},\quad h^2 = 22.7,\quad h = 4.77''$$

Try 2×6 ($1\frac{5}{8}\times 5\frac{5}{8}$)

$$\text{Shear} = s_s = \frac{3}{2}\frac{V}{bh} = 92\times 7\times\frac{5}{8}\times\frac{3}{2}\div\left(1\tfrac{5}{8}\times 5\tfrac{5}{8}\right) = 66\#/\square''\ (<70,\ \therefore O.K.)$$

Purlins

6×8
S.Y.P.
1800 lb. f.

Trusses spaced 12' c.-c. Assume sheathing takes component of load parallel to roof. Purlins, therefore, need be designed for normal load only.

Purlin	16#/' ±
Normal load = 46×7 =	322
	338

Purlin continuous over 2 spans.

$$M = \frac{1}{8}\times 338\times \overline{12}^2\times 12 = 73{,}000''\#$$

$$S = \frac{bh^2}{6} = \frac{M}{s} = \frac{73{,}000}{1800} = 40.6, \text{ Reqd.}$$

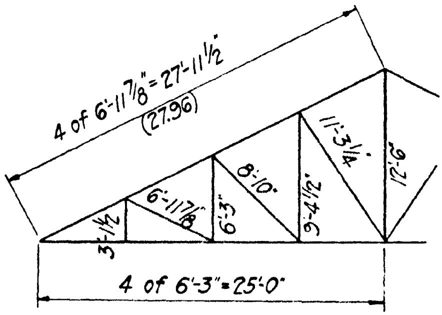
Try 6×8 ($5\frac{1}{2}\times 7\frac{1}{2}$)
$S = 51.6$

$$\text{Shear } s_s = \frac{3}{2}\frac{V}{bh} = \frac{338\times 12\times\frac{5}{8}}{5\frac{1}{2}\times 7\frac{1}{2}}\times\frac{3}{2} = 92\#/\square''$$

($<120\#/\square''$ ∴ O.K.)

Timber Roof Truss — Design Sheet- TRT 3

Truss Member Lengths

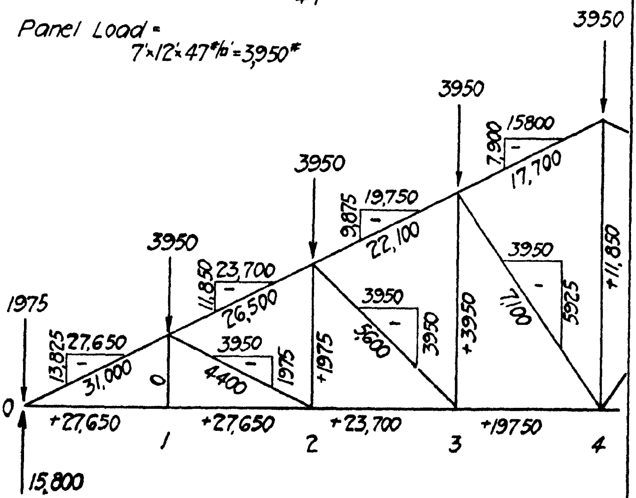


Truss Stresses (pounds)

Design for equivalent vertical loads.

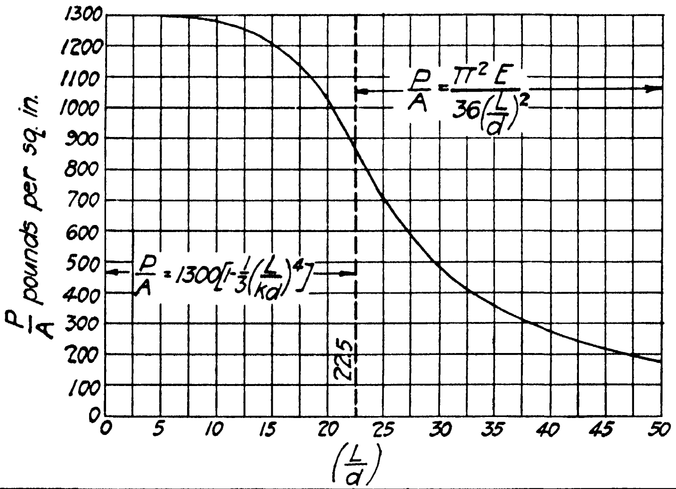
Snow and wind	25#/□'	"Theory," p 62
Slate	10	
Sheathing	4	
Rafters	2	
Purlins	2	
Truss	4	
	47	

Panel Load = 7'×12'×47#/□' = 3,950#



TRT 3

Timber Roof Truss — *Design Sheet-TRT 4*



Allowable Unit Compressive Stress in Southern Pine 1300 lb. c.

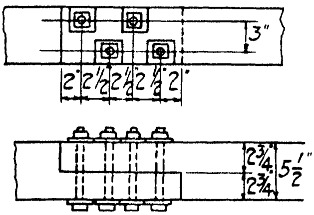
Design of Members

Member	Length	Stress, lbs.	Size	$\frac{L}{d}$	Allowable Unit Stress Lb. per Sq. In.	Area Reqd. Sq. In.	Area Furnished Sq. In.
Top Chord	$6'\text{-}11\frac{7}{8}''$	−31,000	6×6 ($5\frac{1}{2}\times5\frac{1}{2}$)	15.3	1200	25.8	30.2
Bottom Chord	6'-3"	+19,750 (a)	6×6 ($5\frac{1}{2}\times5\frac{1}{2}$)	—	1800	11.0 (a)	30.2 (a)
U_1L_2	$6'\text{-}11\frac{7}{8}''$	−4,400	3×6 ($2\frac{5}{8}\times5\frac{5}{8}$)	32.0	430	10.2	14.8
U_2L_3	8'-10"	−5,600	4×6 ($3\frac{5}{8}\times5\frac{5}{8}$)	29.2	510	11.0	20.4
U_3L_4	$11'\text{-}3\frac{1}{4}''$	−7,100	6×6 ($5\frac{1}{2}\times5\frac{1}{2}$)	24.6	720	9.9	30.2
U_1L_1	$3'\text{-}1\frac{1}{2}''$	0	3/4"φ Not Upset	—	—	—	—
U_2L_2	6'-3"	+1975	3/4"φ Not Upset	—	16,000	0.13	0.30 (b)
U_3L_3	$9'\text{-}4\frac{1}{2}''$	+3950	3/4"φ Not Upset	—	16,000	0.25	0.30 (b)
U_4L_4	12'-6"	+11,850	1"φ Upset to $1\frac{3}{8}''$φ	—	16,000	0.74	0.78 1.05 (b)

(a) See design of bottom-chord splice.
(b) Area at base of threads.

Timber Roof Truss — *Design Sheet-TRT 5*

Top Chord Splice



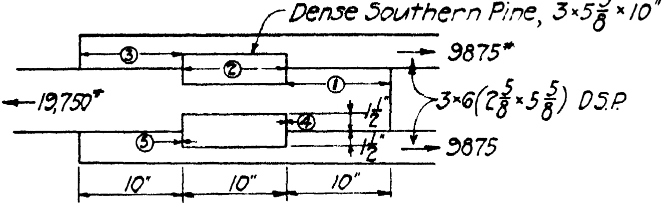
4-Bolts $\frac{3}{4}$" $\phi \times 7\frac{1}{4}$"

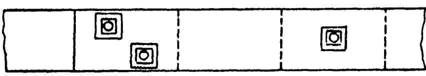
8-Washers $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$

Place in panel 2-3 near joint U_2

Bottom Chord Splice

In panel 3-4, $S = 19,750$#





3-Bolts $\frac{3}{4}$" $\phi \times$ 1'-0½"

6-Washers $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$

Shear on planes 1, 2, 3 $= \frac{19750}{2} = 9875$#

S_s allowable $= 120 \times \frac{3}{2} = 180$ #/□"

See spec. for shear in joints (Par. 6 "Notes on the Use of Stress Grades")

$\frac{9875}{180 \times 5\frac{1}{2}} = 10$"; Use 10"

Compression on planes 4, 5 = 9875#

$\frac{9875}{1300 \times 5\frac{1}{2}} = 1.38$" Use $1\frac{1}{2}$"

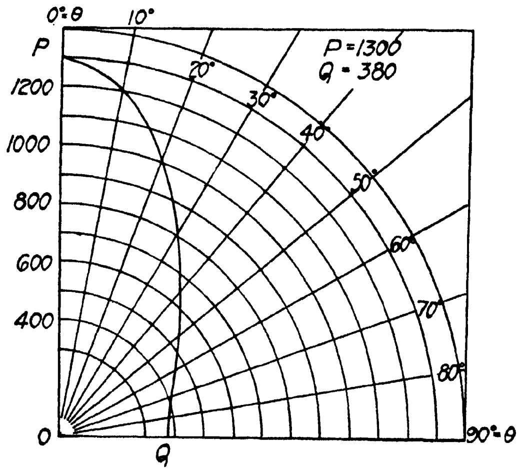
Net area in chord $= 5\frac{1}{2}(5\frac{1}{2} - 3) = 13.75$ □"

Net area in One-half splice $= 5\frac{5}{8}(2\frac{5}{8} - 1\frac{1}{2}) = 6.33$ □"

$S_t = \frac{9875}{6.33} = 1560$ #/□" ($<$ 1800 #/□" allowable) O.K.

Timber Roof Truss — *Design Sheet - TRT 6*

Allowable Bearing Pressure on Inclined Grain

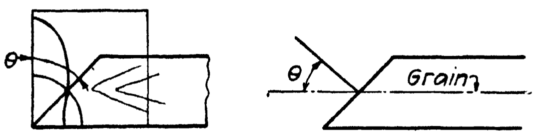


N, compressive stress,
lb. per sq. in.

Hankinson Formula

$$N = \frac{PQ}{P \sin^2\theta + Q \cos^2\theta}$$

For actual use, this diagram, shown here for illustration, should be made on tracing cloth or transparent paper in order that values may be obtained by placing it directly over the surface in question.

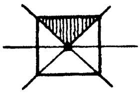


Timber Roof Truss — *Design Sheet—TRT 7*

L4

$$\frac{11{,}850}{380} = 31.20^{\square''}$$

$$\begin{array}{l} 1.62 \ (=1\tfrac{7}{16}''\phi) \\ \hline 32.82 \ (=5.72^2), \quad 6'' \times 6'' = 36^{\square''} \end{array}$$



$$\frac{\frac{11{,}850}{4} \times \frac{2}{3} \times 3}{16{,}000\left(\frac{1}{6} \times 4\frac{9}{16}\right)} = t^2 = 0.49''$$

$$t = 0.70 \quad \left(\tfrac{3}{4}''\right)$$

Washers — L4: $6 \times 6 \times \frac{3}{4}$ Steel

L3

$$\frac{3950}{380} = 10.40^{\square''}$$

$$\begin{array}{l} 0.52 \ (=\tfrac{13}{16}''\phi) \\ \hline 10.92 \ (=3.30^2), \quad 3\tfrac{1}{2}'' \times 3\tfrac{1}{2}'' = 12.25^{\square''} \end{array}$$

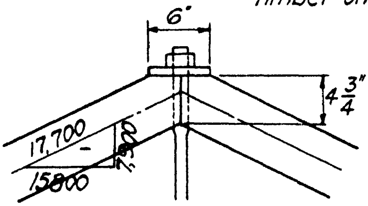
$$\frac{\frac{3950}{4} \times \frac{2}{3} \times 1.75}{16{,}000\left(\frac{1}{6} \times 2\frac{11}{16}\right)} = t^2 = 0.16$$

$$t = 0.40 \left(\tfrac{7}{16}''\right)$$

Use same at L_2 and L_1

Washers — L_3, L_2, L_1: $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{7}{16}$ Steel

U4 Use same as for L_4 if remaining bearing area, timber on timber, is sufficient.



$$\frac{15{,}800}{4\frac{3}{4}\left(5\frac{1}{2} - 1\frac{3}{8}\right)} = 805\#/\square''$$

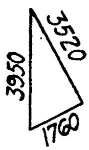
Allowable $= 890\#/\square''$ $\therefore$ O.K.

Washers — U_4: $6 \times 6 \times \frac{3}{4}$ Steel

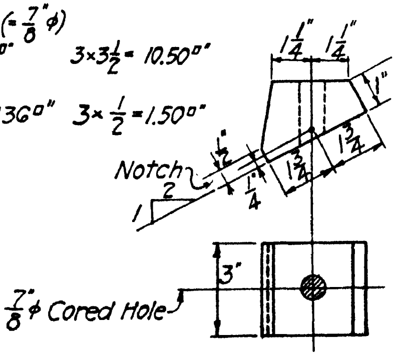
U_3

$$\frac{3520}{380} = 9.28$$

$$\begin{array}{l} 0.60 \ (=\tfrac{7}{8}''\phi) \\ \hline 9.88^{\square''} \end{array} \quad 3 \times 3\tfrac{1}{2} = 10.50^{\square''}$$

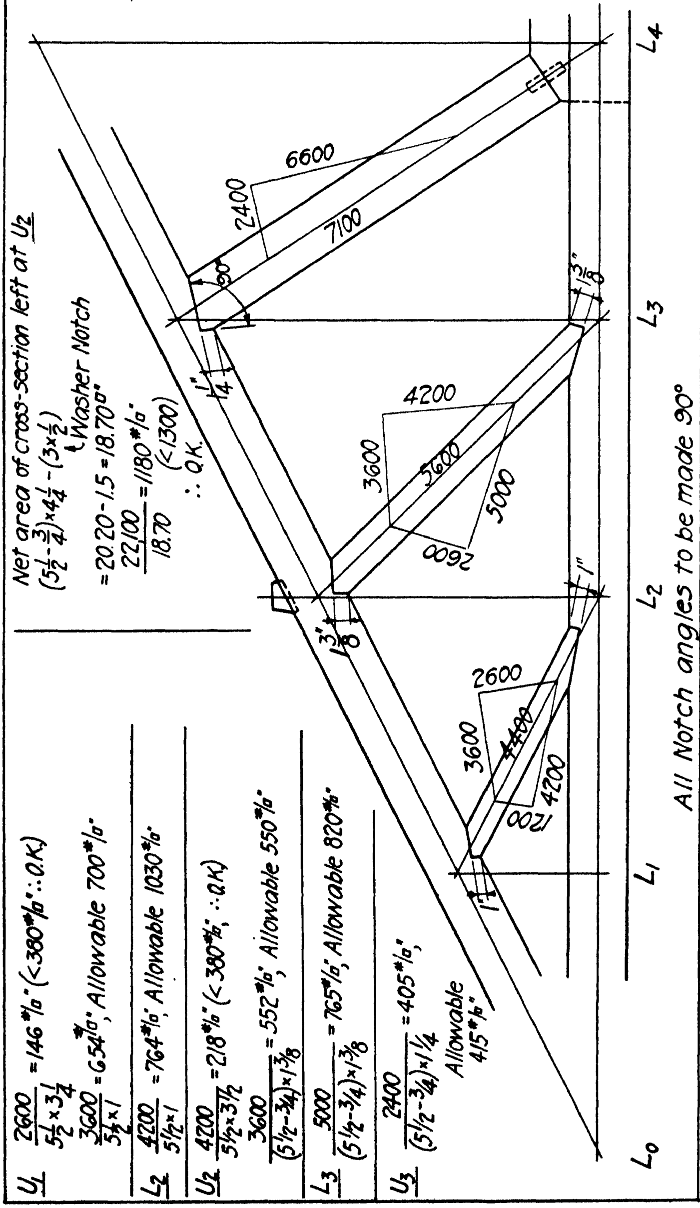


$$\frac{1760}{1300} = 1.36^{\square''} \quad 3 \times \tfrac{1}{2} = 1.50^{\square''}$$



Washers — U_3, U_2, U_1: Special, Cast Iron

Timber Roof Truss — Design Sheet-TRT8



U_1 $\dfrac{2600}{5\frac{1}{2}\times 3\frac{1}{4}} = 146\,\#/\square''\ (<380\,\#/\square''\ \therefore \text{O.K.})$

$\dfrac{3600}{5\frac{1}{2}\times 1} = 654\,\#/\square''$; Allowable $700\,\#/\square''$

L_2 $\dfrac{4200}{5\frac{1}{2}\times 1} = 764\,\#/\square''$; Allowable $1030\,\#/\square''$

U_2 $\dfrac{4200}{5\frac{1}{2}\times 3\frac{1}{2}} = 218\,\#/\square''\ (<380\,\#/\square'';\ \therefore \text{O.K.})$

$\dfrac{3600}{(5\frac{1}{2}-\frac{3}{4})\times 1\frac{3}{8}} = 552\,\#/\square''$; Allowable $550\,\#/\square''$

L_3 $\dfrac{5000}{(5\frac{1}{2}-\frac{3}{4})\times 1\frac{3}{8}} = 765\,\#/\square''$; Allowable $820\,\#/\square''$

U_3 $\dfrac{2400}{(5\frac{1}{2}-\frac{3}{4})\times 1\frac{1}{4}} = 405\,\#/\square''$; Allowable $415\,\#/\square''$

Net area of cross-section left at U_2

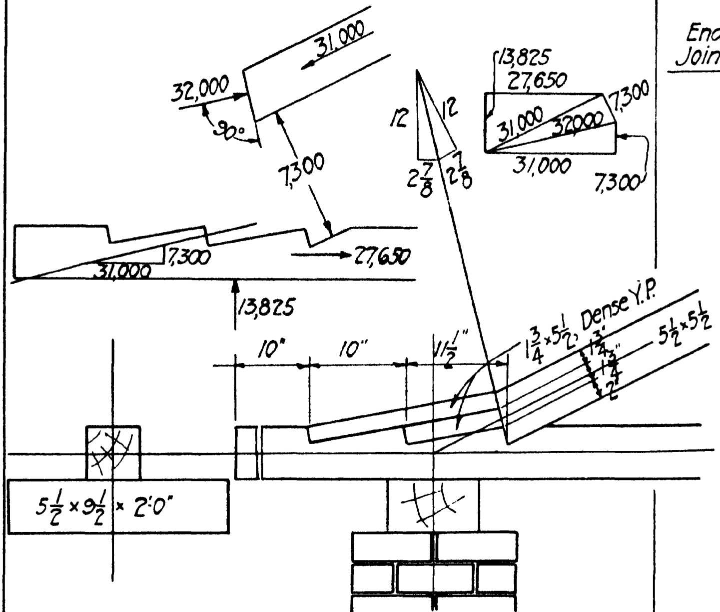
$(5\frac{1}{2}-\frac{3}{4})\times 4\frac{1}{4} - (3\times\frac{1}{2})$ ← Washer Notch

$= 20.20 - 1.5 = 18.70\,\square''$

$\dfrac{22{,}100}{18.70} = 1180\,\#/\square''$ (<1300) $\therefore$ O.K.

Timber Roof Truss — Design Sheet - TRT 9

End Joint



End Pressure, top chord,

$$\frac{32{,}000}{5\frac{1}{2}\times 5.66} = 1028\,\text{\#/□"} \qquad \text{Allowable} = 1150\,\text{\#/□"}$$

$$\frac{7{,}300}{5\frac{1}{2}\times 4\frac{1}{2}} = 295\,\text{\#/□"}\ (<380\,\text{\#/□"})$$

Shear, bottom chord,
$$\frac{31{,}000}{5\frac{1}{2}(120\times 1.5)} = 31.3'' \qquad (31.5''\text{ used})$$

Tension, bottom chord,
$$\frac{31{,}000}{5\frac{1}{2}\times 3\frac{1}{2}} = 1610\,\text{\#/□"} \qquad \text{Allowable } 1800\,\text{\#/□"}$$

Bearing under chord, Sheet 3
$$\frac{15{,}800}{380} = 42\,\text{□" Reqd.}$$
$$(5\tfrac{1}{2}\times 9\tfrac{1}{2} = 52.2\,\text{□"})$$

Bearing on wall,
$$\frac{15{,}800}{111} = 142\,\text{□" Reqd.}$$
$$(9\tfrac{1}{2}\times 24 = 228\,\text{□"})$$

Timber Roof Truss — Design Sheet - TRT 10

Sizes from drawing, for weight computation only.
Not a complete list.

One Truss — **Weights**

Lumber

$2-5\frac{1}{2}\times5\frac{1}{2}\times14'-11\frac{1}{4}''$ @ 48#/c.f.	302	
$2-5\frac{1}{2}\times5\frac{1}{2}\times13'-6\frac{1}{2}''$	273	
$2-5\frac{1}{2}\times5\frac{1}{2}\times23'-9''$	479	
$2-5\frac{1}{2}\times5\frac{1}{2}\times10'-8''$	215	
$2-3\frac{5}{8}\times5\frac{5}{8}\times8'-7''$	117	
$2-2\frac{5}{8}\times5\frac{5}{8}\times6'-7''$	65	
$2-2\frac{5}{8}\times5\frac{5}{8}\times11'-6''$	113	
2 - End Blocks	30 ±	
$1-5\frac{1}{2}\times9\frac{1}{2}\times1'-0''$	18	
$2-1\frac{5}{8}\times5\frac{5}{8}\times1'-0''$	6	
		1618

Metal

$1-1''\phi\times13'-4\frac{1}{2}''$ (with nuts)	40	
$2-\frac{3}{4}''\phi\times10'-3''$	32	
$2-\frac{3}{4}''\phi\times7'-1\frac{1}{2}''$	22	
$2-\frac{3}{4}''\phi\times4'-0''$	13	
$2-\frac{3}{4}''\phi\times1'-7\frac{1}{2}''$	6	
$2-\frac{3}{4}''\phi\times1'-4\frac{1}{2}''$	6	
$10-\frac{3}{4}''\phi\times1'-0\frac{1}{2}''$	18	
$4-\frac{3}{4}''\phi\times10\frac{1}{2}''$	6	
$8-\frac{3}{4}''\phi\times7\frac{1}{4}''$	11	
2 - Pl. $6\times\frac{3}{4}\times6''$	15	
6 - Pl. $3\frac{1}{2}\times\frac{7}{16}\times3\frac{1}{2}''$	9	
48 - Pl. $2\frac{1}{2}\times\frac{5}{16}\times2\frac{1}{2}''$	27	
10 - C.I. Washers	30	
		235
	Total	1853

$$\frac{1853}{12\times50\times\frac{9}{8}} = 2.8^{\#}/\square' \text{ of roof surface}$$

Assumed value, $4^{\#}/\square'$ O.K.

APPENDIX A

MOMENTS AND PRODUCTS OF INERTIA

A-1. Moment of Inertia. In the design of beams and columns it is necessary to obtain the value of the quantity $\int y^2\, dA$, that is, the summation of the products of the elementary areas into which the section may be divided, multiplied each by the square of the distance of the element from an axis in the plane of the figure. The quantity thus computed is called the *moment of inertia*, and is designated by the symbol I, with a subscript to indicate the axis from which the distances are measured. That is,

$$I_X = \int y^2\, dA \tag{1}$$

$$I_Y = \int x^2\, dA \tag{2}$$

Where the dimensions of an area are given in inches, moment of inertia will be expressed in inches to the fourth power. Since moment of inertia is the product of an area, which is positive, and a distance squared, which is also positive, I will always be positive.

A-2. Transfer Formula for Moment of Inertia. A property that is much used in computing the moment of inertia of compound shapes is the following: *The moment of inertia of an area about any axis is equal to the moment of inertia about a parallel axis passing through the centroid of the area, plus the area multiplied by the square of the distance between axes.*

Consider any area with axis X passing through the centroid and a parallel axis X_1 distant d from X. By definition,

$$\begin{aligned} I_{X1} &= \int (y + d)^2\, dA \\ &= \int y^2\, dA + \int 2\, yd\, dA + \int d^2\, dA \end{aligned}$$

The first term, $\int y^2\, dA$, is I_X; the second, $2\, d \int y\, dA$, equals zero since the axis from which y is measured passes through the centroid of the area; and the third term, $d^2 \int dA$, equals $d^2 A$. Therefore,

$$I_{X1} = I_X + Ad^2 \tag{3}$$

It is evident that, for any series of parallel axes, I is least about that axis which passes through the centroid of the area.

Problem A-1. Demonstrate that the moment of inertia (*a*) of a rectangle, width B and height H, about the centroidal axis parallel to B, is $BH^3/12$; (*b*) of a triangle, base B, altitude H, about the centroidal axis parallel to the base, is $BH^3/36$; (*c*) of a circle, radius R, about a diameter, is $\pi R^4/4$.

A–3. Product of Inertia. Later, in transferring moments of inertia to inclined axes, it will be found necessary to evaluate the term $\int xy\,dA$ to which the name *product of inertia* has been applied and for which the symbol K is used. This term is the summation, or integral, of the products of the elementary areas into which the section may be divided, multiplied each by the product of the distances of the element from two rectangular axes. Product of inertia will be positive for areas in the first and third quadrants, and negative for those in the second and fourth. In this respect, product of inertia is unlike moment of inertia, which is always positive.

In evaluating product of inertia, use may often be made of the fact that *product of inertia equals zero when one axis is an axis of symmetry.* For example, consider an area which is symmetrical about a horizontal axis. For each elementary area above the axis for which y is positive, there will be a similar area below the axis for which y is negative, and obviously $\int xy\,dA$ must be zero.

A–4. Transfer Formula for Product of Inertia. Product of inertia may be transferred readily to any pair of rectangular axes from a parallel pair of axes intersecting at the centroid of the area.

Consider any area with centroidal axes X and Y and a pair of parallel axes X_1 and Y_1, with intersection in the third quadrant of X and Y, X and X_1 being distance b apart, Y and Y_1 a apart. We have

$$K_{XY} = \int xy\,dA$$

$$\begin{aligned} K_{X_1Y_1} &= \int (x + a)\ (y + b)\ dA \\ &= \int xy\,dA + \int bx\,dA + \int ay\,dA + \int ab\,dA \\ &= K_{XY} + abA \end{aligned} \tag{4}$$

That is, *product of inertia about any pair of rectangular axes equals product of inertia about a parallel pair of axes intersecting at the centroid of the area plus the area multiplied by the product of the coordinates of the centroid measured from the first pair of axes.*

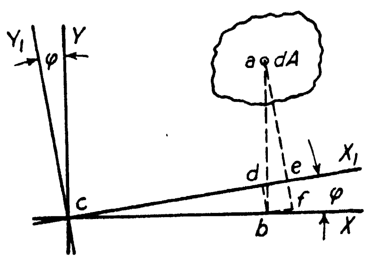


FIG. A–1

A–5. Inclined Axes. In Fig. A–1, X and Y are a pair of rectangular axes and X_1 and Y_1 are another pair intersecting at the same point as the first pair and inclined at an angle ϕ to them. Consider an element of area whose coordinates are x,y and x_1,y_1. From the figure it is seen that

$$x_1 = x \cos\phi + y \sin\phi \ (= cd + bf)$$

$$y_1 = y \cos\phi - x \sin\phi \ (= af - db)$$

Therefore,

$$I_{X1} = \int y_1^2 \, dA$$
$$= \int (y \cos\phi - x \sin\phi)^2 \, dA$$
$$= \cos^2\phi \int y^2 \, dA + \sin^2\phi \int x^2 \, dA - 2 \sin\phi \cos\phi \int xy \, dA$$
$$= \cos^2\phi \, I_X + \sin^2\phi \, I_Y - 2 \sin\phi \cos\phi \, K_{XY} \tag{5}$$

Similarly

$$I_{Y1} = \cos^2\phi \, I_Y + \sin^2\phi \, I_X + 2 \sin\phi \cos\phi \, K_{XY} \tag{6}$$

and

$$K_{X_1Y_1} = \int x_1 y_1 \, dA$$
$$= (\cos^2\phi - \sin^2\phi) K_{XY} + \sin\phi \cos\phi \, (I_X - I_Y) \tag{7}$$

It is evident from Eq. (5), (6), and (7) that, I_X, I_Y and K_{XY} being known, corresponding values for an inclined pair of axes, making a given angle with the first pair, may be found without difficulty. Moreover, *for any given point of intersection the sum of the I's about any pair of rectangular axes is constant.* To show this, add (5) and (6) above.

$$I_{X1} + I_{Y1} = I_X \, (\sin^2\phi + \cos^2\phi) + I_Y \, (\sin^2\phi + \cos^2\phi)$$
$$I_{X1} + I_{Y1} = I_X + I_Y \tag{8}$$

A-6. Maximum and Minimum Values for *I*. Examination of Eq. (5) will show that I_{X1} varies as ϕ varies and, obviously, must have a maximum value for some value of ϕ. Also, since the sum of the I's is constant for every pair of axes, the value of ϕ which makes one I a maximum will make the I about the corresponding rectangular axis a minimum. To obtain this value, differentiate with respect to ϕ and set the result equal to zero:

$$\frac{dI_{X1}}{d\phi} = -2 \sin\phi \cos\phi I_X + 2 \sin\phi \cos\phi I_Y - 2 \, K_{XY}(\cos^2\phi - \sin^2\phi) = 0$$

Therefore

$$2 \sin\phi \cos\phi (I_Y - I_X) - 2 \, K_{XY} \, (\cos^2\phi - \sin^2\phi) = 0 \tag{A}$$

which leads to

$$\tan 2\phi = \frac{2 \, K_{XY}}{I_Y - I_X} \tag{9}$$

There are two angles differing by 180° whose tangents have the same value. Therefore, the above formula will give two values of ϕ differing by 90°. As stated before, these will be the positions of the axes such that for one, I is a maximum, and for the other, a minimum. The pair of axes thus located are known as *principal axes*, and the moments of inertia about these axes are called *principal moments of inertia.*

The value of ϕ having been found, the value of K for the principal axes might be found by substitution in Eq. (7). However, a result may be obtained more easily by comparing Eq. (7) and the line marked (A) above. Line (A) says that for principal axes the left side of the equation equals zero. But this is also the right side of Eq. (7). Therefore, it is evident that *for principal axes K is equal to zero.* It has already been shown that K is zero where one axis is an axis of symmetry. It follows directly that *an axis of symmetry is a principal axis.*

For the case where the I's are known for the principal axes and it is desired to transfer to inclined axes, Eqs. (5), (6), and (7) take the following simple forms:

$$I_{X1} = \cos^2 \phi I_X + \sin^2 \phi I_Y \tag{5A}$$

$$I_{Y1} = \cos^2 \phi I_Y + \sin^2 \phi I_X \tag{6A}$$

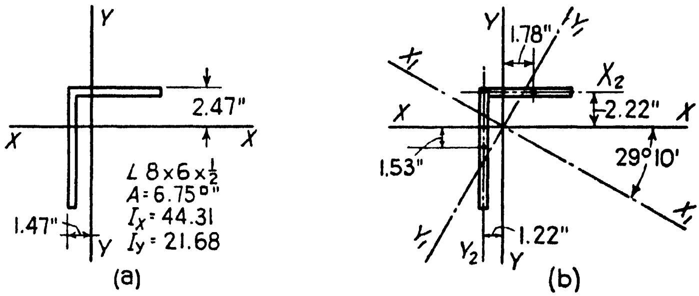
$$K_{X1Y1} = \sin \phi \cos \phi (I_X - I_Y) \tag{7A}$$

A-7. Radius of Gyration. Most column formulas contain a term, r, called *radius of gyration* and defined mathematically as

$$r^2 = \frac{I}{A} \qquad r = \sqrt{\frac{I}{A}} \tag{10}$$

That is, from a given axis, r is the distance at which the entire area of a section might be concentrated without altering the value of the moment of inertia about the axis.

Example A-1. Check the information given in the steel handbooks which states that r minimum for an angle $8 \times 6 \times \frac{1}{2}$ is 1.30. The other handbook values for this angle are given in the figure.



Ex. A-1

Solution. In order to find the angle which the principal axes make with the X and Y axes it is necessary to have, in addition to the values given in the handbook, the value of K_{XY}. The angle was divided into two rectangles as shown. The coordinates of the centroid of each rectangle are also shown. K of each rectangle is zero about the axes through its center of gravity since they are axes of symmetry. Hence

$$K = 0 + (8 \times \tfrac{1}{2})(-1.53)(-1.22) + 0 + (5\tfrac{1}{2} \times \tfrac{1}{2})(+1.78)(+2.22)$$
$$= +18.34$$

Or, perhaps better, since axis X_2 or Y_2 is an axis of symmetry for each of the rectangles into which the angle is divided

$$K_{X_2Y_2} = 0 = K_{XY} + 6.75\,(+1.22)\,(-2.22)$$
$$K_{XY} = +18.34$$

By Eq. (9),
$$\tan 2\phi = \frac{2K}{I_Y - I_X} = \frac{2 \times 18.34}{21.68 - 44.31} = -1.620$$

From the slide rule,
$$2\phi = -(90^\circ - 31^\circ\,40') = -58^\circ\,20'$$
$$\phi = -29^\circ\,10'$$
$$\sin\phi = -0.487$$
$$\cos\phi = +0.873$$
$$\tan\phi = -0.558$$[1]

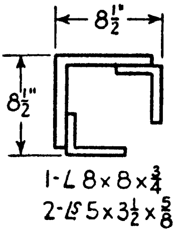
Next, the value of I_{X1} was computed by means of Eq. (5)

$$I_{X1} = \cos^2\phi\, I_X + \sin^2\phi\, I_Y - 2\sin\phi\cos\phi\, K_{XY}$$
$$= (+0.873)^2\, 44.31 + (-0.487)^2\, 21.68 - 2(-0.487)\,(+0.873)\,(+18.34)$$
$$= +54.49$$

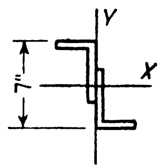
By Eq. (8)
$$I_{Y1} = 44.31 + 21.68 - 54.49 = 11.50$$
$$r_{Y1} = \sqrt{\frac{11.50}{6.75}} = 1.305. \quad \text{Check.}$$

Problem A-2. Here is shown a built-up section of a type used for the legs in high radio towers. Compute the minimum r for this section.

Ans. $r_{\text{min.}} = 2.49$



PROB. A-2



PROB. A-3

Problem A-3. The section shown is made of 2 angles $5 \times 3\frac{1}{2} \times \frac{1}{2}$. Find the least value of r.

Ans. $I_X = 47.06$
$I_Y = 14.72$
$K_{XY} = -20.75$
$r_{\text{min.}} = 0.75$

[1] The "Cambria Steel Handbook" and also KETCHUM'S "Structural Engineers' Handbook" give the value of the tangent of the angle which the principal axes make with the X and Y axes for all angles with unequal legs. Each book lists this tangent for an angle $8 \times 6 \times \frac{1}{2}$ as 0.558.

A-8. Ellipse of Inertia. It may be shown that, if an ellipse is constructed with its axes coinciding with the principal centroidal axes of a section, with its long and short semi-diameters equal respectively to the maximum and minimum radii of gyration, the radius of gyration for any other centroidal axis equals the normal distance from the centroid to a tangent to the ellipse drawn parallel to that axis. Such an ellipse is called an *ellipse of inertia.*

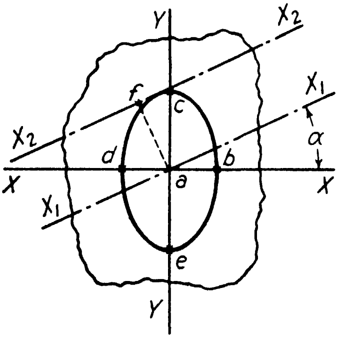


FIG. A-2

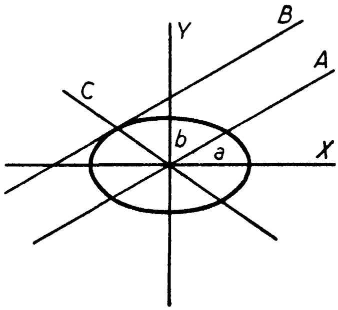


FIG. A-3

Assume that, for the section of Fig. A-2, axes X and Y are principal centroidal axes and that the values of I_X and I_Y are known. Compute the radii of gyration, r_X and r_Y, and lay off

$$ab = ad = r_Y$$

and

$$ac = ae = r_X$$

Pass an ellipse whose axes are X and Y through points b, c, d, e. Next draw an axis X_1X_1 making any angle, α, with XX and also draw a tangent X_2X_2 parallel to X_1X_1. The distance from X_1 to X_2 is af. By analytic geometry it may be shown that a property of the ellipse is:

$$\overline{af}^2 = \overline{ac}^2 \cos^2 \alpha + \overline{ab}^2 \sin^2 \alpha$$

$$= r_X^2 \cos^2 \alpha + r_Y^2 \sin^2 \alpha \tag{B}$$

If both sides of Eq. (5A) are divided by A, the area of the section, there is obtained

$$r_{X1}^2 = \cos^2 \phi \, r_X^2 + \sin^2 \phi \, r_Y^{2'}$$

By comparing this with (B) it is made evident that

$$af = r_{X1}$$

It is clear that from an ellipse of this character, called an ellipse of inertia, the value of r and, therefore, of I about any axis may be measured.

Bending. The student will recall from his study of analytic geometry that if in an ellipse, whose semi-diameters are a and b, B is a tangent, A a diameter parallel to B, and C a diameter through B's point of tangency, A and C are called *conjugate diameters* (Fig. A-3). Furthermore, the rela-

tion between these conjugate diameters is given by the equation

$$m_A m_C = -\frac{b^2}{a^2} \tag{C}$$

where m_A is the slope of line A, and m_C that of C.

If both numerator and denominator of the right side of Eq. (1–3), p. 10, be divided by A, the area of the section, there is obtained (see Fig. 1–3, p. 8)

$$\tan \alpha = -\frac{r_X^2}{r_Y^2} \cot \theta$$

Therefore

$$\tan \alpha \tan \theta = -\frac{r_X^2}{r_Y^2}$$

However, in the notation of analytic geometry,

$$\tan \theta = m_A, \quad \tan \alpha = m_C$$

It is seen by comparison with Eq. (C) above that the plane of loading and the neutral axis are conjugate diameters of an ellipse whose semi-diameters are the radii of gyration of the section about the principal axes. Therefore, *if an ellipse of inertia be constructed for any section subjected to simple bending the neutral axis will be parallel to the tangent at the point where the plane of loading cuts the ellipse.*

The properties just given are of interest mathematically and are of some practical value in visualizing results. The graphical work necessary to obtain dependable numerical results is, however, extremely tedious. In general, accuracy and increased speed will be secured by other methods. The student should convince himself of this by attempting for Ex. 1–4, p. 16, a solution in which he locates the position of the neutral axis and the value of I_{NA} from an ellipse of inertia.

A useful construction for evaluating K and I for inclined axes, when K and I are given for some pair of rectangular axes of common origin, is the circle of inertia. A description of this is given by Professor L. J. Johnson, in a paper published in the *Journal* of the Associated Engineering Societies, May, 1902: see also *Bulletin* 211, University of Illinois Engineering Experiment Station, p. 55.

APPENDIX B

AMERICAN INSTITUTE OF STEEL CONSTRUCTION SPECIFICATION FOR THE DESIGN, FABRICATION AND ERECTION OF STRUCTURAL STEEL FOR BUILDINGS

As adopted June 1, 1923, and revised November, 1928, with editorial revision, January, 1934. This specification was replaced by a revision on June 24, 1936 (editorially revised, 1937), which differs radically in several respects from the provisions here printed. Some of the major changes are reported here in footnotes. The specification may be obtained from the A.I.S.C., 200 Madison Avenue, New York, N. Y.

Section 1. This Specification defines the practice adopted by the American Institute of Steel Construction for the design, fabrication, and erection of structural steel for buildings.

Section 2. General. To obtain a satisfactory structure, the following major requirements must be fulfilled.

(**a**) The material used must be suitable, of uniform quality, and without defects affecting the strength or service of the structure.

(**b**) Proper loads and conditions must be assumed in the design.

(**c**) The unit stresses must be suitable for the material used.

(**d**) The workmanship must be good, so that defects or injuries are not produced in the manufacture.

(**e**) The computations and design must be properly made so that the unit stresses specified shall not be exceeded, and the structure and its details shall possess the requisite strength and rigidity.

Section 3. Material. Structural steel shall conform to the Standard Specifications of the American Society for Testing Materials for Steel for Buildings, Serial Designation A 9 (or, if so specified by the Buyer, for Steel for Bridges, Serial Designation A 7), as amended to date. (Ultimate tensile strength 60,000-72,000 pounds per square inch.)

Section 4. Loading. (**a**) Steel structures shall be designed to sustain the dead weight imposed upon them, including the weight of the steel frame itself, and, in addition, the maximum live load as specified in each particular case. Proper provision shall be made for temporary stresses caused by erection.

(**b**) In cases where live loads have the effect of producing impact or vibration, a proper percentage shall be added to the static live load stresses to provide for such influences, so that the total stress found in any member is an equivalent static stress.

(**c**) Proper provision shall be made for stresses caused by wind both during erection and after completion of the building. The wind pressure

is dependent upon the conditions of exposure, but the allowable stresses specified in Section 5 (f) and (g) are based upon the steel frame being designed to carry a wind pressure of not less than twenty (20) pounds per square foot on the vertical projection of exposed surfaces during erection, and fifteen (15) pounds per square foot on the vertical projection of the finished structure.

(d) Proper provision shall be made to securely fasten the reaction points of all steel construction and transmit the stresses to the foundations of the structure.

Section 5. Allowable stresses.[1] All parts of the structure shall be so proportioned that the sum of the maximum static stresses in pounds per square inch shall not exceed the following:

(a) Tension.

		1936
Rolled Steel, on net section....................	18,000	(20,000)
On the area of the nominal diameter of rivets under the limitations defined in Section 13................	13,500	(15,000)

(b) Compression.

Rolled Steel, on short lengths or where lateral deflection is prevented............................	18,000	(for $\frac{L}{r} > 120$ only)

On gross section of columns,

$$\frac{18{,}000}{1 + \frac{l^2}{18{,}000\,r^2}}$$

with a maximum of..............................	15,000

in which l is the unbraced length of the column, and r is the corresponding least radius of gyration of the section, both in inches.

For main compression members, the ratio l/r shall not exceed 120, and for bracing and other secondary members, 200.

[1] Certain of the 1936 values have been shown above in parallel column with the 1934 stresses. The 1934 Rankine column formula is retained in the 1936 specification only for values of L/r greater than 120, being replaced for smaller slenderness ratios by the parabolic formula

$$P/A = 17{,}000 - 0.485(L/r)^2$$

The 18,000 compressive limit for compression on short lengths is replaced by 20,000 for plate girder stiffeners, gross section, and by 24,000 for the webs of rolled sections at toe of fillet. In this last connection a new provision for web crippling is of interest (Section 19h). The constants in the Rankine type formula for compression on extreme fibers of beams and girders are made 22,500 and 1800 instead of 20,000 and 2000, an upper limit of 20,000 being set. Hand-driven rivets being obsolete, no mention is made of them in the 1936 specification. The 13,000 unit stress specified in 1936 for shear on webs of beams and plate girders replaces the dual limit of the 1934 edition.

(c) **Bending.**

		1936
On extreme fibers of rolled sections, and built-up sections, net section, if lateral deflection is prevented	18,000	(20,000)

When the unsupported length l exceeds 15 times b, the width of the compression flange, the stress in pounds per square inch in the latter shall not exceed

$$\frac{20{,}000}{1+\dfrac{l^2}{2000\,b^2}}$$

The laterally unsupported length of beams and girders shall not exceed 40 times b, the width of the compression flange.

On extreme fibers of pins, when the forces are assumed as acting at the center of gravity of the pieces	27,000	(30,000)

(d) **Shearing.**

On pins	13,500	(15,000)
On power-driven rivets	13,500	(15,000)
On turned bolts in reamed holes with a clearance of not more than 1/50 of an inch	13,500	(15,000)
On hand-driven rivets	10,000	
On unfinished bolts	10,000	(10,000)
On the gross area of the webs of beams and girders, where h, the clear distance between flanges in inches, is not more than 60 times t, the thickness of the web in inches	12,000	(13,000)

On the gross area of the webs of beams and girders if the web is not stiffened where h is more than 60 times t, the greatest average shear per square inch, $\frac{V}{A}$, shall not exceed

$$\frac{18{,}000}{1+\dfrac{h^2}{7200\,t^2}}$$

in which V is the total shear, and A is gross area of web in square inches.

(e) **Bearing.**

	Double Shear	Single Shear	1936 Double Shear	1936 Single Shear
On pins	30,000	24,000	32,000	32,000
On power-driven rivets	30,000	24,000	40,000	32,000
On turned bolts in reamed holes	30,000	24,000	40,000	32,000
On hand-driven rivets	20,000	16,000		
On unfinished bolts	20,000	16,000	25,000	20,000
On expansion rollers per linear inch, 600 times the diameter of the roller in inches.				

(f) **Combined Stresses.** For combined stresses due to wind and other loads, the permissible working stress may be increased $33\frac{1}{3}$ per cent, provided the section thus found is not less than that required by the dead and live loads alone.

(g) **Members Carrying Wind Only.** For members carrying wind stresses only, the permissible working stresses may be increased $33\frac{1}{3}$ per cent.

Section 6. Symmetrical Members. Sections shall preferably be symmetrical.

Section 7. Beams and Girders. (a)[1] **Rolled Beams** shall be proportioned by the moment of inertia of their net section. Plate girders with webs fully spliced for tension and compression shall be so proportioned that the unit stress on the net section does not exceed the stresses specified in Section 5 as determined by the moment of inertia of the net section.

(b)[2] **Plate girder webs** shall have a thickness of not less than 1/160 of the unsupported distance between the flanges.

(c) **Web splices** shall consist of plates on each side of the web capable of transmitting the full stress through the splice connections.

(d)[3] **Stiffeners.** Stiffeners shall be required on the webs of rolled beams and plate girders at the ends, and at points of concentrated loads, and at other points where h, the clear distance between flanges, is greater than $85\,t\,\sqrt{18{,}000\,(A/V) - 1}$, in which t is the thickness of the web. When stiffeners are required, the distance in inches between them shall not be greater than $85\,t\,\sqrt{18{,}000\,(A/V) - 1}$, or not greater than 6 feet. When h is greater than 60 times t, the thickness of the web of a plate girder, stiffeners shall be required at distances not greater than 6 feet apart. Stiffeners under or over concentrated loads shall be proportioned to distribute such loads into the web.

Plate girder stiffeners shall generally be in pairs, one on each side of the web, and shall have a close bearing against the flange angles at points of concentrated loading; stiffeners over the end bearings shall be on plate fillers. The pitch of rivets in stiffeners shall not exceed 6 inches.

(e) **Flange plates** of all girders shall be limited in width so as not to extend more than 6 inches or more than 12 times the thickness of thinnest plate beyond the outer row of rivets connecting them to the angles.

(f) **Crane runway girders** and the supporting framework shall be proportioned to resist the greatest horizontal stresses caused by the operation of the cranes.

[1] The 1936 rules make no deductions for ordinary rivet holes and in general proportion beams and girders by their gross moment of inertia. Allowance is to be made for rivet hole area in excess of 15 per cent of the gross flange area, and for holes for bolts, pins, and countersunk rivets.

[2] 1/170 (1936 rules).

[3] The 1936 specification replaces the criterion for the use of stiffeners by $8000\,t/\sqrt{s}$, where s is the maximum intensity of shearing stress in the web in the panel under investigation. The clear distance between intermediate stiffeners is restricted to 84 in. or less as given by the formula

$$d = (270{,}000\,\sqrt[3]{st/h})/s$$

(g) **Rivets** connecting the flanges to the web at points of direct load on the flange between stiffeners shall be proportioned to carry the resultant of the longitudinal and transverse shears.

(h) **Rivets** connecting the flanges to the webs of plate girders and of columns subjected to bending shall be so spaced as to carry the increment of the flange stress between the rivets.

Section 8. Column Bases. (a) Proper provision shall be made to distribute the column loads on the footings and foundations.

(b) The top surface of all column bases, except rolled steel bearing plates 4 inches or under in thickness, shall be planed for the column bearing.

(c) Column bases shall be set true and level, with full bearing on the masonry, and be properly secured to the footings.

Section 9. Eccentric Loading. Full provision shall be made for stresses caused by eccentric loads.

Section 10. Combined Stresses. (a) Members subject to both direct and bending stresses shall be so proportioned that the greatest combined stresses shall not exceed the allowed limits.

(b) All members and their connections which are subject to stresses of both tension and compression due to the action of live loads shall be designed to sustain stress giving the largest section, with 50 per cent of the smaller stress added to it. If the reversal of stress is due to the action of wind, the member shall be designed for the stress giving the largest section and the connections proportioned for the largest stress.

Section 11. Abutting Joints. Compression members when faced for bearings shall be spliced sufficiently to hold the connecting members accurately in place. Other joints in riveted work, whether in tension or compression, shall be fully spliced.

Section 12. Net Sections. (a)[1] In calculating tension members, the net section shall be used, and in deducting the rivet holes they shall be taken $\frac{1}{8}$ inch greater in diameter than the nominal diameter of the rivets.

(b) In pin connected tension members, the net section through the pin hole, transverse to the axis of the member, shall be at least 25 per cent greater than the net section of the member. The net section beyond the pin hole, parallel with the axis of the member, shall be not less than 75 per cent of the net section required through the pin hole.

Section 13. Rivets and Bolts. (a) In proportioning rivets, the nominal diameter of the rivet shall be used.

(b) Rivets carrying calculated stresses, and whose grip exceeds five diameters, shall have their number increased 1 per cent for each additional 1/10 inch in the rivet grip. Special care shall be used in heating and driving such rivets.

(c) Rivets shall be used for the connections of main members carrying live loads which produce impact, and for connections subject to reversal of stresses.

[1] The 1936 specification adopts the same rule for the calculation of net section along a diagonal or zigzag as that enjoined by the 1935 A.R.E.A. specification for railway bridges, reprinted herein on p. 348.

(d) Finished bolts in reamed holes may be used in shop or field work where it is impracticable to obtain satisfactory power-driven rivets. The finished shank shall be long enough to provide full bearing, and washers used under the nuts to give full grip when turned tight.

Unfinished bolts may be used in shop or field work for connections in small structures used for shelters, and for secondary members of all structures such as purlins, girts, door and window framing, alignment bracing and secondary beams in floor.

(e) The end reaction stresses of trusses, girders, or beams, and the axial stresses of tension or compression members which are carried on rivets, shall have such stresses developed by the shearing and bearing values of the rivets; but where rivets are used for shelf or bracket supports or for connections that also provide rigidity to the structure, the rivets may in addition to their shearing and bearing stresses, carry tension as defined in Section 5 (a).

Section 14. Rivet Spacing. (a) The minimum distance between centers of rivet holes shall be three diameters of the rivet; but the distance shall preferably be not less than $4\frac{1}{2}$ inches for $1\frac{1}{4}$ inch rivets, 4 inches for $1\frac{1}{8}$ inch rivets, $3\frac{1}{2}$ inches for 1 inch rivets, 3 inches for $\frac{7}{8}$ inch rivets, $2\frac{1}{2}$ inches for $\frac{3}{4}$ inch rivets, 2 inches for $\frac{5}{8}$ inch rivets, and $1\frac{3}{4}$ inches for $\frac{1}{2}$ inch rivets. The maximum pitch in the line of stress of compression members composed of plates and shapes shall not exceed 16 times the thinnest outside plate or shape, nor 20 times the thinnest enclosed plate or shape with a maximum of 12 inches, and at right angles to the direction of stress the distance between lines of rivets shall not exceed 30 times the thinnest plate or shape. For angles in built sections with two gage lines, with rivets staggered, the maximum pitch in the line of stress in each gage line shall not exceed 24 times the thinnest plate with a maximum of 18 inches.

(b) In tension members composed of two angles, a pitch of 3′-6″ will be allowed, and in compression members, 2′-0″, but the ratio l/r for each angle between rivets shall not be more than $\frac{3}{4}$ of that for the whole member.

(c) The pitch of rivets at the ends of built compression members shall not exceed four diameters of the rivets for a length equal to $1\frac{1}{2}$ times the maximum width of the member.

(d) The minimum distance from the center of any rivet hole to a sheared edge shall be $2\frac{1}{4}$ inches for $1\frac{1}{4}$ inch rivets, 2 inches for $1\frac{1}{8}$ inch rivets, $1\frac{3}{4}$ inches for 1 inch rivets, $1\frac{1}{2}$ inches for $\frac{7}{8}$ inch rivets, $1\frac{1}{4}$ inches for $\frac{3}{4}$ inch rivets, $1\frac{1}{8}$ inches for $\frac{5}{8}$ inch rivets and 1 inch for $\frac{1}{2}$ inch rivets. The maximum distance from any edge shall be 12 times the thickness of the plate, but shall not exceed 6 inches.

Section 15. Connections. (a) Connections carrying calculated stresses, except for lacing, sag bars, or hand rails, shall have not fewer than 2 rivets.

(b) Members meeting at a joint shall have their lines of center of gravity meet at a point if practicable; if not, provision shall be made for any eccentricity.

(c) The rivets at the ends of any member transmitting the stresses into that member should have their centers of gravity in the line of the center of gravity of the member; if not, provision shall be made for the effect of the resulting eccentricity. Pins may be so placed as to counteract the effect of bending due to dead load.

(d) When a beam or girder "A" is connected to another member in such a manner that "A" acts as a continuous or fixed end beam, proper provision shall be made for the bending moments at such a connection.

(e) Where stress is transmitted from one piece to another, through a loose filler, the number of rivets shall be such as to properly develop the stresses; tight-fitting fillers shall be preferred.

Section 16. Lattice. (a) The open sides of compression members shall be provided with lattice having tie plates at each end and at intermediate points if the lattice is interrupted. Tie plates shall be as near the ends as practicable. In main members carrying calculated stresses the end tie plates shall have a length of not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones of not less than one-half of this distance. The thickness of tie plates shall be not less than one-fiftieth of the distance between the lines of rivets connecting them to the segments of the members, and the rivet pitch shall be not more than four diameters. Tie plates shall be sufficient in size and number to equalize the stress in the parts of the members.

(b) Lattice bars shall have neatly finished ends. The thickness of lattice bars shall be not less than one-fortieth for single lattice and one-sixtieth for double lattice of the distance between end rivets; their minimum width shall be as follows:

For 15″ channels, or built sections with $3\frac{1}{2}''$ and 4″ angles — $2\frac{1}{4}''$ ($\frac{3}{4}''$ rivets), or $2\frac{1}{2}''$ ($\frac{7}{8}''$ rivets).

For 12″, 10″, and 9″ channels, or built sections with 3″ angles — $2\frac{1}{4}''$ ($\frac{3}{4}''$ rivets).

For 8″ and 7″ channels, or built sections with $2\frac{1}{2}''$ angles — 2″ ($\frac{5}{8}''$ rivets), or $2\frac{1}{4}''$ ($\frac{3}{4}''$ rivets).

For 6″ and 5″ channels, or built sections with 2″ angles — $1\frac{1}{2}''$ ($\frac{1}{2}''$ rivets), or $1\frac{3}{4}''$ ($\frac{5}{8}''$ rivets).

(c) The inclination of lattice bars to the axis of the members shall preferably be not less than 45 degrees. When the distance between the rivet lines in the flanges is more than 15 inches, the lattice shall be double and riveted at the intersection if bars are used, or else shall be made of angles.

(d) Lattice bars shall be so spaced that the ratio l/r of the flange included between their connections shall be not over $\frac{3}{4}$ of that of the member as a whole.

Section 17. Expansion. Proper provision shall be made for expansion and contraction.

Section 18. Minimum Thickness. No steel less than $\frac{5}{16}$ inch thick shall be used for exterior construction, nor less than $\frac{1}{4}$ inch for interior construction, except for linings or fillers and rolled structural sections.

These provisions do not apply to light structures such as skylights, marquees, fire-escapes, light one-story buildings, or light miscellaneous steel work.

For trusses having end reactions of 35,000 pounds or over, the gusset plates shall be not less than $\frac{3}{8}$ inch thick.

Section 19. Adjustable Members. The total initial stress in adjustable members shall be assumed as not less than 5000 pounds.

Section 20. Workmanship. (**a**) All workmanship shall be equal to the best practice in modern structural shops.

(**b**) Drifting to enlarge unfair holes shall not be permitted.

(**c**) The several pieces forming built sections shall be straight and fit close together; and finished members shall be free from twists, bends, or open joints.

(**d**) Rolled sections, except for minor details, shall not be heated.

(**e**) Wherever steel castings are used, they shall be properly annealed.

(**f**) **Punching.** Material may be punched $\frac{1}{16}$ inch larger than the nominal diameter of the rivets, whenever the thickness of the metal is equal to or less than the diameter of the rivets, plus $\frac{1}{8}$ inch. When the metal is thicker than the diameter of the rivet, plus $\frac{1}{8}$ inch, the holes shall be drilled, or sub-punched and reamed.

(**g**) Rivets are to be driven hot and, wherever practicable, by power. Rivet heads shall be of hemispherical shape and uniform size throughout the work for the same size rivet, full, neatly finished, and concentric with the holes. Rivets, after driving, shall be tight, completely filling the holes, and with heads in full contact with the surface. Rivets shall be heated uniformly and their temperature before driving should not exceed 1950° F., which is a light yellow color. An air hammer should not be used for driving after the temperature is below 1000° F., which is a blood red color.

(**h**) Compression joints depending upon contact bearing shall have the bearing surfaces truly faced and plane after the members are riveted. All other joints shall be cut or dressed true and straight, especially where exposed to view.

(**i**) The use of a cutting torch is permissible if the metal being cut is not carrying stresses during the operation. The radius of re-entrant flame cut fillets shall be as large as possible, but never less than 1 inch. To determine the net area of members so cut, $\frac{1}{8}$ inch shall be deducted from the flame cut edges. Stresses shall not be transmitted through a flame cut surface.

Section 21. Painting. (**a**) Parts not in contact, but inaccessible after assembling, shall be properly protected by paint. Surfaces to be riveted in contact shall not be painted.

(**b**) All steel work, except where encased in concrete, shall be thoroughly cleaned and given one coat of acceptable metal protection well worked into the joints and open spaces.

(**c**) Machine finished surfaces shall be protected against corrosion.

(**d**) Field painting is a phase of maintenance, but it is important that unless otherwise properly protected, all steel work shall after erection be protected by a field coat of good paint applied by a competent painter.

Section 22. Erection. (**a**) The frame of all steel skeleton buildings shall be carried up true and plumb, and temporary bracing shall be introduced wherever necessary to take care of all loads to which the structure may be subjected, including erection equipment, and the operation of same. Such bracing shall be left in place as long as may be required for safety.

(**b**) As erection progresses the work shall be securely bolted up to take care of all dead load, wind and erection stresses.

(**c**) Wherever piles of material, erection equipment, or other loads are

carried during erection, proper provision shall be made to take care of stresses resulting from the same.

(d) No riveting shall be done until the structure has been properly aligned.

(e) Rivets driven in the field shall be heated and driven with the same care as those driven in the shop.

Section 23. Inspection. (a) Material and workmanship at all times shall be subject to the inspection of experienced engineers representing the purchaser.

(b) Material or workmanship not conforming to the provisions of this Specification shall be rejected at any time defects are found during the progress of the work.

(c) The Contractor furnishing such material or doing such work shall promptly replace the same.

(d) All inspection as far as possible shall be maae at the place of manufacture, and the Contractor or Manufacturer shall co-operate with the Inspector, permitting access for inspection to all places where work is being done.

APPENDIX C

A PORTION OF THE STANDARD SPECIFICATIONS FOR HIGHWAY BRIDGES OF THE AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS

Second Edition, 1935

with parallel portions of the first edition (1931) where important differences occur

This specification is divided into five " Divisions " as follows: I. General Provisions. II. Materials. III. General Construction. IV. Special Construction. V. Design. Certain articles referred to on the design sheets of Chapter VII are here given only by title as the details of their provisions are not of immediate importance to the student. Copies of the Specification may be obtained from the Association, National Press Building, Washington, D. C.

IV. SPECIAL CONSTRUCTION

BITUMINOUS CARPETS

4. 7. 22. Materials.

4. 7. 23. Preparation of Subfloor.

4. 7. 24. Construction of Tar Mat Surface. (Here the details of the application of the tar and aggregate are given. The total thickness of the tar mat coat is definitely limited to $\frac{3}{4}$ in.)

4. 7. 25. Construction of Asphalt Mat Surface (Mixed Method).

4. 7. 26. Application of Seal Coat to Tar and Asphalt Mats.

4. 7. 27. Construction of Asphalt Mat Surface (Penetration Method).

4. 7. 28. Opening to Traffic.

V. DESIGN

5. 1. 1. Determination of Waterway Area. (Requires that the bridge shall provide sufficient waterway area to discharge flood flows safely.)

5. 1. 2. Restricted Waterways. (Names the protective construction required if the waterway is so restricted that erosive velocities will result.)

5. 1. 3. Channel Openings. (Relates to navigation requirements.)

5. 1. 4. Pier Spacing and Location.

5. 1. 5. Size of Culvert Openings.

5. 1. 6. Materials.

5. 1. 7. Width of Roadway and Sidewalk. The width of roadway shall be the clear width measured at right angles to the longitudinal center line of the bridge.

5. 1. 8. Length of Culverts.

5. 1. 9. Clearances. The horizontal clearance shall be the clear width, and the vertical clearance the clear height, available for the passage of vehicular traffic, as shown on the clearance diagrams.

Unless otherwise provided the several parts of the structure shall be constructed to secure the following limiting dimensions or clearances for traffic:

The clearances and width of roadway for 2-lane traffic shall be not less than those shown in Figure 1. The roadway width shall be increased at least 9 feet for each additional lane of traffic.

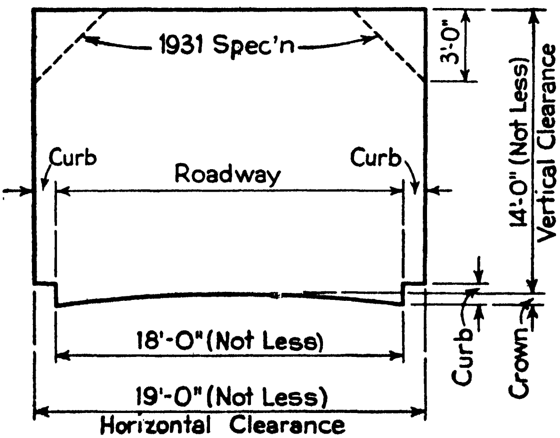


FIG. 1

In cases involving curved tracks, the horizontal clearances shall be increased an amount corresponding to that required to maintain the specified clearances. If the outer rail is superelevated, the clearances shall be correspondingly increased.

5. 1. 12. Drainage. The transverse drainage of roadways shall be secured by means of a suitable crown in the roadway surface. If necessary, longitudinal drainage shall be secured by means of scuppers, which shall be of sufficient size and number to drain the gutters adequately. If drainage gutters and downspouts are required, the downspouts shall be of cast or wrought-iron pipe not less than 4 inches in diameter, provided with suitable clean-out fixtures. The details of floor drains shall be such as to prevent the discharge of drainage water against any portion of the structure. Overhanging portions of concrete and timber floors, preferably, shall be provided with drip beads.

5. 1. 13. Paved Floors. Pavements other than wood block shall be supported by reinforced concrete slabs carried on steel or reinforced concrete floor members. Wood block pavements may be supported by a creosoted plank base.

5. 1. 16. Classification of Bridges.[1] The classification of bridges with reference to traffic shall be as follows:

[1] This article was numbered 5. 1. 17 in 1931 Edition.

Class AA. Bridges for specially heavy traffic units in locations where the passage of such loads is frequent.

Class A. Bridges for normally heavy traffic units and the occasional passage of specially heavy loads.

Class B. Bridges for light traffic units and the occasional passage of normally heavy loads. Class B bridges shall be considered as temporary or semi-temporary structures.

Class C. Bridges for electric railway traffic in addition to highway traffic. The latter may correspond to any one of the classes described above.

LOADS

5. 2. 1. Loads. Structures shall be proportioned for the following loads and forces:

(a) Dead load.
(b) Live load.
(c) Impact or dynamic effect of the live load.
(d) Lateral forces.
(e) Other forces, when they exist, as follows:
Longitudinal force, centrifugal force, and thermal forces.

Members shall be proportioned for the combination of loads and forces producing the maximum total stress, except as otherwise provided herein.

Upon the stress sheets a diagram of the assumed live loads shall be shown and the stresses due to the various loads shall be shown separately.

5. 2. 2. Dead Load. The dead load shall consist of the weight of the structure complete, including the roadway, sidewalks, and car tracks, pipes, conduits, cables and other public utility services.

The snow and ice load is considered to be offset by an accompanying decrease in live load and impact and shall not be included except under special conditions.

In the case of structures having concrete slab floors an adequate allowance shall be made in the design dead load to provide for the weight of a wearing surface. This allowance will depend upon the type of wearing surface contemplated; it shall be in addition to the weight of any monolithically placed concrete wearing surface; and shall be not less than 15 pounds per square foot of roadway.

The following weights are to be used in computing the dead load:

	Weight per cubic foot, pounds
Steel	490
Cast iron	450
Aluminum alloys	175
Timber (treated or untreated)	60
Concrete, plain or reinforced	150
Loose sand and earth	100
Rammed sand or gravel, and ballast	120
Macadam or gravel, rolled	140
Cinder filling	60
Pavement, other than wood block	150
Railway rails and fastenings (per linear foot of track)	150

5. 2. 3. Live Load. The live load shall consist of the weight of the applied moving load of vehicles, cars and pedestrians.

5. 2. 4. Highway Live Loads. The highway live load on the roadway portion of the bridge shall consist of trains of motor trucks, or equivalent loads, as hereinafter specified. Each loading is designated by the letter H, followed by a numeral indicating the gross weight in tons of the heaviest loaded truck in the train.

5. 2. 5. Traffic Lanes. The truck trains or equivalent loads shall be assumed to occupy traffic lanes, each having a width of 9 feet corresponding to the standard truck clearance width. Within the curb to curb width of the roadway, the traffic lanes shall be assumed to occupy any position which will produce the maximum stress, but which will not involve overlapping of adjacent lanes, nor place the center of the lane nearer than 4 feet 6 inches to the roadway face of the curb.

5. 2. 6. Trucks. The wheel spacing, weight distribution, and clearance of the trucks used for design purposes shall be as shown in Figure 4.

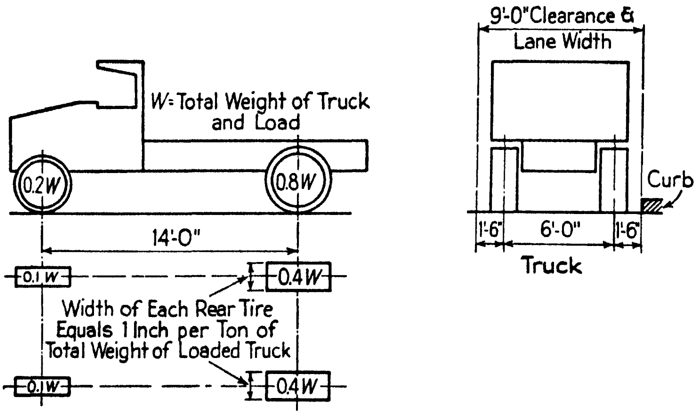


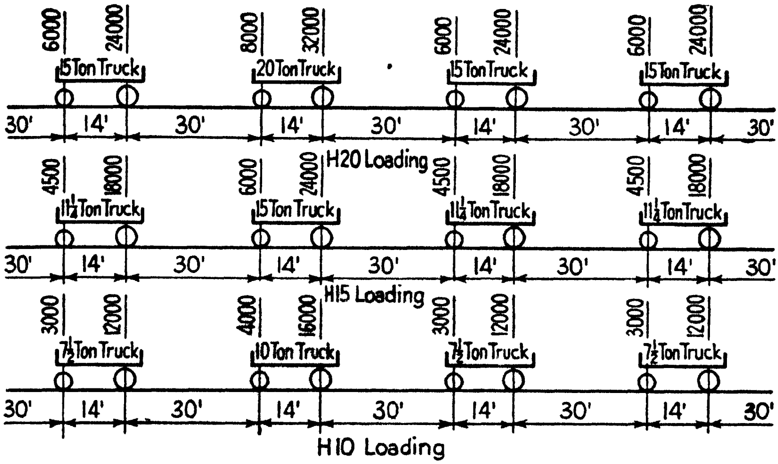
FIG. 4

5. 2. 7. Highway Loadings. The highway loading shall be of three classes, namely, H20, H15, and H10. Loadings H15 and H10 are 75 per cent and 50 per cent, respectively, of loading H20. These loadings shall consist of either truck trains or equivalent loadings as described below.

(*a*) *Truck Train Loadings.* The truck train loadings shall be as shown in Figure 5 and shall be used for loaded lengths of less than 60 feet (but may be used for greater loaded lengths. The loaded length for transverse members such as floor beams shall be considered as the combined lengths of the adjacent panels).[1] Each train shall consist of one truck of the gross weight indicated by the loading class followed by, or preceded by, or both followed and preceded by, a line of trucks of indefinite length, each of the following or preceding trucks having a gross weight of three fourths of the gross weight indicated by the loading class.

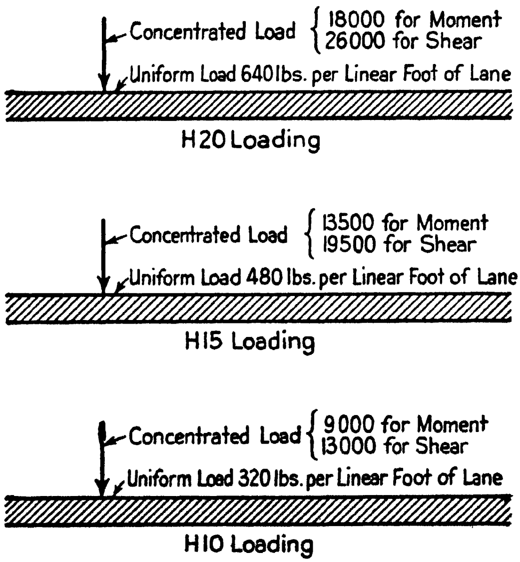
The trucks in adjacent lanes shall be considered as headed in the same direction.

[1] Not in 1931 Edition.



TRUCK TRAIN LOADING

Fig. 5



EQUIVALENT LOADING

Fig. 6

(*b*) *Equivalent Loadings.* The equivalent loadings shall be as shown in Figure 6, and shall be used only for loaded lengths of 60 feet or greater. Each lane loading shall consist of a uniform load per linear foot of traffic lane combined with a single concentrated load so placed on the span as to produce maximum stress. The concentrated load shall be considered as uniformly distributed across the lane on a line normal to the center line of the lane. For the computation of moments and shears, different concentrated loads shall be used as indicated in Figure 6. (The lighter concentrated load shall be used in computing the stresses in members in which the greater part of the stress is produced by bending moments. The heavier concentrated load shall be used when the greater part of the stress in a member is produced by shearing forces.)[1]

5. 2. 8. Selection of Loadings. Bridges of the different classes shall be designed for the loadings as follows:

Class of bridge	Loading
AA	H20
A	H15
B	H10

5. 2. 9. Application of Loadings. The loadings shall be applied by that one of the following methods which produces the greater maximum stress in the member considered, due allowance being made for the reduced load intensities hereinafter specified for roadways having loaded widths in excess of 18 feet.

(1) Each traffic lane loading shall be considered as a unit, and the number and position of the loaded lanes shall be such as will produce maximum stress.

(2) The roadway shall be considered as loaded over its entire width with a load per foot of width equal to one ninth of the load of one traffic lane. (This shall apply to both uniform and concentrated loads.)[1]

5. 2. 10. Reduction in Load Intensity. If the loaded width of the roadway exceeds the two lane width of 18 feet, the specified loads shall be reduced one per cent for each foot of loaded roadway width in excess of 18 feet with a maximum reduction of 25 per cent, corresponding to a loaded roadway width of 43 feet. If the loads are lane loads, the loaded width of the roadway shall be the aggregate width of the lanes considered; if the loads are distributed over the entire width of the roadway, the loaded width of the roadway shall be the full width of roadway between curbs.

(When there are more than two main trusses or girders, as in deck structures of wide roadway, the loaded width for lane loadings shall be considered as the aggregate width of lanes which may be placed between the trusses or girders adjacent to the one in which the stresses are being computed. The loaded width in the case of a uniformly distributed load shall be the entire distance center to center of the trusses or girders adjacent to the one under consideration.

(The above provisions for reduction of intensity of load shall apply to slab spans with main reinforcement parallel to the center line of the road-

[1] Not in 1931 Edition.

way only when maximum stresses are produced by loading the entire width of roadway.

(The reduction in intensity of floor beam loads shall be determined as in the case of main trusses or girders, using the width of roadway which must be loaded to produce maximum stresses in the floor beam.)[1]

5. 2. 12. Sidewalk and Foot Bridge Loading. Sidewalk floors, stringers, and their immediate supports shall be designed for a live load of not less than 100 pounds per square foot of sidewalk area.

Girders or trusses of bridges with sidewalks shall be designed for a sidewalk live load determined by the following formula:

$$P = \left(40 + \frac{3000}{L}\right)\left(\frac{55 - W}{50}\right) \text{in which:}$$

P = live load in pounds per square foot of sidewalk area, but not to exceed 100 pounds per square foot.

L = loaded length of sidewalk in feet.

W = width of sidewalk in feet.

In calculating stresses in structures which support cantilevered sidewalks, the sidewalk shall be considered as fully loaded on only one side of the structure if this condition produces maximum stress.

All parts of foot bridges shall be designed for a live load of not less than 100 pounds per square foot.

5. 2. 13. Impact. Live load stresses, except those due to sidewalk loads and centrifugal, tractive, and wind forces, shall be increased by an allowance for dynamic vibratory and impact effects (provided, however, that this impact allowance shall not be applied to stresses in *timber* since the working stresses for timber given in these specifications are chosen sufficiently low to allow for impact effects).[1]

The amount of this allowance or increment is expressed as a fraction of the live load stress, and for both electric railway and highway loadings shall be determined by the formula:

$$I = \frac{50}{L + 125} \text{in which:}$$

I = impact fraction.

L = the length in feet of the portion of the span which is loaded to produce the maximum stress in the member considered.

5. 2. 14. Longitudinal Force. Provision shall be made for the effect of a longitudinal force of 10 per cent of the live load on the structure, acting 4 feet above the floor.

5. 2. 15. Lateral Forces. (a) The wind force on the structure shall be assumed as a moving horizontal load equal to 30 pounds per square foot on $1\frac{1}{2}$ times the area of the structure as seen in elevation, including the floor system and railings and on one half the area of all trusses or girders in excess of two in the span.

[1] Not in 1931 Edition.

(b) The lateral force due to the moving live load and the wind force against this load shall be considered as acting 6 feet above the roadway and shall be as follows:

Highway bridges, 200 pounds per linear foot.

Highway bridges carrying electric railway traffic, 300 pounds per linear foot.

(c) The total assumed wind force shall be not less than 300 pounds per linear foot in the plane of the loaded chord and 150 pounds per linear foot in the plane of the unloaded chord on truss spans, and not less than 300 pounds per linear foot on girder spans.

(d) In calculating the uplift, due to the foregoing lateral forces, in the posts and anchorages of viaduct towers, highway viaducts shall be considered as loaded on the leeward traffic lane with a uniform load of 400 pounds per linear foot of lane, and viaducts carrying electric railway traffic in addition to highway traffic shall be considered as loaded on the leeward track with a uniform load of 800 pounds per linear foot of track.

(e) A wind pressure of 50 pounds per square foot on the unloaded structure, applied as specified above in paragraph (a), shall be used if it produces greater stress than the combined wind and lateral forces of paragraphs (a) and (b).

5. 2. 16. Centrifugal Force. If the electric railway track is curved, the structure shall be designed to resist a lateral force equal to 10 per cent of the moving railway loading. This lateral force shall be considered as acting 4 feet above the top of rail.

5. 2. 17. Thermal Forces. Provision shall be made for the stresses or movements resulting from the following variations in temperature:

Moderate climate, from 0° to 120° F.
Cold climate, from −30° to 120° F.

The rise and fall in temperature shall be figured from an assumed temperature at the time of erection.

DISTRIBUTION OF LOADS

5. 3. 1. Distribution of Wheel Loads to Stringers and Floor Beams. *Shear.* In calculating end shears and end reactions in transverse floor beams and longitudinal beams and stringers, no lateral or longitudinal distribution of the wheel load shall be assumed.

Bending Moment in Stringers. In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel loads shall be assumed. The lateral distribution shall be determined as follows:

a. Interior Stringers.

Interior stringers shall be proportioned for loads determined in accordance with the following table, except that when the limiting stringer spacings are exceeded the stringer loads shall be determined by the reactions of the truck wheels, assuming the flooring between stringers to act as a simple beam.

Kind of floor	Floor designed for one traffic lane: Fraction of a wheel load to each stringer	Floor designed for one traffic lane: Limiting stringer spacing, in feet	Floor designed for two or more traffic lanes: Fraction of a wheel load to each stringer	Floor designed for two or more traffic lanes: Limiting stringer spacing, in feet
Plank	$\frac{S}{4.0}$	4.0	$\frac{S}{3.5}$	5.0
Strip 4 inches in thickness or wood block on 4 inches plank subfloor	$\frac{S}{4.5}$	4.5	$\frac{S}{3.75}$	5.5
Strip 6 inches or more in thickness	$\frac{S}{5.0}$	5.0	$\frac{S}{4.0}$	6.0
Concrete	$\frac{S}{6.0}$	6.0	$\frac{S}{4.5}$	10.0

S = spacing of stringers in feet.

(*b*) *Outside Stringers.*

The live load supported by outside stringers shall be the reaction of the truck wheels, assuming the flooring to act as a simple beam between stringers.

(*c*) *Total Capacity of Stringers.*

The combined load capacity of the beams in a panel shall not be less than the total live and dead load in the panel.

Bending Moment in Floor Beams. In calculating bending moments in floor beams no transverse distribution of the wheel loads shall be assumed.

If longitudinal stringers are omitted and the floor is supported directly on the floor beams, the latter shall be proportioned for a fraction of the wheel loads, as indicated in the following table, except that when the limiting floor beam spacing is exceeded the floor beam loads shall be determined by the reactions of the truck wheels, assuming the flooring between floor beams to act as a simple beam.

Kind of floor	Fraction of wheel loads to each floor beam	Limiting floor beam spacing, in feet
Plank	$\frac{S}{4.0}$	4.0
Strip 4 inches in thickness or wood block on 4 inches plank subfloor	$\frac{S}{4.5}$	4.5
Strip 6 inches or more in thickness	$\frac{S}{5.0}$	5.0
Concrete	$\frac{S}{6.0}$	6.0

S = spacing of floor beams in feet.

5. 3. 2. Distribution of Wheel Loads on Concrete Slabs (1935 Edition). *Bending Moment.* In calculating bending stresses due to wheel loads on floor slabs no distribution in the direction of the span of the slab shall be assumed. In the direction perpendicular to the span of the slab, the wheel load shall be considered as distributed uniformly over a width of slab which is termed the " effective width " and is computed as follows:

Where S = span of slab in feet.
W = width of tire with a maximum value of 1.25 feet.
E = effective width of slab in feet for one wheel load.

Case I. Single load at center of span.

$$E = 0.6S + 2W$$

Case II. More than one load on the same element of slab.

In calculating the bending moment for more than one load on the same element of a slab, the loads shall be placed as in calculating the maximum moment for a simple beam. This process determines the number of loads that may occur on the element. The moment shall then be calculated for a single load at the center of the span as in Case I and increased for each additional load on the element as follows, where D = distance between the load nearest the center of the span and each additional load:

For $\frac{D}{S} = 0$ .... 100 per cent $\qquad \frac{D}{S} = 0.1$......... 40 per cent

$\frac{D}{S} = 0.3$........ 15 per cent $\qquad \frac{D}{S} = 0.6$......... 0 per cent

Increases for intermediate values of $\frac{D}{S}$ may be obtained by interpolation in the above table.

Case III. Loads on parallel elements of a slab.

The maximum bending moment shall be calculated as in Case I or Case II and increased by the following percentages where B = distance between the parallel loaded elements:

For $\frac{B}{S} = 0$ .. 100 per cent, $\frac{B}{S} = 0.1$.. 60 per cent, $\frac{B}{S} = 0.4$.. 30 per cent

$\frac{B}{S} = 1.0$......... 10 per cent, $\frac{B}{S} = 1.4$.......... 0 per cent

Increases for intermediate values of $\frac{B}{S}$ may be obtained by interpolation in the above table.

The design assumptions of this article do not provide for the effect of loads near unsupported edges. Therefore, at the ends of the bridge and at intermediate points where the continuity of the slab is broken, the edges of the slab shall be supported by diaphragms or other suitable means.

5. 3. 2. Distribution of Wheel Loads on Concrete Slabs (1931 Edition). *Bending Moment.* In calculating bending stresses due to wheel loads on concrete slabs no distribution in the direction of the span of the slab shall be assumed. In the direction perpendicular to the span of the slab, the wheel load shall be considered as distributed uniformly over a width of slab which is termed the " effective width " and is obtained from the following formulas in which:

S = span of slab in feet.
W = Width of wheel or tire in feet.
D = distance in feet from the center of the near support to the center of the wheel.
E = Effective width in feet for *1 wheel.*

Case I. Main Reinforcement Parallel to Direction of Traffic.

$E = 0.7S + W$, in which " E " shall have a maximum value of 7.0 feet

When 2 wheels are so located on a transverse element of the slab that their effective widths overlap, the effective width for each wheel shall be $\frac{1}{2}(E + C)$, in which " E " is the value determined by the formula above and " C " is the distance between centers of wheels.

Case II. Main Reinforcement Perpendicular to Direction of Traffic.

$E = 0.7\ (2D + W)$

For this case the bending moment on a strip of slab 1 foot in width shall be determined by placing the wheel loads in the position to produce the maximum bending, assuming no distribution, determining the effective width for each wheel, and assuming the load of each wheel on the 1-foot strip to be the wheel load divided by its respective effective width.

The design assumption of Case II does not provide for the effect of loads near unsupported edges. Therefore at the ends of the bridge and at intermediate points where the continuity of the slab is broken, the edges of the slab shall be supported by diaphragms or other suitable means.

Shear.

Slabs designed for bending moment in accordance with the foregoing rules and for the wheel loads contemplated by these specifications may be considered adequate for *shear* without special reinforcement.

UNIT STRESSES

5. 4. 1. General (1935 Edition). Except as modified elsewhere in these specifications, the several parts of a steel or concrete structure shall be so proportioned that the unit stresses shall not exceed those given below.

Members subject to stresses produced by a combination of dead load, live load and impact and with either lateral or longitudinal forces, or with bending due to lateral or longitudinal forces may be proportioned for unit stresses 25 per cent greater than those given below.

Unless otherwise noted, unit stresses are given in pounds per square inch.

5. 4. 1. General (1931 Edition). Except as modified elsewhere in these specifications, the several parts of a structure shall be so proportioned that the unit stresses will not exceed those specified below.

Members of structural steel and of concrete shall be so proportioned that an increase of the highway live load by 100 per cent or the electric railway live load by 50 per cent will not produce combined unit stresses in the members more than those specified for dead load.

Unless otherwise noted, unit stresses are given in pounds per square inch.

STEEL STRUCTURES

5. 4. 2. Structural Grade and Rivet Steel (1935 Edition).

Tension:

Axial tension, structural members, net section	18,000
Bolts, area at root of thread	10,000

Axial compression:

Axial compression, gross section, for values of L/r not greater than 140.

Riveted ends	$15{,}000 - \frac{1}{4}\left(\frac{L}{r}\right)^2$
Pin ends	$15{,}000 - \frac{1}{3}\left(\frac{L}{r}\right)^2$

L = length of members in inches
r = radius of gyration of member in inches

Compression splice material, gross section	18,000

Bending on extreme fiber:

Compression on flanges of beams and plate girders	$18{,}000 - 5\left(\frac{L}{b}\right)^2$

L = length in inches of the unsupported flange between lateral connections or knee braces
b = flange width in inches

Tension in rolled shapes, built sections and girders, net section	18,000
Pins	27,000

Diagonal tension:

In webs of girders and rolled beams, at sections where maximum shear and bending occur simultaneously	18,000

Shear:

Girder webs, gross section	11,000
Pins and shop driven rivets	13,500
Power driven field rivets and turned bolts	11,000
Hand driven rivets and unfinished bolts	9,000

Bearing:

Pins, steel parts in contact and shop driven rivets	27,000
Power driven rivets and turned bolts	22,500
Hand driven rivets and unfinished bolts	18,000
Expansion rollers, pounds per linear inch	600d

d = diameter of roller in inches

In proportioning rivets the nominal diameter shall be used.

The effective bearing area of a pin, a bolt, or a rivet shall be its diameter multiplied by the thickness of the metal on which it bears.

In metal $\frac{3}{8}$ inch thick and over, one half the depth of countersink shall be omitted in calculating bearing area. In metal less than $\frac{3}{8}$ inch thick countersunk rivets shall not be assumed to carry stress.

5. 4. 2. Structural Grade and Rivet Steel (1931 Edition).

	For live load and lateral forces	For dead load
Tension:		
Axial tension, structural members, net section	16,000	24,000
Bolts, area at root of thread...............	10,000	15,000
Axial compression:		
Axial compression, gross section............	$\dfrac{16{,}000}{1+\frac{1}{13{,}500}(L/r)^2}$	$\dfrac{24{,}000}{1+\frac{1}{13{,}500}(L/r)^2}$
but not to exceed the value for $L/r = 40$ L = length of the member, in inches r = least radius of gyration of the member, in inches		
Compression splice material, gross section..	16,000	24,000
Bending on extreme fiber:		
Compression in flanges of beams and plate girders..............................	$\dfrac{16{,}000}{1+\frac{1}{2{,}000}(L/b)^2}$	$\dfrac{24{,}000}{1+\frac{1}{2{,}000}(L/b)^2}$
L = length in inches of the unsupported flange between lateral connections or knee braces b = flange width, in inches		
Tension in rolled shapes, built sections and girders, net sections....................	16,000	24,000
Pins..................................	24,000	36,000
Diagonal tension:		
In webs of girders and rolled beams, at sections where maximum shear and bending occur simultaneously..................	16,000	24,000
Shear:		
Girder webs, gross section.................	10,000	15,000
Pins and shop driven rivets................	12,000	18,000
Power driven field rivets and turned bolts..	10,000	15,000
Hand driven rivets and unfinished bolts.....	8,000	12,000
Bearing:		
Pins, steel parts in contact and shop driven rivets..............................	24,000	36,000
Power driven field rivets and turned bolts..	20,000	30,000
Hand driven rivets and unfinished bolts....	16,000	24,000
Expansion rollers, pounds per linear inch ..	600d	900d
d = diameter of roller in inches		

5. 4. 3. Steel Castings. *For steel castings, three fourths of the unit stresses specified above for structural grade steel shall apply.*

5. 4. 6. Bearing on Masonry.

Bearing on granite masonry	800
Bearing on sandstone and limestone masonry	400
Bearing on concrete	600

CONCRETE STRUCTURES

5. 4. 7. Concrete.

	1931 and 1935 For live load and lateral forces	1931 only. For dead load temperature and shrinkage
Direct Compression:		
Columns reinforced with longitudinal bars and separate lateral ties	$600-15\frac{L}{D}$	$900-22.5\frac{L}{D}$
but not to exceed	450	675
Compressive stress due to bending*	800 (900 in 1935)	1,200
Tension	Zero	Zero
Shear (diagonal tension)		
Beams without shear reinforcement:		
Longitudinal bars not anchored	60	80
Longitudinal bars anchored	90	120
Beams with shear reinforcement and anchorage	160	240
Punching shear	160	240

5. 4. 8. Reinforcement.

	1931 and 1935 For live load and lateral forces	1931 only. For dead load temperature and shrinkage
Steel Reinforcement:		
Tension	16,000	24,000
Compression	10 times the compression in the surrounding concrete	
Bond:		
Bars not anchored	80	120
Bars adequately anchored by hooks or otherwise	120	180

The above unit stresses are based upon the use of concrete having an ultimate compressive strength at 28 days of 3,000 pounds per square inch. For concrete having a less strength, the unit stresses shall be proportionately reduced.

For floor slabs, where it is particularly desired to lessen the dead weight of the floor, and for arch rings or ribs, the above unit stresses may be modified as follows:

For special mixes, designed for high strength, the compressive stress may be taken as 30 per cent of the ultimate of the concrete at 28 days for live

* The combined dead and normal live load compressive stress shall not exceed 1,000 pounds per square inch.

load (and 40 per cent for stresses due to)[1] dead load, temperature and shrinkage. The extreme upper limit for such ultimate strengths shall not exceed 4,500 pounds per square inch.

STRUCTURAL STEEL DESIGN

5. 6. 6. Alternate Stresses. Members subject to alternate stresses of tension and compression, due to the combination of dead, live, impact and centrifugal stresses shall be proportioned for the kind of stress requiring the larger section.

If the alternate stresses occur in succession during one passage of the live load, each shall be increased by 50 per cent of the smaller. The connections of such members shall be proportioned for the sum of the net alternate stresses not so increased.

If the live load and dead load stresses are of opposite sign, only 70 per cent of the dead load stress shall be considered as effective in counteracting the live load stress.

5. 6. 7. Combined Stresses. Members subject to both axial and bending stresses shall be proportioned so that the combined fibre stresses will not exceed the specified axial unit stress. If members are continuous over panel points, three-fourths of the bending stress, computed as for a simple beam, shall be added to the axial stress.

5. 6. 8. Secondary Stresses. Designing and detailing shall be done so as to avoid secondary stresses as far as practicable. In ordinary trusses without sub-paneling, no account usually need be taken of the secondary stresses in any member whose width measured in the plane of the truss is less than 1/10 of the length of the member. If the width is greater than 1/10 of the length, or if subpaneling is used, secondary stresses due to deflection of the truss shall be considered.

5. 6. 9. Rolled Beams. Rolled beams shall be proportioned by the moments of inertia of their net sections.

5. 6. 10. Limiting Lengths of Members. For compression members, the ratio of unsupported length to the least radius of gyration shall not exceed 120 for main and stiffening members nor 140 for laterals and sway bracing. In proportioning the top chords of half-through trusses, the unsupported length shall be assumed as the length between laterally supported panel points.

For main riveted tension members, the ratio of length to least radius of gyration shall not exceed 200.

5. 6. 11. Depth Ratios. The ratio of the depth to the length of spans, preferably, shall be not less than the following:

For trusses	1/10
For plate girders (a) and rolled beams (b) used as girders	1/25

For continuous spans, the span length shall be considered as the distance between dead load points of contraflexure (c).

If depths less than these are used, the sections shall be so increased that the maximum deflection will be not greater than if these ratios had not been exceeded.

Note. In 1931 Edition, ratio for (a) was 1/15, for (b) 1/20; item (c) did not appear.

[1] Not in 1935 Edition.

5. 6. 13. Effective Area of Angles in Tension. The effective area of a single angle tension member, or of each angle of a double tension member in which the angles are connected back to back on the same side of a gusset plate, shall be assumed as the net area of the connected leg plus one-half of the area of the unconnected leg.

If a double angle tension member is connected with the angles back to back on opposite sides of a gusset plate, the full net area of the angles shall be considered as effective. If the angles connect to separate gusset plates, as in the case of a double-webbed truss, and the angles are connected by stay plates located as near the gussets as practicable, or by other effective means, the full net area of the angles shall be considered as effective. If the angles are not so connected, only 80 per cent of the net area shall be considered as effective.

Lug angles shall not be considered as effective in transmitting stress.

5. 6. 14. Minimum Thickness of Metal. Gusset plates shall be not less than $\frac{3}{8}$ inch in thickness. Other structural steel, except for fillers and in railings, shall be not less than 5/16 inch in thickness.

Metal subjected to marked corrosive influence shall be increased in thickness or specially protected against corrosion.

5. 6. 15. Plates in Compression. The thickness of web plates of compression members shall be not less than 1/30 of the transverse distance between the lines of rivets connecting them to the flanges. The thickness of cover plates of compression members and cover plates on the compression flanges of plate girders, preferably, shall be not less than 1/40 of the transverse distance between the lines of rivets connecting them to the flanges, but the minimum may be 1/50 of this distance, provided that the width of the plate between the connecting lines of rivets in excess of 40 times the thickness shall not be considered as effective in resisting stress.

5. 6. 16. Outstanding Legs of Angles. The widths of the outstanding legs of angles in compression (except where reinforced by plates) shall not exceed the following:

In girder flanges, 12 times the thickness.
In main members carrying axial stress, 12 times the thickness.
In bracing and other secondary members, 16 times the thickness.

GENERAL DETAILS OF DESIGN

Note: In general these matters are sufficiently covered in the excerpts printed from the A.R.E.A. Specifications in Appendix D.

5. 6. 34. Stay Plates. The open sides of compression members shall be provided with lacing bars and shall have stay plates as near each end as practicable. Stay plates shall be provided at intermediate points where the lacing is interrupted. In main members, the length of the end stay plates between end rivets shall be not less than $1\frac{1}{4}$ times the distance between the inner lines of rivets connecting them to the flanges; and the length of intermediate stay plates between end rivets, not less than $\frac{3}{4}$ of that distance. In lateral struts and other secondary members, the over-all length of end and intermediate stay plates shall be not less than $\frac{3}{4}$ of the distance between the inner lines of rivets connecting them to the flanges.

The separate segments of tension members composed of shapes may be

connected by stay plates or end stay plates and lacing. End stay plates shall have the same minimum length as specified for end stay plates on main compression members and intermediate stay plates shall have a minimum length of $\frac{3}{4}$ of that specified for intermediate stay plates on main compression members. The clear distance between stay plates on tension members shall not exceed 3 feet.

The thickness of stay plates shall be not less than 1/50 of the distance between the inner rivet lines connecting them to the flanges. Stay plates shall be connected by not less than three rivets on each side and in members having lacing bars, the last rivet in the stay plate, preferably, shall also pass through the end of the adjacent bar.

5. 6. 35. Lacing Bars. The lacing of compression members shall be proportioned to resist shearing stresses normal to the member not less than those calculated by the formula:

$$V = \frac{P}{100}\left(\frac{100}{\frac{L}{r} + 10} + \frac{\frac{L}{r}}{100}\right) \text{ in which}$$

V = normal shearing stress in pounds

P = allowable compressive axial load on member, in pounds

L = length of member, in inches

r = radius of gyration of section about the axis perpendicular to the plane of the lacing, in inches.

Note: The 1931 edition replaced the above formula by these two limitations, the one giving the greater value being used,

$$V = 4\,I(24{,}000 - p)/cL$$

or

$$= 0.4\,pI/cL$$

where p is the actual average compression in the member, c the distance from neutral axis to extreme fiber in inches, and I the moment of inertia of the section about the centroidal axis perpendicular to the plane of the lacing, with the other letters as before.

If the lacing of a horizontal or inclined compression member is in a vertical plane, the shear in the lacing caused by the weight of the member shall be added to the shear calculated by the formula above.

The shear shall be considered as divided equally among all shear-resisting elements in parallel planes, whether made up of continuous plates or of lacing. The size of the bar shall be determined by the formula for axial compression in which "L" shall be taken as the distance between the connections to the main sections.

The minimum width of lacing bars shall be:

For 1-inch rivets, $2\frac{3}{4}$ inches.
For $\frac{7}{8}$-inch rivets, $2\frac{1}{2}$ inches.
For $\frac{3}{4}$-inch rivets, $2\frac{1}{4}$ inches.
For $\frac{5}{8}$-inch rivets, 2 inches.

The minimum thickness of bars shall be 1/40 of the distance between connections for single lacings, and 1/60 for double lacing, but not less than 5/16 inch.

Lacing bars of compression members shall be so spaced that the L/r of the portion of the flanges included between lacing-bar connections will be not greater than 40, and not greater than $\frac{2}{3}$ of the L/r of the member.

The angle between the lacing bars and the axis of the member shall be approximately 45 degrees for double lacing and 60 degrees for single lacing. If the distance between rivet lines in the flanges is more than 15 inches, and a bar with a single rivet in the connection is used, the lacing shall be double and riveted at the intersections. Lacing bars having at least 2 rivets in each end shall be used on flanges 5 inches or more in width.

Shapes of equal strength may be used instead of flats.

5. 6. 37. Net Section of Riveted Tension Members. In calculating the required section of riveted tension members, net sections shall be used in all cases and in deducting rivet holes, the holes shall be taken as $\frac{1}{8}$ inch larger than the nominal diameter of the rivet.

The net section shall be the least area which can be obtained by deducting from the gross sectional area, the area of holes cut by any straight or zigzag section across the member, counting the full area of the first hole and a fractional part of each succeeding hole, the fractional part being determined by the formula:

$$X = 1 - \frac{S^2}{4gh}\text{, in which}$$

X = fraction of rivet hole to be deducted.

S = stagger or longitudinal spacing of rivet with respect to rivet on last gauge line.

g = distance between gauge lines, or transverse spacing.

h = diameter of rivet holes, or nominal diameter of rivet plus $\frac{1}{8}$ inch.

5. 6. 64. Minimum Size of Angles. The smallest angle used in bracing shall be 3 by $2\frac{1}{2}$ inches. There shall be not less than three rivets in each end connection of the angles.

5. 6. 65. Lateral Bracing. Bottom lateral bracing shall be provided in all spans except I-beam spans and deck plate girder spans of 50 feet or less. Bottom laterals shall be supported at their intersections by rigid hangers, if necessary, to prevent excessive deflection.

Top lateral bracing shall be provided in deck spans, and in through spans having sufficient headroom.

The lateral bracing of compression chords, preferably, shall be as deep as the chords and effectively connected to both flanges.

5. 6. 66. Portal and Sway Bracing. Through truss spans shall have portal bracing, preferably, of the 2-plane or box type, rigidly connected to the end post and the top chord flanges, and as deep as the clearance will allow. If a single plane portal is used, it shall be located, preferably, in the central transverse plane of the end posts, with diaphragms between the webs of the posts to provide for a distribution of the portal stresses. The portal bracing shall be designed to take the full end reaction of the top chord lateral system and the end posts shall be designed to transfer this reaction to the truss bearings.

Deck truss spans shall have sway bracing in the plane of the end posts and at all intermediate panel points. This bracing shall extend the full depth of the trusses below the floor system. The end sway bracing shall

be proportioned to carry the entire upper lateral stress to the supports through the end posts of the truss.

Through truss spans shall have sway bracing at each intermediate panel point if the height of the trusses is such as to permit a depth of 5 feet or more for the bracing. When the height of the trusses will not permit of such depth, the top lateral struts shall be provided with knee braces. Top lateral struts shall be at least as deep as the top chord.

CONCRETE DESIGN

5. 7. 5. Moments in Floor Slabs. Concrete floor slabs built continuously over supporting beams or joist shall be designed for 80 per cent of the maximum live load bending moment of a simply supported slab of the same span.

5. 7. 6. Expansion Joints. Provision for end expansion shall be made in all concrete slab or girder bridges having a clear span length in excess of 40 feet. When multiple span construction is used, with spans 40 feet or less in length, expansion joints shall be provided at intervals of not more than 80 feet. When the superstructure is cast integrally with the abutments, the reinforcement in the slab or girders shall be increased to provide for the thermal forces induced by a temperature drop of 40° F.

In concrete floors on metal structures, expansion joints shall be provided at both fixed and expansion ends of the span.

APPENDIX D

AMERICAN RAILWAY ENGINEERING ASSOCIATION

1931

GENERAL SPECIFICATIONS FOR STEEL RAILWAY BRIDGES

For Fixed Spans Less than 300 Feet in Length

Fourth Edition — May, 1931

(I) PROPOSALS AND DRAWINGS

(II) GENERAL FEATURES OF DESIGN

Materials Used

8. Structures shall be made wholly of structural steel except where otherwise specified. Rivet steel shall be used for rivets only. Forged steel shall be used for large pins, large expansion rollers and other parts if specified by the Engineer. Cast steel preferably shall be used for shoes and bearings. Cast iron may be used only where specifically authorized by the Engineer.

Types of Bridges

9. The different types of bridges may be used as follows:
Rolled beams for spans up to 35 feet.
Plate girders for spans from 30 feet to 125 feet.
Riveted trusses for spans from 100 feet to 300 feet.
Pin-connected trusses for spans from 150 feet to 300 feet.

Dimensions for Calculation

11. The dimensions for the calculation of stresses shall be as follows:

SPAN LENGTH

For trusses and girders, the distance center to center of end bearings.
For floor-beams, the distance center to center of trusses or girders.
For stringers, the distance center to center of floor-beams.

DEPTH

For riveted trusses, the distance between centers of gravity of chord sections.

For pin-connected trusses, the distance center to center of chord pins.

For plate girders, floor-beams, and stringers, the distance between centers of gravity of flanges, but not to exceed the distance back to back of the flange angles.

Spacing Trusses, Girders, and Floor-beams

12. The width center to center of girders or trusses shall be not less than one-fifteenth of the effective span, and not less than is necessary to prevent overturning under the assumed lateral loading. Panel lengths shall not exceed $1\frac{1}{2}$ times the width, c. to c. of trusses or girders.

(III) LOADS

Loads

17. The structures shall be proportioned for the following loads:
   a. Dead load.
   b. Live load.
   c. Impact or dynamic effect of the live load.
   d. Lateral loads and forces.
   e. Centrifugal force, including impact.
   f. Longitudinal force.

Stresses due to these loads and forces shall be shown separately on the stress sheets.

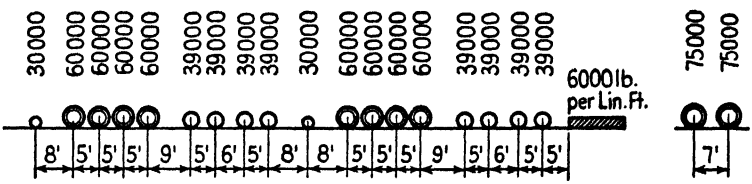
Dead Load

19. The dead load shall consist of the estimated weight of the entire suspended structure. Timber shall be assumed to weigh $4\frac{1}{2}$ pounds per foot B. M.; ballast, assumed level with the base of rail and including track ties embedded therein, 120 pounds per cubic foot; reinforced concrete, 150 pounds per cubic foot; waterproofing, 150 pounds per cubic foot; and rails and fastenings, 150 pounds per linear foot of track.

Live Load

20. The minimum live load for each track shall be Coopers E-60 as shown in Figs. 2 and 3, except as modified in Section 21.

The loading that gives the larger stresses shall be used.



Figs. 2–3

21. In special locations, where conditions limit the loading to light engines, a lighter loading, as stipulated by the Engineer, may be used, but in no case less than three-fourths of that specified in Section 20. The live load assumed shall be proportional to the loading specified in Section 20, with the same wheel spacing.

Floors

24. Timber cross-ties shall be designed for the maximum wheel load distributed over three ties and with 100 per cent impact added. The fiber

stress shall not exceed 2000 pounds per square inch. The ties shall be not less than 10 feet in length. They shall be spaced with openings not exceeding 4 inches and shall be secured against bunching. The maximum dap of ties shall be $1\frac{1}{4}$ inches.

25. Floors consisting of beams transverse to the axis of the structure shall be designed for a uniform live load of 15,000 pounds per linear foot for each track, with 100 per cent impact added, when the minimum live load specified in Section 20 is used. When heavier loadings are used, this uniform load shall be increased proportionately.

27. In ballasted floor bridges, the live load shall be considered as uniformly distributed laterally over a width of 10 feet.

Impact

28. The dynamic increment of the live load shall be added to the maximum computed live load stresses and shall be determined by the formula

$$I = S\frac{300}{300 + \frac{L^2}{100}}, \text{ in which}$$

I = impact or dynamic increment to be added to the live-load stress.
S = computed maximum live-load stress.
L = the length in feet of the portion of the span which is loaded to produce the maximum stress in the member.

Lateral Forces

32. The wind force on the structure shall be a moving load of 30 pounds per square foot on $1\frac{1}{2}$ times its vertical projection on a plane parallel with its axis, but not less than 200 pounds per linear foot at the loaded chord or flange, and 150 pounds per linear foot at the unloaded chord or flange.

The wind force on the train shall be a moving load of 300 pounds per linear foot on one track, applied 8 feet above the base of rail.

33. The lateral force to provide for the effect of the sway of the engines and train in addition to the wind loads specified in Section 32, shall be a moving load equal to 5 per cent of the specified live load on one track, but not more than 400 pounds per linear foot, applied at the base of rail.

Longitudinal Force

37. Provision shall be made in the design for the effect of a longitudinal force of 20 per cent of the live load on one track only, applied 6 feet above the top of the rail. In structures (such as ballasted deck bridges of only three or four spans), where, by reason of continuity of members or frictional resistance, the longitudinal force will be largely directed to the abutments, its effect on the superstructure shall be taken as one-half that specified above.

(IV) UNIT STRESSES AND PROPORTIONING OF PARTS

38. The several parts of structures shall be so proportioned that the unit stresses will not exceed the following, except as modified in Sections 46 and 47:

	Pounds per sq. inch
Axial tension, net section	16,000
Axial compression, gross section	$15{,}000 - 50\frac{l}{r}$
but not to exceed	12,500
l = the length of the member in inches.	
r = the least radius of gyration of the member in inches.	
Tension in extreme fibers of rolled shapes, built sections and girders, net section	16,000
Tension in extreme fibers of pins	24,000
Shear in plate girder webs, gross section	10,000
Shear in power-driven rivets and pins	12,000
Bearing on power-driven rivets, pins, outstanding legs of stiffener angles, and other steel parts in contact	24,000
Rivets driven and bucked by pneumatically or electrically operated hammers are considered power driven.	
The above mentioned values for shear and bearing shall be reduced 25 per cent for countersunk rivets, hand driven rivets and turned bolts.	
Bearing on expansion rollers, per linear inch	$600d$
d = the diameter of the roller in inches.	
Bearing on granite masonry	800
Bearing on sandstone and limestone masonry	400
Bearing on concrete masonry	600

39. For cast steel in shoes and bearings, the above mentioned unit stresses shall apply.

Effective Bearing Area

41. The effective bearing area of a pin, a bolt or a rivet shall be its diameter multiplied by the thickness of the piece, except that for counter-sunk rivets, half the depth of the countersink shall be omitted.

Effective Diameter of Rivets

42. In proportioning rivets, the nominal diameter of the rivet shall be used.

Combined Stresses

46. Members subject to stresses produced by a combination of dead load, live load, impact, and centrifugal force, with either lateral or longitudinal forces, or bending due to lateral action, may be proportioned for unit stresses 25 per cent greater than those specified in Section 38; but the section shall not be less than that required for dead load, live load, impact, and centrifugal force.

Compression Flanges

48. The gross area of the compression flanges of plate girders and rolled beams shall not be less than the gross area of the tension flanges, but the stress per square inch of gross area shall not exceed

$$16{,}000 - 150\frac{l}{b}, \text{ in which}$$

l = the length in inches of the unsupported flange between lateral connections or knee braces.
b = the flange width in inches.

(V) DETAILS OF DESIGN

Slenderness Ratios

49. The ratio of length to least radius of gyration shall not exceed:

100 for main compression members.
120 for wind and sway bracing.
140 for single lacing, and for double lacing not riveted at intersections.
170 for double lacing riveted at intersections.
200 for riveted tension members.

Depth Ratios

50. The depth of trusses preferably shall be not less than one-tenth of the span. The depth of plate girders preferably shall be not less than one-twelfth of the span. The depth of rolled beams used as girders and the depth of solid floors preferably shall be not less than one-fifteenth of the span. (If smaller depths than these are used, the section shall be so increased that the maximum deflection will not be greater than if these limiting ratios had not been exceeded.)[1]

Parts Accessible

51. Details shall be so designed that all parts will be accessible for inspection, cleaning and painting. Closed sections shall be avoided wherever possible.

Effective Area of Angles

54. The effective area of single angles in tension shall be assumed as the net area of the connected leg plus 50 per cent of the area of the unconnected leg. Single angles connected by lug angles shall be considered as connected by one leg.

Strength of Connections

56. Connections shall have a strength at least equal to that of the members connected, regardless of the computed stress. Connections shall be made, as nearly as practicable, symmetrical about the axes of the members.

Limiting Thickness of Metal

57. Metal shall be not less than $\frac{3}{8}$ inch thick, except for fillers. Metal subject to marked corrosive influences shall be increased in thickness or protected against such influences.

Sizes of Rivets

58. Rivets shall be $\frac{3}{4}$ inch, $\frac{7}{8}$ inch or 1 inch in diameter as specified.

Pitch of Rivets

59. The minimum distance between centers of rivet holes shall be three diameters of the rivet, but the distance preferably shall be not less than $3\frac{1}{4}$ inches for 1-inch rivets, 3 inches for $\frac{7}{8}$-inch rivets and $2\frac{1}{2}$ inches for $\frac{3}{4}$-inch rivets. The maximum pitch in the line of stress for members composed of plates and shapes shall be 7 inches for 1-inch rivets, 6 inches for $\frac{7}{8}$-inch rivets and 5 inches for $\frac{3}{4}$-inch rivets. For angles with two gage lines and rivets staggered, the maximum pitch in each line shall be twice the amounts given

[1] Not included in 1935 Specification.

above. If two or more web plates are used in contact, stitch rivets shall be provided to make them act in unison. In compression members, the stitch rivets shall be spaced not more than 24 times the thickness of the thinnest plate in the direction perpendicular to the line of stress, and not more than 12 times the thickness of the thinnest plate in the line of stress. In tension members, the stitch rivets shall be spaced not more than 24 times the thickness of the thinnest outer plate in either direction. In tension members composed of two angles in contact, a pitch of 12 inches may be used for riveting the angles together.

Edge Distance

60. The minimum distance from the center of any rivet hole to a sheared edge shall be: $1\frac{3}{4}$ inches for 1-inch rivets, $1\frac{1}{2}$ inches for $\frac{7}{8}$-inch rivets and $1\frac{1}{4}$ inches for $\frac{3}{4}$-inch rivets; to a rolled edge $1\frac{1}{2}$ inches, $1\frac{1}{4}$ inches and $1\frac{1}{8}$ inches, respectively. The maximum distance from any edge shall be eight times the thickness of the plate, but shall not exceed 6 inches.

Long Rivets

62. Rivets which carry calculated stress and whose grip exceeds four and one-half diameters shall be increased in number at least one per cent for each additional $\frac{1}{16}$ inch of grip. If the grip exceeds six times the diameter of the rivet, specially designed rivets shall be used.

Pitch of Rivets at Ends

63. The pitch of rivets at the ends of built compression members shall not exceed four diameters of the rivet for a distance equal to one and one-half times the maximum width of the member.

Outstanding Legs of Angles

65. The width of the outstanding legs of angles in compression (except when reinforced by plates) shall not exceed the following:

(a) For stringer flange angles, ten times the thickness.
(b) For main members carrying axial stress, twelve times the thickness
(c) For bracing and other secondary members, fourteen times the thickness.

Expansion

87. Provision shall be made for expansion and contraction at the rate of one inch for every 100 feet in length. The expansion ends shall be secured against lateral movement. In spans more than 250 feet in length, provision shall be made for expansion in the floor.

Expansion Bearings

88. Spans more than 70 feet in length shall have rollers at one end. Shorter spans will be arranged to slide on smooth surfaces.

Fixed Bearings

89. Bearings and ends of spans shall be secured against lateral motion.

Pedestals and Shoes

91. Pedestals and shoes preferably shall be made of cast steel. The difference between the top and bottom bearing widths shall not exceed

twice the depth. For hinged bearings, the depth shall be measured from the center of the pin. Where built pedestals and shoes are used, the web plates and the angles connecting them to the base plate shall be not less than $\frac{3}{4}$ inch thick. If the size of the pedestal permits, the webs shall be rigidly connected transversely. The minimum thickness of the metal in cast steel pedestals shall be one inch. Pedestals and shoes shall be so constructed that the load will be distributed uniformly over the entire bearing. Spans more than 70 feet in length shall have hinged bearings at each end.

(VI) FLOORS

Ballasted Floors

95. Ballasted floors shall have not less than 6 inches of ballast under the ties.

Spacing of Stringers

98. Stringers usually shall be spaced 6 feet 6 inches center to center. If four stringers are used under one track, each pair shall be placed symmetrically about the rail.

I-Beam Girders

99. Rolled beams supporting timber decks shall be arranged with not more than four, and preferably not less than two, beams under each rail. The beams in each group shall be placed symmetrically about the rail, and shall be spaced far enough apart to permit cleaning and painting. They shall be connected by solid web diaphragms near the ends and at intermediate points, spaced not over twelve times the flange width. Bearing plates shall be continuous under each group of beams. End stiffeners shall be used if required by Section 38.

End Connection Angles

101. The legs of stringer connection angles shall be not less than 4 inches in width, and not less than $\frac{5}{8}$ inch in thickness before facing. Shelf angles shall be provided to support the stringers during erection, but the connection angles shall be sufficient to carry the whole load. Stringers in through spans shall be riveted between the floor-beams.

(VII) BRACING

Cross-Frames

112. Deck plate girder spans shall be provided with cross-frames at each end proportioned to resist centrifugal and lateral forces, and shall have intermediate cross-frames at intervals not exceeding 18 feet.

Laterals

113. The smallest angle to be used in lateral bracing shall be $3\frac{1}{2}$ by 3 by $\frac{3}{8}$ inches. There shall be not less than three rivets at each end connection of the angles. Angles shall be connected at their intersections by plates.

(VIII) PLATE GIRDERS

Spacing of Girders

115. The girders of deck bridges usually shall be spaced 6 feet 6 inches between centers, except that:

(a) In single-track deck spans 75 feet or more in length, the girders shall be spaced in accordance with Section 12, but not less than 7 feet 6 inches between centers.
(b) In bridges on curves, the girders shall be spaced as shown on the plans.

Design of Plate Girders

116. Plate girders shall be proportioned either by the moment of inertia of their net section including compression side; or by assuming that the flanges are concentrated at their centers of gravity. In the latter case, one-eighth of the gross section of the web, if properly spliced, may be used as flange section. For girders having unusual sections, the moment of inertia method shall be used.

Flange Section

118. Flange plates shall be equal in thickness, or shall diminish in thickness from the flange angles outward. No plate shall have a thickness greater than that of the flange angles.

119. Where flange cover plates are used, one cover plate of the top flange shall extend the full length of the girder. Other flange plates shall extend at least 18 inches beyond the theoretical end.

Thickness of Web Plates

120. The thickness of web plates shall be not less than $\frac{1}{20}\sqrt{D}$, where D represents the distance between flanges in inches.

Flange Rivets

121. The flanges of plate girders shall be connected to the web with a sufficient number of rivets to transfer to the flange section the horizontal shear at any point combined with any load that is applied directly on the flange. Where ties rest on the flange, one wheel load shall be assumed to be distributed over 3 feet.

Web Splices

123. Web plates shall be spliced symmetrically by plates on each side. The splice plates for shear shall be the full depth of the girders between flanges. The splice shall be equal to the web in strength in both shear and moment. There shall be not less than two rows of rivets on each side of the joint.

End Stiffeners

124. Plate girders shall have stiffener angles over end bearings, the outstanding legs of which will extend as nearly as practicable to the outer edge of the flange angles. These end stiffeners shall be proportioned for bearing of the outstanding legs on the flange angles, and shall be arranged to transmit the end reaction to the pedestals or distribute it over the masonry bearings. They shall be connected to the web by enough rivets to transmit the reaction. End stiffeners shall not be crimped.

Intermediate Stiffeners

125. The webs of plate girders shall be stiffened by angles at intervals not greater than:

(a) Six feet.
(b) The depth of the web.
(c) The distance given by the formula

$$d = \frac{t}{40}(12{,}000 - S)$$

d = the distance between rivet lines of stiffeners in inches.
t = the thickness of the web in inches.
S = web shear in pounds per square inch at the point considered.

126. If the depth of the web between the flange angles or side plates is less than 50 times the thickness of the web, intermediate stiffeners may be omitted.

128. Intermediate stiffeners shall be riveted in pairs to the web of the girder. The outstanding leg of each angle shall be not less than 2 inches plus one-thirtieth of the depth of the girder, nor more than 16 times its thickness.

Gusset Plates in Through Girders

129. In through plate girder spans, the top flanges shall be braced by means of gusset plates or knee-braces with solid webs connected to the floor-beams and extending usually to the clearance line. If the unsupported length of the inclined edge of the gusset plate exceeds 18 inches, the gusset plate shall have one or two stiffening angles riveted along its edge. The gusset plate shall be riveted to a stiffener angle on the girder. Preferably it shall form no part of the floor-beam web.

130. In through plate girder spans with solid floors, there shall be knee-braces with $\frac{3}{8}$-inch webs, extending usually to the clearance line, at intervals of about 12 feet. Each knee-brace shall be well riveted to the floor and the girder, especially at the top, and shall have its edge reinforced by one or two angles.

Ends of Through Girders

131. If plate girders project two feet or more above the base of the rail, the upper corners shall be rounded. In multiple span bridges, usually only the extreme ends shall be rounded. Exposed ends of girders shall be neatly finished with end plates.

Spans Shipped Riveted

132. Deck plate girder spans less than 50 feet in length shall be shipped riveted complete, unless otherwise specified.

Masonry Bearings

133. End bearings on masonry preferably shall be raised above the bridge by metal pedestals.

134. Sole plates shall be not less than $\frac{3}{4}$ inch thick and not less in thickness than the flange plus $\frac{1}{8}$ inch. Preferably they shall not be more than 18 inches long.

(IX) TRUSSES

(X) VIADUCTS

(XI) MATERIALS

(XII) WORKMANSHIP

(XIII) WEIGHING AND SHIPPING

(XIV) SHOP PAINTING

(XV) SHOP INSPECTION

(XVI) FULL-SIZE TESTS

AMERICAN RAILWAY ENGINEERING ASSOCIATION

1935

SPECIFICATIONS FOR STEEL RAILWAY BRIDGES

For Fixed Spans Not Exceeding 400 Feet in Length

PART I — DESIGN AND MANUFACTURE

SECTION A — PROPOSALS AND DRAWINGS

SECTION 1 — GENERAL FEATURES OF DESIGN

Spacing of Trusses, Girders, and Stringers

103. The distance between centers of trusses or girders shall be sufficient to prevent overturning by the specified lateral forces. In no case shall it be less than one-twentieth of the span for through spans, nor one-fifteenth of the span for deck spans.

The girders of deck spans and the stringers of through spans shall be spaced not less than 6 feet 6 inches between centers, except that if there are four stringers or girders under one track they shall be in pairs, one pair symmetrical about each rail.

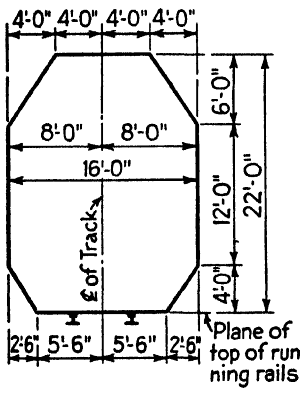


Fig. 1

Depth Ratios

104. The depth of trusses preferably shall be not less than one-tenth of the span. The depth of plate girders preferably shall be not less than one-twelfth of the span. The depth of rolled beams used as girders and the depth of solid floors preferably shall be not less than one-fifteenth of the span.

Clearances

105. The clearances on straight track shall not be less than those shown in Fig. 1. On curved track the clearance shall be increased to allow for the overhanging and the tilting of a car 85 feet long, 60 feet between centers of trucks, and 14 feet high.

The superelevation of the outer rail shall be as specified by the Engineer. The distance from the top of rail to the top of tie shall be taken as 8 inches. Where legal requirements provide greater clearances, they shall govern.

Dimensions for Calculation

106. For the calculation of stresses:

The length shall be:

For trusses and girders, the distance between centers of bearings.

For floor beams, the distance between centers of trusses or girders.

For stringers, the distance between centers of floor-beams.

For timber cross-ties, the distance between centers of stringers or deck girders.

The depth shall be:

For pin-connected trusses, the distance between centers of pins.

For riveted trusses, the distance between centers of gravity of chords.

Floor

109. Timber cross-ties shall be not less than ten feet long, and spaced not more than six inches apart. They shall be secured against bunching.

SECTION 2 — LOADS AND STRESSES

Dead Load

202. In estimating the weight for the purpose of computing dead load stresses, the following unit weights shall be used:

	Pounds per Cubic Foot
Steel	490
Concrete	150
Sand, gravel, and ballast	120
Asphalt-mastic and bituminous macadam	150
Granite	170
Paving bricks	150
Timber	60

The track rails, inside guard rails, and fastenings shall be assumed to weigh 200 pounds per linear foot for each track.

Live Load

203. The recommended live load for each track is Coopers E–72. (See A.R.E.A., 1931, Par. 20.)

Impact

206. To the maximum computed static live load stresses, there shall be added the impact, consisting of:

(a) The lurching effect:

A percentage of the static live load stress equal to . . . . $\frac{100}{S}$

S = spacing, in feet, between centers of longitudinal girders, stringers, or trusses; or length, in feet, of floor beams or transverse girders.

(b) The direct vertical effect:

With steam locomotives (hammer blow, track irregularities, and car impact), a percentage of the static live load stress equal to:

For L less than 100 feet. . . . . $100-0.60L$

For L 100 feet or more. . . . . $\dfrac{1800}{L-40}+10$

With electric locomotives (track irregularities and car impact), a percentage of the static live load stress equal to. . . . . $\dfrac{360}{L}+12.5$

L = length, in feet, center to center of supports for stringers, longitudinal girders, and trusses (chords and main members);

or, L = length of floor beams or transverse girders, in feet, for floor beams, floor beam hangers, subdiagonals of trusses, transverse girders, and supports for transverse girders.

The impact shall not exceed 100 per cent. of the static live load.

Wind on Loaded Bridge

209. The wind force shall be considered as a moving load acting in any horizontal direction. The wind force on the bridge shall be taken at 30 pounds a square foot of:

(a) one and one-half times the vertical projection of the floor system and girders,

(b) the vertical projection of all trusses,

but not less than 200 pounds a linear foot of loaded chord or flange and 150 pounds a linear foot of unloaded chord or flange,

(c) the vertical projection of the columns and tower bracing.

The wind force on the train shall be taken as 300 pounds a linear foot on one track, applied 8 feet above the top of the rail.

Wind on Unloaded Bridge

210. If a wind force of 50 pounds a square foot of surface as defined above, on the unloaded bridge, would produce greater stresses than the wind forces specified in Article 209 combined with the live load, the members where such greater stresses occur shall be designed therefor.

Sway of Locomotives

212. The lateral force to provide for the effect of the sway of locomotives (in addition to the other lateral forces specified) shall be a moving concentrated load of 20,000 pounds applied at the top of rail, in either horizontal direction, at any point of the span.

Bracing between Compression Members

213. The lateral bracing of the compression chords or flanges of trusses and deck girders and between the posts of viaduct towers shall be proportioned for a transverse shear in any panel equal to $2\frac{1}{2}$ per cent. of the total axial stress in both members in that panel, in addition to the shear from the specified lateral forces.

Reversal of Stress

215. Members subject to reversal of stress under the passage of the live load shall be proportioned as follows:

Determine the resultant tensile stress and the resultant compressive stress and increase each by 50 per cent of the smaller; then proportion the member so that it will be capable of resisting either increased resultant stress. The connections shall be proportioned for the sum of the resultant stresses.

Combined Stresses

216. Members subject to both axial and bending stresses shall be so proportioned that the combined fiber stresses will not exceed the allowed axial stress. In members continuous over panel points, only three-fourths of the bending stress computed as for simple beams shall be added to the axial stress.

217. Members subject to stresses produced by a combination of dead load, live load, impact, and centrifugal force, with other lateral forces and with longitudinal force, or with bending due to such forces, may be proportioned for unit stresses 25 per cent. greater than those specified in Article 301; but the section of the member shall not be less than that required for the combination of dead load, live load, impact, and centrifugal force.

SECTION 3 — UNIT STRESSES

Unit Stresses

301. The allowable unit stresses to be used in proportioning the parts of a bridge shall be as follows:

	Pounds Per Square Inch
(a) Structural and Rivet Steel.	
Axial tension, structural steel, net section	18,000
Tension in extreme fibers of rolled shapes, girders, and built sections, subject to bending	18,000
Axial compression, gross section:	
For stiffeners of plate girders	18,000
For columns centrally loaded and with values of l/r not greater than 140:	
Riveted ends	$15{,}000 - \frac{1}{4}\frac{l^2}{r^2}$
Pins ends	$15{,}000 - \frac{1}{3}\frac{l^2}{r^2}$

l = length of member, in inches.
r = least radius of gyration of member, in inches.

For columns with values of l/r greater than 140 and for columns of known eccentricity, see Appendix A.[1]

Compression in extreme fibers of rolled shapes, girders, and built sections, subject to bending (for values of l/b not greater than 40)	$18{,}000 - 5\frac{l^2}{b^2}$

l = length, in inches, of unsupported flange between lateral connections or knee braces.
b = flange widths, in inches.

[1] Appendix A of A.R.E.A. Specification. Not reproduced here.

	Pounds per sq. in.
Stress in extreme fibers of pins	27,000
Shear in plate girder webs, gross section	11,000
Shear in power-driven rivets and pins	13,500
Shear in turned bolts and hand-driven rivets	11,000
Bearing on pins	24,000
Bearing on power-driven rivets, milled stiffeners, and other steel parts in contact	27,000
Rivets driven by pneumatically or electrically operated hammers are considered power-driven.	
Bearing on rocker pins	12,000
Bearing on turned bolts and hand-driven rivets	20,000

Bearing on expansion rollers and rockers, pounds per linear inch:

For diameters up to 25 inches.... $\frac{p - 13,000}{20,000} 600d$

For diameters from 25 to 125 inches $\frac{p - 13,000}{20,000} 3000\sqrt{d}$

d = diameter of roller or rocker, in inches.
p = yield point in tension of the steel in the roller or the base, whichever is the lesser.

(b) Cast Steel.

For cast steel shoes and pedestals, the allowable unit stresses in compression and bearing shall be the same as those for structural steel. Other allowable unit stresses shall be three-fourths of those for structural steel.

(c) Masonry.

Bearing pressure:

Granite	800
Concrete	600
Sandstone and limestone	400

(d) Timber Cross Ties.

Extreme fiber stress in bending:

Yellow pine, dense structural grade	1,500
Douglas fir, close grain structural grade	1,400
White oak	1,200
White pine, Norway pine, and spruce	800

In computing the stresses in timber cross ties, the wheel load shall be considered as distributed over three ties and as applied without impact.

Slenderness Ratio

304. The slenderness ratio (ratio of length to least radius of gyration) shall not exceed:

100 for main compression members.
120 for wind and sway bracing in compression.
140 for single lacing.
200 for double lacing.
200 for tension members other than eye bars.

Proportioning Web Members

305. Web members shall be so proportioned that a live load which will increase the total unit stresses in the chords one-third will produce unit

stresses in the web members not more than one-third greater than the designing stresses.

SECTION 4 — DETAILS OF DESIGN

Compression Members

405. Compression members shall be so designed that the main elements of the section will be connected directly to the gusset plates, pins, or other members.

The center of gravity of a built-up section shall coincide as nearly as practicable with the center line of the section. Preferably the segments shall be connected by solid webs.

In members consisting of segments connected by cover plates or lacing, or segments connected by webs, the thickness of the webs of the segments shall be not less than $\frac{1}{32}$ of the unsupported distance between the nearest rivet lines or the roots of the flanges of rolled segments. The thickness of the cover plates or webs connecting the segments shall be not less than $\frac{1}{40}$ of the unsupported distance between the nearest lines of their connecting rivets or the roots of their rolled flanges.

Net Section

409. The net section of a riveted tension member is the sum of the net sections of its component parts. The net section of a part is the product of the thickness of the part multiplied by its least net width.

The net width for any chain of holes extending progressively across the part shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain and adding, for each gage space in the chain, the quantity

$$\frac{s^2}{4g}$$

s = pitch of any two successive holes in the chain.
g = gage of the same holes.

The net section of the part is obtained from that chain which gives the least net width.

For angles, the gross width shall be the sum of the widths of the legs less the thickness. The gage for holes in opposite legs shall be the sum of the gages from back of angle less the thickness.

For splice members, the thickness shall be only that part of the thickness of the member which has been developed by rivets beyond the section considered.

The diameter of the hole shall be taken as $\frac{1}{8}$ inch greater than the nominal diameter of the rivet.

Effective Sections of Angles

410. If angles in tension are so connected that bending cannot occur in any direction, the effective section shall be the net section of the angle. If connected on one side of a gusset plate, the effective section shall be the net section of the connected leg plus one-half the section of the unconnected leg.

Section at Pin Holes

411. In pin-connected riveted tension members the net section beyond the pin hole, parallel with the axis of the member, shall be not less than the net section of the member. The net section through the pin hole, transverse to the axis of the member, shall be at least 40 per cent greater than the net section of the member. The ratio of the net width (through the pin hole transverse to the axis of the member) to the thickness of the segment preferably shall not be more than 12.

Sizes of Rivets in Angles

417. The diameter of the rivets in angles whose size is determined by calculated stress shall not exceed one-fourth of the width of the leg in which they are driven. In angles whose size is not so determined, 1-inch rivets may be used in $3\frac{1}{2}$-inch legs, $\frac{7}{8}$-inch rivets in 3-inch legs, and $\frac{3}{4}$-inch rivets in $2\frac{1}{2}$-inch legs.

Stay Plates

420. On the open sides of compression members, the segments shall be connected by lacing bars and there shall be stay plates as near each end as practicable. There shall be stay plates at intermediate points where the lacing is interrupted. In main members the length of the end stay plates shall be not less than one and one-fourth times the distance between the lines of rivets connecting them to the outer flanges. The length of intermediate stay plates shall be not less than three-fourths of that distance.

The segments of tension members composed of shapes shall be stayed together. The length of the stay plates shall be not less than two-thirds of the lengths specified for stay plates on compression members. They shall be connected to each segment by at least three rivets.

The thickness of stay plates shall be not less than one-fiftieth of the distance between the lines of rivets connecting them to the outer flanges for main members, or one-sixtieth of that distance for bracing members.

Lacing

421. Lacing bars of compression members shall be so spaced that the slenderness ratio of the portion of the flange included between the lacing bar connections will be not more than 40 nor more than two-thirds of the slenderness ratio of the member.

In compression members, the shearing stress normal to the member in the plane of the lacing shall be that obtained by the following formula,

$$V = \frac{P}{100}\left(\frac{100}{\frac{l}{r} + 10} + \frac{\frac{l}{r}}{100}\right)$$

V = normal shearing stress.
P = allowable compressive axial load on member.
l = length of member, in inches.
r = radius of gyration of section about the axis perpendicular to plane of lacing, in inches.

To the shear so determined shall be added any shear due to the weight of the member or to other forces, and the lacing proportioned for the combined shear.

The shear shall be considered as divided equally among all parallel planes in which there are shear resisting elements, whether continuous plates or lacing. The section of the lacing bars shall be determined by the formula for axial compression in which l is taken as the distance along the bar between its connections to the main segments for single lacing, and as 70 per cent of that distance for double lacing.

If the distance across the member between rivet lines in the flanges is more than 15 inches and a bar with a single rivet in the connection is used, the lacing shall be double and riveted at the intersections.

The angle between the lacing bars and the axis of the member shall be approximately 45 degrees for double lacing and 60 degrees for single lacing.

Lacing bars may be shapes or flat bars. For main members the minimum thickness of flat bars shall be one-fortieth of the distance along the bar between its connections for single lacing, and one-sixtieth for double lacing. For bracing members, the limits shall be one-fiftieth for single lacing and one-seventy-fifth for double lacing.

The diameter of the rivets in lacing bars shall not exceed one-third of the width of the bar. There shall be at least two rivets in each end of lacing bars riveted to flanges more than 5 inches in width.

Reinforcing Plates at Pin Holes

422. Where necessary for the required section or bearing area, the section at pin holes shall be increased on each segment by plates so arranged as to reduce the eccentricity of the segment to a minimum. One plate on each side shall be as wide as the outstanding flanges will allow. At least one full width plate on each segment shall extend to the far edge of the stay plate and the others not less than 6 inches beyond the near edge. These plates shall be connected by enough rivets to transmit the bearing pressure and so arranged as to distribute it uniformly over the full section.

Forked Ends

423. Forked ends will be permitted only where unavoidable. There shall be enough pin plates on forked ends to make the section of each jaw equal to that of the member. The pin plates shall be long enough to develop the pin plate beyond the near edge of the stay plate, but not less than the length required by Article 422.

Proportioning Plate Girders

426. Plate girders and other members subject to bending that produces net tension on one face shall be proportioned by the moment-of-inertia method, using the net section of the compression side as well as of the tension side.

Flange Section

427. The gross section of the compression flange of a plate girder or a rolled beam shall not be less than the gross section of the tension flange.

Flanges of plate girders preferably shall be made without cover plates or

side plates unless angles of greater section than 6 inches by 6 inches by $\frac{7}{8}$ inch would otherwise be required.

Cover plates shall be equal in thickness, or shall diminish in thickness from the flange angles outward. No plate shall be thicker than the flange angles. When cover plates are used, at least one plate on each flange shall extend the full length of the girder. Other flange plates shall extend far enough to allow two rows of rivets at each end of the plate, beyond the theoretical end, and there shall be enough rivets to develop the plate between its end and the theoretical end of the next plate outside.

In through bridges, there shall be end and corner cover plates.

Thickness of Web Plates

431. The thickness of web plates shall be not less than 1/170 of the clear distance between the flanges.

Intermediate Stiffeners

433. If the depth of the web between the flanges or side plates of a plate girder exceeds 60 times its thickness, it shall be stiffened by pairs of angles riveted to the web. The clear distance between stiffeners shall not exceed 72 inches nor that given by the formula:

$$d = \frac{255000t}{S}\sqrt[3]{\frac{St}{a}}$$

d = clear distance between stiffeners, in inches.
t = thickness of web, in inches.
a = clear depth of web between flanges or side plates, in inches.
S = unit shearing stress, gross section, in web at point considered.

The width of the outstanding leg of each angle shall be not more than 16 times its thickness and not less than two inches plus $\frac{1}{30}$ of the depth of the girder.

Cross Frames

438. In deck plate girder spans there shall be cross frames at the ends and at intervals not exceeding 18 feet. The end frames shall be proportioned for the centrifugal and lateral forces.

If there are two lines of stringers under each track in panels more than 20 feet in length, they shall be connected by cross frames.

Eye-Bars

440. The thickness of eye-bars shall be not less than one inch nor more than two inches. The section of the head through the center of the pin-hole shall exceed that of the body of the bar by at least 35 per cent. The form of the head shall be submitted to the Engineer for approval before the bars are made. The diameter of the pin shall be not less than eight-tenths of the width of the widest bar attached.

Eye-Bar Packing

441. The eye-bars of a set shall be symmetrical about the central plane of the truss and as nearly parallel as practicable. The inclination of any bar to the plane of the truss shall not exceed $\frac{1}{16}$ inch to the foot. The bars

shall be packed close, held against lateral movement, and so arranged that those in the same panel will not be in contact.

Expansion

442. The design shall be such as to allow for the changes in length of the span, resulting from changes in temperature, at the rate of one inch in 100 feet. Provision shall be made for changes in length of the span resulting from live load stresses. In spans more than 300 feet long, allowance shall be made for the expansion in the floor.

End Bearings

443. In spans more than 70 feet long, there shall be rollers or rockers at one end. Shorter spans shall be designed to slide on bearings with smooth surfaces.

Bearings and ends of spans shall be secured against lateral movement.

End bearings on masonry preferably shall be raised above the bridge seat by metal pedestals or bolsters.

Rollers

444. Expansion rollers may be either cylindrical or segmental and shall be not less than 6 inches in diameter. They shall be coupled together with substantial side bars and geared to the upper and lower plates. The roller nest shall be so designed that the parts may be cleaned readily.

Pedestals and Shoes

445. Pedestals and shoes preferably shall be made of cast steel. Base plates may be rolled slabs. The difference in width between the top and the bottom bearing surfaces shall not exceed twice the vertical distance between them. For hinged bearings, this distance shall be measured from the center of the pin. In built pedestals and shoes, the web-plates and the angles connecting them to the base plate shall be not less than $\frac{3}{4}$ inch thick. If the size of the pedestal permits, the webs shall be rigidly connected by diaphragms. The minimum thickness of the metal in cast steel pedestals shall be one inch. Pedestals and shoes shall be so designed that the load will be distributed uniformly over the entire bearing surface.

In spans more than 70 feet long there shall be hinged bearings at both ends.

Anchor Bolts

447. Anchor bolts shall be not less than $1\frac{1}{4}$ inches in diameter. There shall be washers under the nuts. Anchor bolt holes in pedestals and sole plates shall be $\frac{3}{8}$ inch larger in diameter than the bolts and at expansion points the holes in the sole plates shall be slotted.

Anchor bolts that do not take uplift shall be long enough to extend 12 inches into the masonry. Those that do take uplift shall be designed to engage a mass of masonry the weight of which is one and one-half times the uplift.

Camber

448. The camber of trusses shall be equal to the deflection produced by the dead load plus a load of 3000 pounds per foot of track. The camber of

plate girders more than 90 feet in length shall be equal to the deflection produced by the dead load only. Plate girders 90 feet or less in length and rolled beams need not be cambered.

Pins

450. In pins more than 9 inches in diameter, there shall be a hole not less than 2 inches in diameter bored longitudinally on the center line.

The turned bodies of pins shall be long enough to extend at the ends $\frac{1}{4}$ inch beyond the outside faces of the parts connected. The pins shall be secured by chambered nuts or by solid nuts and washers. If the pins are bored, through rods with cap washers may be used. The screw ends shall be long enough to allow burring the threads.

Pin-connected members shall be secured in such a way as to limit lateral movement on the pin. Filler rings shall be made of metal not less than $\frac{1}{2}$ inch thick.

SECTION 5 — WORKMANSHIP

Web Plates

526. The edges of web plates of girders that have no cover plates shall not be more than $\frac{1}{8}$ inch above or below the backs of the top flange angles. Web plates of girders that have cover plates may be $\frac{1}{2}$ inch less in width than the distance back to back of flange angles.

Eye-Bars

531. Eye-bars shall be straight, true to size, and free from twists, folds in the neck or head, and other defects. The heads shall be made by upsetting, and rolling or forging. Welding shall not be done. The thickness of the head and neck shall not overrun more than:

$\frac{1}{16}$ inch for bars 8 inches or less in width.
$\frac{1}{8}$ inch for bars more than 8 inches and not more than 12 inches wide.
$\frac{3}{16}$ inch for bars more than 12 inches wide.

532. Both ends of eye-bars shall be bored at the same time. Bars that are to be placed side by side in the structure shall be bored so accurately that the pins may be passed through the holes at both ends at the same time without driving when the bars are placed together.

SECTION 6 — FULL-SIZE TESTS OF EYE-BARS

SECTION 7 — WEIGHING AND SHIPPING

PART II — MATERIALS

PART III — ALLOY STEELS [1]

[1] Nickel and silicon steels both have been used in important railway and highway bridges and have proved satisfactory. Other alloy steels have been used successfully.

APPENDIX E

A PORTION OF THE AMERICAN WELDING SOCIETY "CODE FOR FUSION WELDING AND GAS CUTTING IN BUILDING CONSTRUCTION"

PART A — STRUCTURAL STEEL

1937 Edition

Section 1. General Applications.
Section 2. Definitions.
Section 3. Materials. 1. Structural steel to be welded under this code shall conform to Serial Designation A-9 (Steel for Buildings) of the current Standard Specifications of the American Society for Testing Materials.

2. Filler metal (arc-welding electrodes and gas-welding rods) shall conform to all general requirements, and to all special requirements for at least one of the grades of filler metal, provided by Serial Designation A 205-37T (Iron and Steel Filler Metal), as amended to date, issued jointly by the American Society for Testing Materials and the American Welding Society.

Section 4. Permissible Unit Stresses. 1. Welded joints shall be proportioned so that the stresses caused therein by loads specified in the Building Code shall not exceed the following values, expressed in Kips (thousands of pounds) per square inch:

Kind of Stress	For Welds Made with Filler Metal of	
	Grade 2, 4, 10 or 15	Grade 20, 30 or 40
Shear on section through weld throat......	13.6	11.3
Tension on section through weld throat....	15.6	13.0
Compression (crushing) on section through throat of weld........................	18.0	18.0

Fiber stresses due to bending shall not exceed the values prescribed above for tension and compression respectively. Stress in a fillet weld shall be considered as shear, for any direction of the applied stress.

2. In designing welded joints, adequate provision shall be made for bending stresses due to eccentricity, if any, in the disposition or sections of base metal parts.

Section 5. Design.
Section 6. Workmanship.
Section 7. Erection.
Section 8. Gas Cutting.

SPECIFICATIONS FOR DESIGN, CONSTRUCTION, ALTERATION AND REPAIR OF HIGHWAY AND RAILWAY BRIDGES BY FUSION WELDING

AMERICAN WELDING SOCIETY

1936

SECTION 1 — GENERAL PROVISIONS

101. Application. This Specification is to be used in conjunction with the prescribed standard specification for the design and construction of highway or railroad bridges, as required.

102. Fusion Welding. Fusion welding may be substituted in new work, for any riveting and bolting permitted by the standard specification mentioned above; and may be used alone or in combination with riveting, bolting or other means of fastening in the strengthening and repairing of old bridges; provided that the base material shall conform to the provisions of Article 104, and provided that all welds and welding shall conform in design, execution and results obtained, to the provisions of this Specification.

103. Gas Cutting. Gas (flame) cutting may be substituted for shearing or planing in the preparation of ends and edges to be welded, provided that all processes pertaining thereto shall be executed, and results obtained, in accordance with this Specification. This provision shall not be interpreted to amend the provisions of the applicable general specification with respect to use of gas cutting for preparation of bearing or sliding surfaces, or for other purposes prohibited by the general specification.

104. Base Metal. This Specification provides for fusion welding (and gas cutting) of base metal consisting of Steel for Bridges, Serial Designation A7-34 (or current revision thereof) of the American Society for Testing Materials, or similar low carbon steel or wrought iron approved by the engineer. The carbon content shall not exceed 0.25% nor the manganese content 1.00% by check analysis.

Wrought iron shall correspond to A.S.T.M. Standard Specifications as follows: For bars and shapes, A41–30; for plates A42–30 Class A or B; for definitions of terms A81–33; or the current revision thereof.

SECTION 2 — DESIGN OF NEW BRIDGES

201. General. Full and complete information regarding location, type, size and extent of all welds in accordance with the specifications, shall be clearly shown on the plans. The plans shall clearly distinguish between shop, field and erection welds.

TABLE 1 — BASE MATERIAL

Formula No.	*Type of Member*	*Type of Stress*	*Required Area* (Note 1)
1	Beams and Girders	Bending (with reversal)	$\frac{I}{C} = \frac{\text{Max. (Mom.)} - \frac{1}{2}\text{ Min. (Mom.)}}{18000}$ (Note 2)
2	Beams and Girders	Shear (with reversal)	$A = \frac{\text{Max.} - \frac{1}{2}\text{ Min.}}{13500}$
3	Axially Stressed, not connected by Fillet Welding	Tension (Max.) with reversal to Compression (Min.)	$A = \frac{\text{Max.} - \frac{1}{2}\text{ Min.}}{18000}$ but not less than $\frac{\text{Min.} + \frac{1}{2}\text{ Min.}}{p}$ (Note 3)
4	Axially Stressed, not connected by Fillet Welding	Compression (Max.) with reversal to Tension (Min.)	$A = \frac{\text{Max.} - \frac{1}{2}\text{ Min.}}{p}$ (Note 3)
5	Axially Stressed, connected by Fillet Welding	Tension (Max.) Either sign for Min.	$A = \frac{\text{Max.} - \frac{1}{2}\text{ Min.}}{12000}$ but not less than $\frac{\text{Max.}}{15000}$ and not less than $\frac{\text{Min.} + \frac{1}{2}\text{ Min.}}{p}$
6	Axially Stressed, connected by Fillet Welding	Compression (Max.) Either sign for Min.	$A = \frac{\text{Max.} - \frac{1}{2}\text{ Min.}}{12000}$ but not less than $\frac{\text{Max.} - \frac{1}{2}\text{ Min.}}{p}$ and not less than $\frac{\text{Max.}}{p}$

Note 1: The values herein stated assume 33000 lbs. per sq. in. to be the minimum allowable yield point of base material, and assume 18000 lbs. per sq. in. to be the basic allowable tension under the applicable general specifications. Otherwise all denominators to be modified proportionately, except that those of Formulas 5 and 6 are not to be increased.

Note 2: The denominator 18000 is to be reduced, for the compression flange, as directed in the applicable general specifications. Thus for A.A.S.H.O. Hwy. Br. Spec. 1935, and for A.R.E.A. Rwy. Br. Spec. 1935, use 18000 — $5(l/b)^2$.

Note 3: "p" = allowable compressive unit stress from the applicable general specifications. Thus for A.A.S.H.O. Hwy. Br. Spec. 1935, and for A.R.E.A. Rwy. Br. Spec. 1935, $p = 15000 - \frac{1}{4}(l/r)^2$.

202. Maximum and Minimum Stresses. Maximum and Minimum stresses (axial stress, bending moment, shear, etc., respectively) shall be computed in accordance with the requirements of the applicable general specifications. These will hereinafter be referred to as " Max." and " Min.," respectively. " *Max.*" refers to the numerically greater stress, of whichever sign, and *is to be used* in the design formulas of this Section *with a plus sign.* " *Min.*" refers to the numerically smaller stress; *if it be of the same sign as " Max." it shall be used with a plus sign,* and *if it be of opposite sign to " Max." it shall be used with a minus sign,* in the design formulas.

203. Required Base Material. Case 1. For beams, girders and axially-stressed members not spliced or end-connected by fillet welding, *if " Max." and " Min." have the same sign (no reversal)*, the required base material shall be calculated by the unit stresses prescribed in the applicable general specifications.

Case 2. For beams and girders subject to reversal, for axially-stressed members subject to reversal, and for axially-stressed members spliced or end-connected by fillet welding regardless of reversal, the required base material shall be calculated in accordance with the formulas of Table 1 in which —

Max. and Min. have the respective values and signs stated in Art. 202.
Mom. = calculated external bending moment.
I/C = required section modulus for beams and girders (gross areas in the absence of rivet or bolt holes).
A = required cross sectional area (gross section for tension members in the absence of rivet or bolt holes).

204. Required Weld Areas. The required weld areas shall be calculated in accordance with Formulas 7–9 of Table 2, in which

For butt welds, A = minimum cross sectional area of weld transverse to the line of action of the stress.
For fillet welds, A = length of fillet times throat dimension.

TABLE 2 — WELD AREAS

Formula No.	*Type of Weld*	*Type of Stress*	*Required Weld Area*
7	Butt	Tension or Compression	$A = \dfrac{\text{Max.} - \frac{1}{2}\text{ Min.}}{13500}$ but not less than $\dfrac{\text{Max.}}{16000}$
See Art. 218 for penalty in case of single-vee backed-up welds.			
8	Butt	Shear	$A = \dfrac{\text{Max.} - \frac{1}{2}\text{ Min.}}{9000}$ but not less than $\dfrac{\text{Max.}}{12000}$
(Same exception as under Formula 7.)			
9	Fillet	Tension, Compression or Shear	$A = \dfrac{\text{Max.} - \frac{1}{2}\text{ Min.}}{7200}$ but not less than $\dfrac{\text{Max.}}{9600}$

205. Design of Butt Welds. Formulas 7 and 8 for butt welds assume that there are no abrupt changes of stress distribution on the opposite sides of the weld, that the reinforcement is very moderate and merges smoothly into the base metal, and that the parts are so arranged and held at the time of welding that the weld metal may contract with practically entire freedom. Otherwise, the area of butt weld shall be increased according to the provisions of Article 218 where same apply, and otherwise according to the judgment of the Engineer. A transverse butt weld shall not be assumed in computations to be longer than the width of the joint or piece, perpendicular to the direction of stress.

206. Design of Fillet Welds. Fillet welds placed transversely to the direction of stress shall be calculated as carrying shear.

By special design of fillet welded splices or end connections, proven by tests to alleviate or remove the susceptibility to fatigue failure of the base material adjacent to the ends of the fillets, the area of base material required by Formulas 5 and 6 may be reduced, the lower limit being the area required by Art. 203, Case 1, or by Formulas 3 and 4, whichever may apply.

207. Combined Stresses. In all cases of combined stress, as axial plus bending, etc., the allowable unit stress of each kind shall be derived from the respective foregoing Formula, and the maximum combined stresses compared therewith as would be done under the applicable general specifications.

208. Increased Unit Stresses. When the specifications covering the design of members for stresses due to combined loads, secondary and erection stresses, permit the use of increased unit stresses, a corresponding increase shall be applied to the allowable unit stresses in welds.

209. Eccentricity. In designing welded joints, adequate provisions shall be made for bending stresses due to eccentricity, if any, in the disposition and section of base metal parts and location and types of welded joints.

For members having symmetrical cross sections, the welds shall be arranged symmetrically about the axis of the member, or proper allowance shall be made for unsymmetrical distribution of stresses.

For members having unsymmetrical cross section, as angles, etc., the lengths of welds shall be determined by moments about the gravity axis of the member.

210. Residual Stresses. Unit stresses specified herein are for welded joints in which residual stresses have been regarded in accordance with the intent of this Specification.

Joints shall be designed so as to minimize, as far as practicable, stresses due to shrinkage, bending and eccentricity.

In the case of a member or part which must, in fabrication or erection, be welded at both ends into a rigid structure or assemblage, the design shall provide that the end connection last made shall be of a type to avoid or minimize contraction in the direction of the length of such member or part.

The designer shall note on the drawings those joints or groups of joints in which it is especially important that the sequence and technique of welding be carefully controlled to minimize locked-up stresses.

211. Effective Size of Weld. The strength of a weld shall for purposes of strength computations be based on the minimum throat dimensions of the diagrammatic weld and on the length, with no allowance for that portion of the weld area included in either penetration, reinforcement or tapered ends. Weld lengths shown on the drawings shall be the net effective lengths.

212. Composite Joints. On new work welds and rivets combined in one connection shall not be considered as sharing the stress, and welds shall be provided to carry the entire stress for which the connection is designed.

213. Ambiguity of Stresses. Structures shall be designed so as to avoid, as far as practicable, ambiguity in the determination of stresses.

214. Arrangement of Parts. Members and joints shall be so designed that the component parts may be readily assembled and securely held in place by means of clamps or other suitable devices, and with the welds so located as to be readily accessible for performing the welding operations, inspecting, painting and maintenance.

215. Shop and Field Welds. In the design of members and joints, shop welds shall be used as much as possible and the use of field welds shall be kept a minimum.

216. Position of Weld. In the design of a member or joint, the position in which the welds are made shall have the following order of preference: positioned flat, flat, vertical, horizontal and overhead.

217. Fillet Welds. Except as specified for intermittent welding the minimum size of a strength weld shall in general be one-quarter inch and the minimum length shall be four times the size, or else the size of the weld shall be considered not to exceed one-fourth of its length.

Fillet welds shall be returned without discontinuity around corners, an appropriate length but not less than four times the size.

218. Butt Welds. All butt welds shall be reinforced by depositing additional metal on the weld and the adjoining base metal extending beyond the surface of the thinnest part joined. The depth of reinforcement shall be not less than $\frac{1}{16}$ inch, nor more than $\frac{3}{16}$ inch, up to and including 1 inch in thickness, and not less than $\frac{1}{8}$ inch nor more than $\frac{1}{4}$ inch on thicknesses of more than 1 inch.

All butt welds shall be designed so that reinforcement contours are equal on both sides of the member or symmetrical with the axis of stress and show easy transition curves.

Butt welds between plates of unequal thicknesses shall have the thicker plate beveled to a slope of 1 to 5 when the thicker plate exceeds the thinner plate by 25 per cent or $\frac{1}{4}$ inch. The weld surface shall have a slope not exceeding 1 to 5 when the thicker plate is not beveled.

Ends of butt welds shall be extended on temporary bars and the ends cut off and ground to dimensions of plates joined, or the ends of the welds shall be chipped down to solid metal and side welds applied to fill out ends to same reinforcement as tops of welds.

Single-vee butt welds, except when applied to a backing-up plate, shall have roots of weld chipped out and rewelded as indicated above. Single-vee butt welds made on a backing-up plate shall have weld metal thor-

oughly fused with the backing-up plate. Backed-up welds subjected to tension shall have stress reduced 15 per cent.

All butt welds in tension members shall be restricted to rectangular sections which can be welded in accordance with this specification.

219. Slot Welds. When welding inside a slot or hole in a plate or section, in order to join it to an abutting part, fillet welding may be used along the walls of the slot or hole, but the latter shall not be filled with weld-metal nor partially filled in such a manner as to form a direct weld metal connection between opposite walls; except that fillet welds along opposite walls may overlap each other for a distance of one-fourth their size.

The width of holes for slot welds shall be at least equal to the thickness of the base metal plus one-quarter inch, with a minimum width of $\frac{7}{8}$ inch. For half-round holes on plate edges, the diameter shall be not less than fifteen-sixteenths inch. Corners of rectangular holes shall be rounded with a radius of at least three-eighths inch.

For full-round holes the diameter shall be not less than 3 times the prescribed length of fillet weld, and not less than $1\frac{1}{2}$ times the thickness of the material.

220. Saddle Welds.[1] Saddle welds shall not be used.

221. Intermittent Welds. Intermittent fillet welds may be used in cases where a continuous weld is not required for strength and where the use of a continuous weld would produce excessive shrinkage or warping. The minimum length of a segment of intermittent weld shall be 2 inches, but not less than four times the size of the weld.

When intermittent fillet welding is required on main members chain intermittent welding shall be used, and in all cases chain intermittent welding shall be given preference over staggered intermittent welding.

Intermittent butt welds shall not be used.

222. Sealing Welds. Sealing, when desired, shall preferably not be accomplished by running a bead of minimum dimensions along the intervals between intermittent strength welds. Preferably a continuous weld shall be used, combining the functions of sealing and strength, and changing section only gradually and as changes in the required strength may necessitate.

223. Noncontinuous Beams. The web connection at the ends of noncontinuous beams shall be designed so as to avoid excessive secondary stresses due to bending. Where web connections lack flexibility, as where angles are not used, the length of the welded connection shall not be greater than two-thirds the depth of the beam and shall preferably be symmetrical about the gravity axis of the beam. Where such a connection is made with fillet welds, they shall be used on both sides of the web, and the size of each shall be not less than four-fifths of the web-thickness.

224. Stringer Connections. Stringer connections shall be so arranged as to minimize the participation of stringers in the chord or flange stresses.

[1] A superposed weld having its base or contiguous fused sides substantially in the same plane, the limits of their angular disposition being from 135° to 225°.

225. Built Sections. If two or more plates or rolled shapes are used in contact, sufficient welds shall be provided to make the parts act in unison.

The length of fillet welds at the ends of built up members shall be not less than the width of the member.

In compression members, the maximum spacing center to center of welds shall be not more than 24 times the thickness of the thinnest plate, in the direction perpendicular to the line of stress, and not more than 16 times the minimum size of the weld in the direction of the line of stress. In tension members, the welds shall be spaced not more than 24 times the thickness of the thinner outer plate, in either direction.

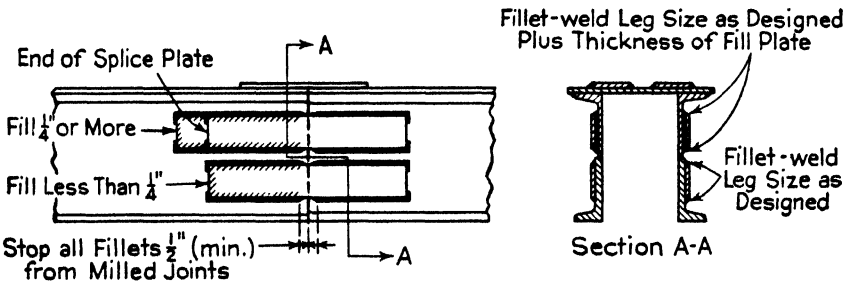
Sufficient weld strength shall be provided to transmit the shearing stresses between joined parts caused by flexure due to long-column action, applied bending moments and any beam reactions, or other loads tending to stress the parts unequally.

226. Girders. Girders shall be proportioned by their moments of inertia and their webs shall be fully spliced.

Stiffeners may be either shapes or plates, welded at their end on the compression flange by continuous welds and to the web by continuous or intermittent welds, designed to transmit the stresses. Ends of stiffeners shall be welded to the tension flanges only when necessary for bearing, in which case transverse welds shall be avoided by use of fills or pads.

227. Strength of Connections. Unless otherwise provided, welded connections at ends of tension or compression members, shall be designed to develop the full effective strength of the member.

Where compression members are spliced by full-milled bearing, and unless otherwise stipulated in the applicable general specifications, the splice material and its welding shall be arranged to hold all parts in line and shall be proportioned for 50 per cent of the computed stress; where compression members are in full-milled bearing on base plates, there shall be sufficient welding to hold all parts securely in place.



COMPRESSION MEMBER (MILLED FOR BEARING)

FIG. 1

228. Indirect Splices and Fillers. If splice material is not in direct contact with the part to which it connects, the amount of welding on the side of the joint not in direct contact, shall be in excess of the amount required for a contact splice, to the extent necessary to develop the section of the

intervening fillers. The intervening fillers shall be extended and developed outside the area of the splice plate except that if the intervening filler is less than $\frac{1}{4}$ inch thick, the splice material shall be extended with it. (See Fig. 1.)

229. Faced Joints. Welds shall not be extended continuously across the plane of a faced joint, but shall be kept back at least one-half inch therefrom. (See Fig. 1.)

230. Lap Joints. The minimum width of laps, on lap joints, shall be four times the thickness of the thinner part joined.

231. Interpenetration of Members. Where a member is cut to permit interpenetration of another member, sufficient welding or other reinforcement shall be applied to develop the required strength of the original sections cut, care being exercised to avoid excessive residual stress.

232. Continuity of Beams. Beams may be made continuous by providing connections capable of resisting the full moments and shears.

SECTION 3 — STRENGTHENING AND REPAIRING OF EXISTING BRIDGES

SECTION 4 — FILLER METAL

401. Filler Metal — General. Filler Metal shall conform to the requirements for Grade E15 or G15 of the "American Welding Society Tentative Specifications for Filler Metal" Revised June 1, 1933, and Feb. 1, 1936, the essential requirements for which are:

1. Qualifications joint and all-weld-metal specimen shall be made with steel A.S.T.M. A7, as modified in Art. 104.
2. Qualification joint and all-weld-metal specimen shall not be stress-relieved.
3. Minimum joint strength shall be 60,000 lbs. per sq. in.
4. Minimum joint free-bend elongation shall be 25%.
5. Minimum all-weld-metal strength shall be 60,000 lbs. per sq. in.
6. Minimum all-weld-metal elongation shall be 17% in 2 in.

402. Old Work. Filler metal for use in repair or strengthening of old bridges, and for use with wrought iron, may depart from the foregoing by special arrangement with the Engineer in charge.

SECTION 5 — EQUIPMENT AND PROCESSES

SECTION 6 — WORKMANSHIP

SECTION 7 — TECHNIQUE OF ARC WELDING

SECTION 8 — TECHNIQUE OF GAS WELDING AND GAS CUTTING

SECTION 9 — INSPECTION

DESIGN TABLES 5.2.3-4

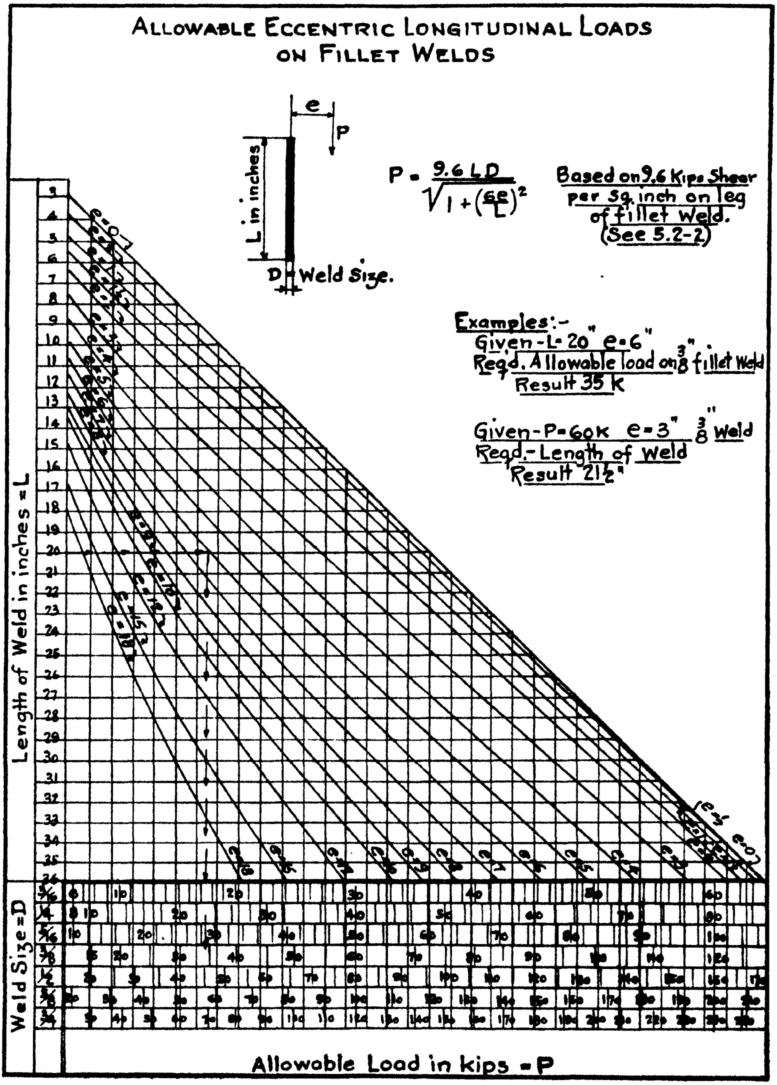
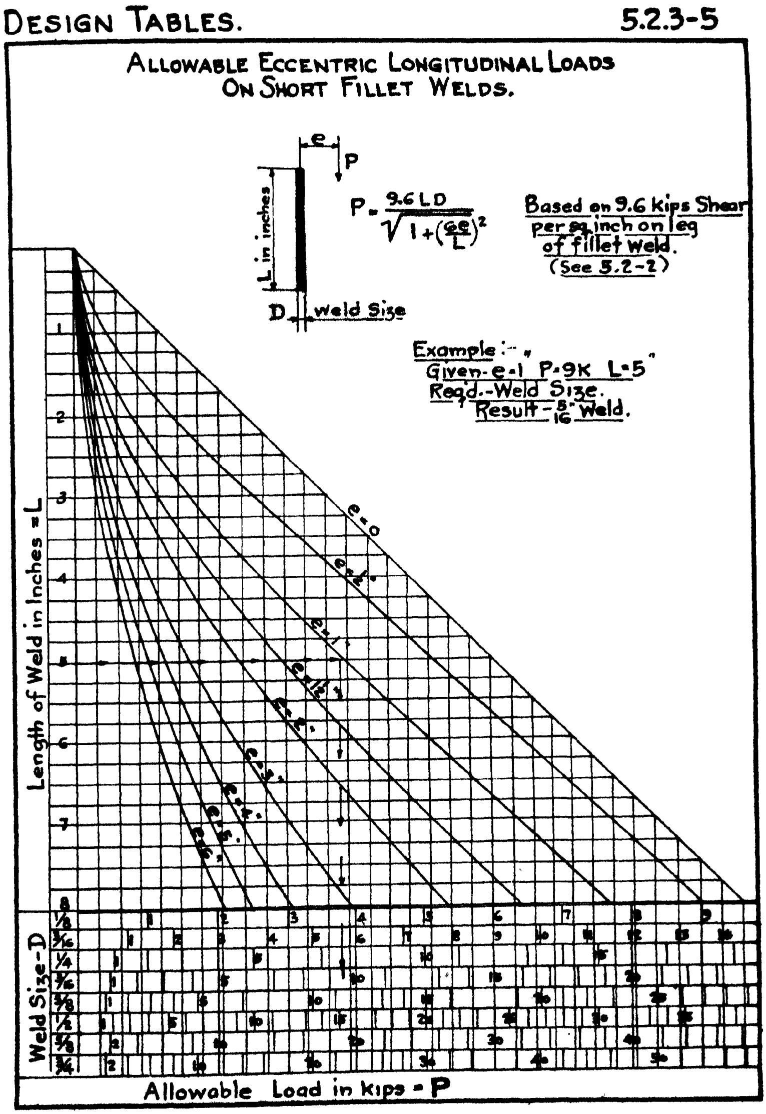


PLATE E1



Design Tables.
5.2.3-5
Allowable Eccentric Longitudinal Loads
On Short Fillet Welds.
L in inches
Weld Size
P = 9.6 L D / √(1+(6e/L)²)
Based on 9.6 kips Shear per sq. inch on leg of fillet weld. (See 5.2-2)
Example:- Given- e=1" P=9K L=5"
Req'd.-Weld Size.
Result-5/16" Weld.
Length of Weld in Inches = L
Weld Size-D
Allowable Load in kips = P

PLATE E2

DESIGN TABLES 5.2.3-6

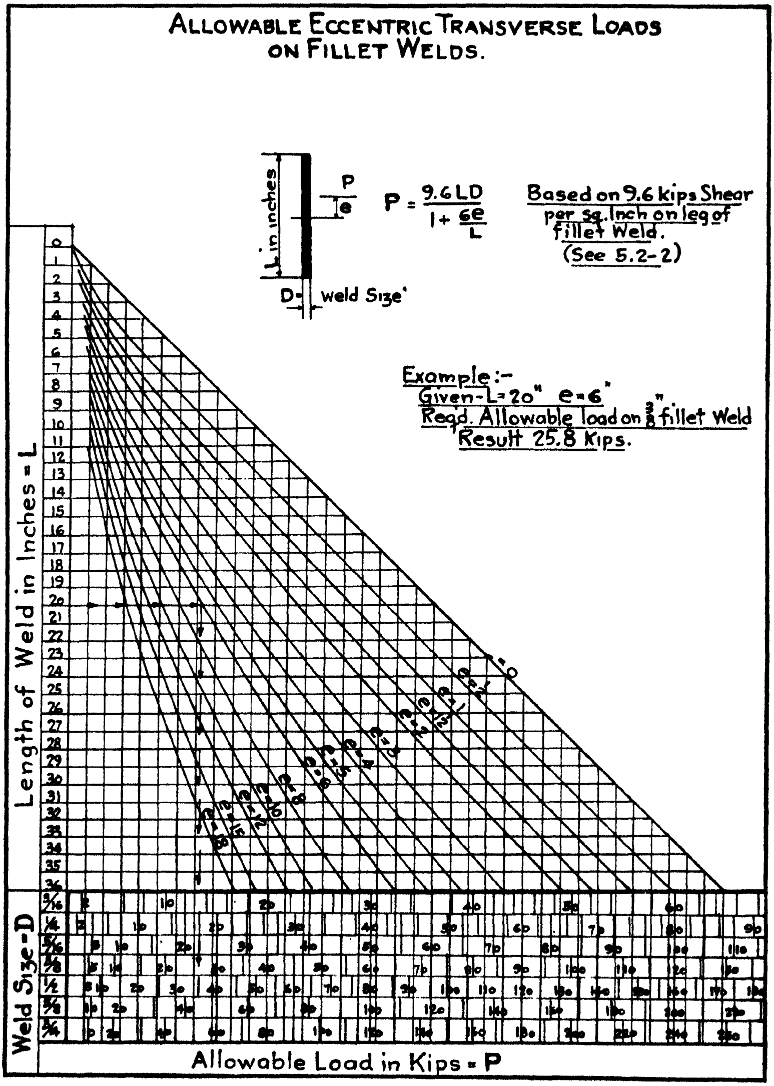


PLATE E3

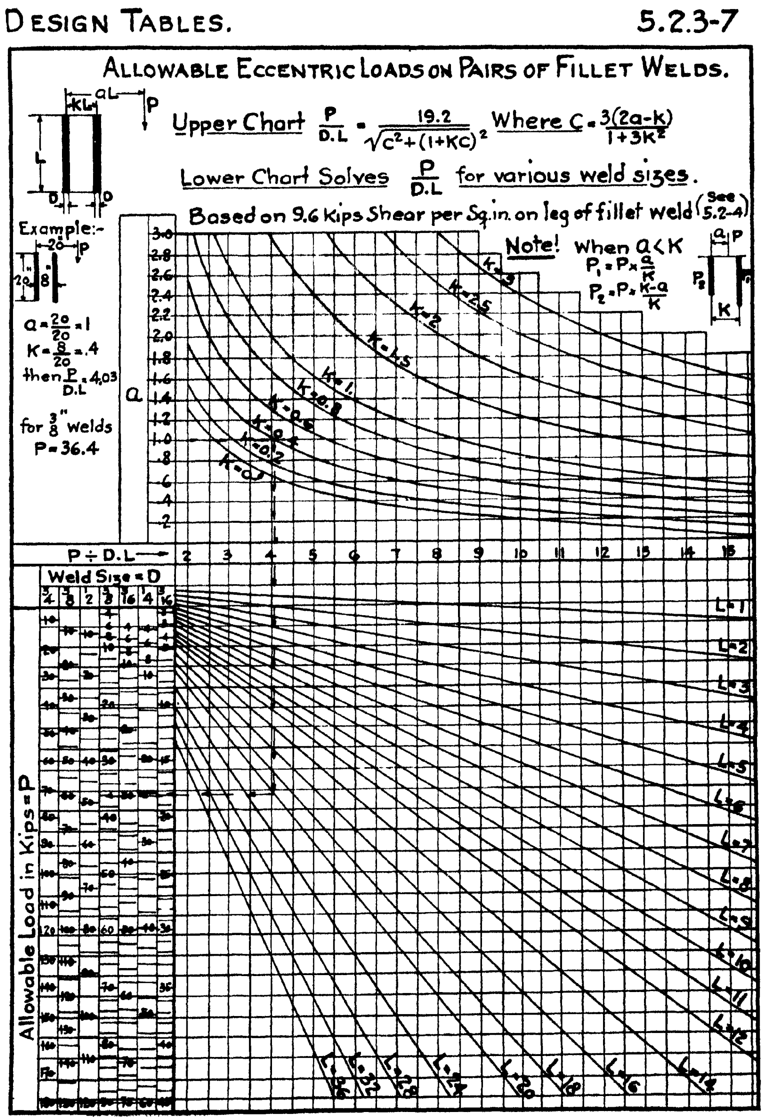

DESIGN TABLES.
5.2.3-7
ALLOWABLE ECCENTRIC LOADS ON PAIRS OF FILLET WELDS.
Upper Chart P/D.L = 19.2/√(C²+(1+KC)²) Where C = 3(2a-k)/(1+3K²)
Lower Chart Solves P/D.L for various weld sizes.
Based on 9.6 Kips Shear per Sq.in. on leg of fillet weld (See 5.2-4)
Note! When a<K P₁ = P×a/K P₂ = P×(K-a)/K
Example:-
a = 20/20 = 1
K = 8/20 = .4
then P/D.L = 4.03
for 3/8" welds P = 36.4
P ÷ D.L
Weld Size = D
Allowable Load in Kips = P

PLATE E4

DESIGN TABLES 5.2.3-8

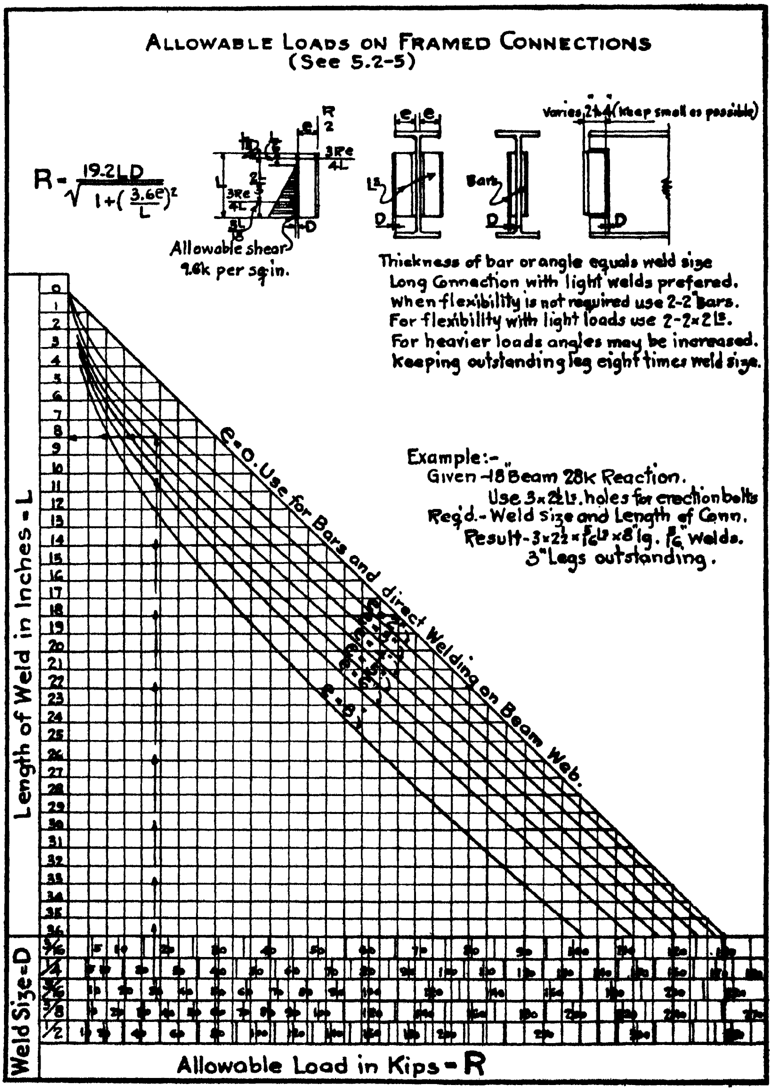
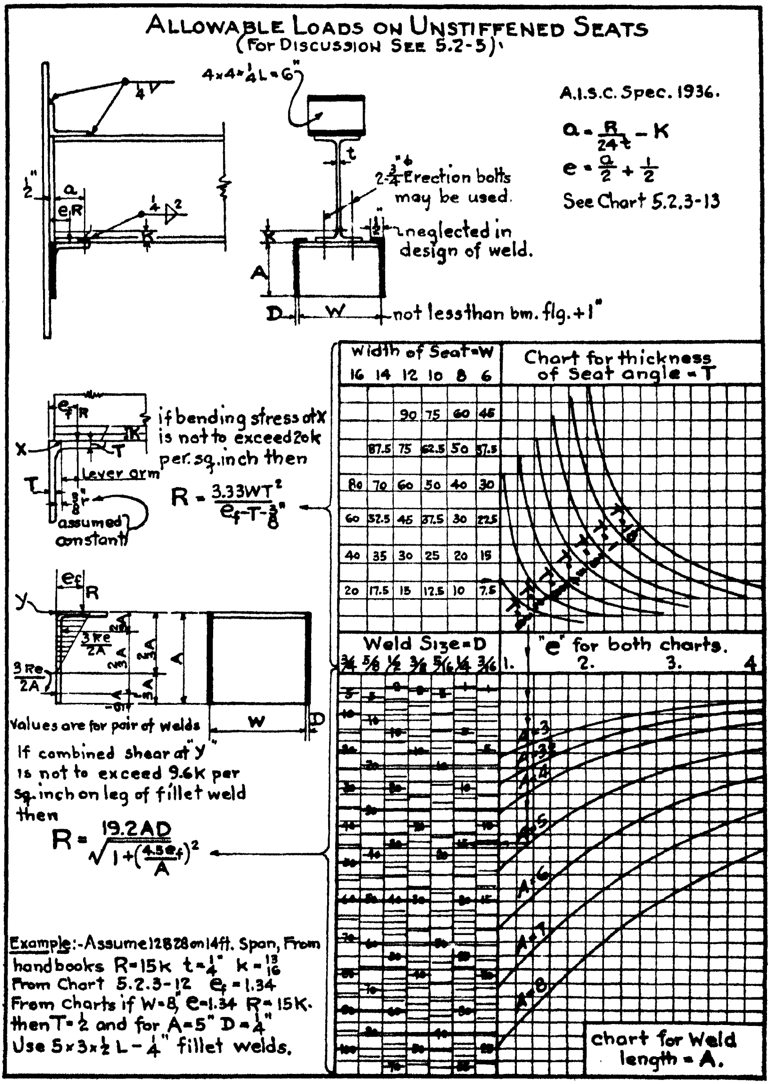


PLATE E5

DESIGN TABLES 5.2.3-9



DESIGN TABLES 5.2.3-10

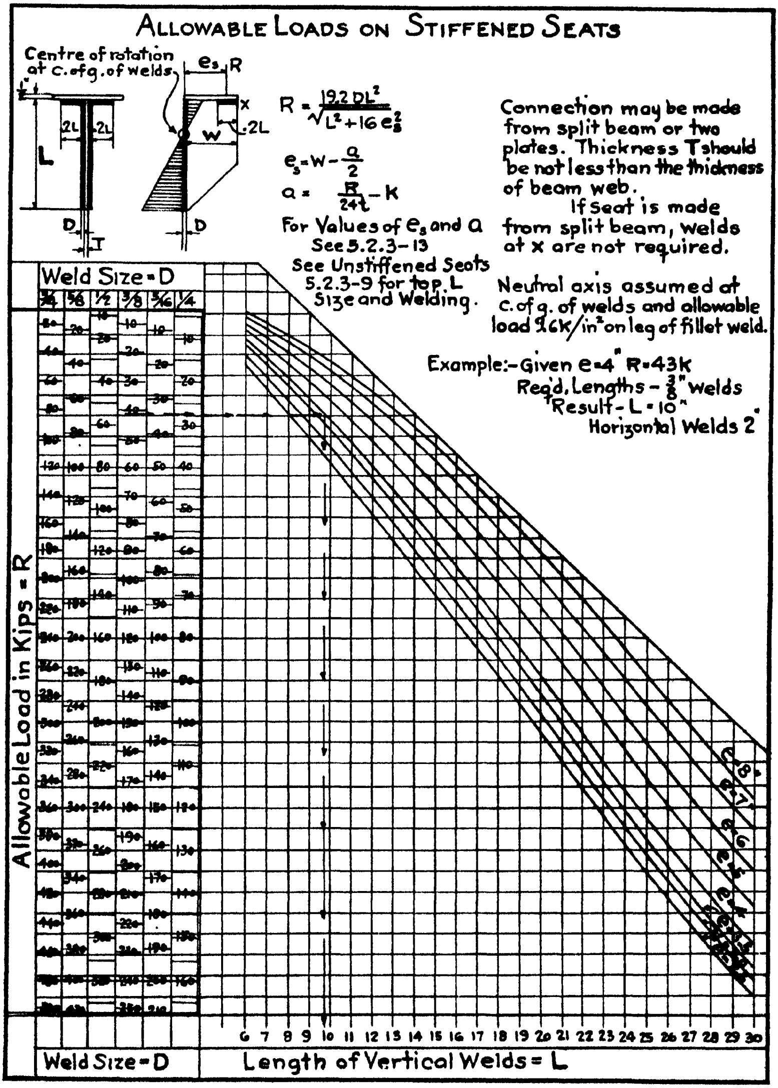
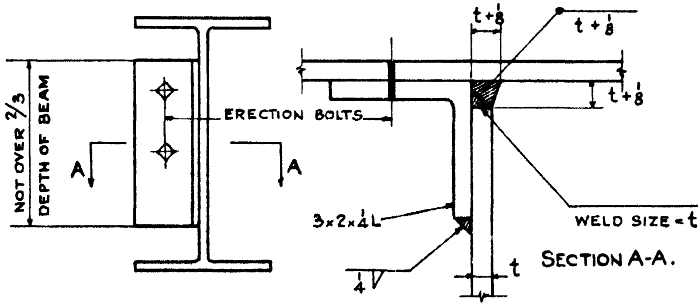


PLATE E7

DESIGN TABLES 5.2.3-11

SIDE ANGLE CONNECTION WITH BUTT WELD.

SEE 5.2-5 a



ALLOWABLE LOAD IN KIPS

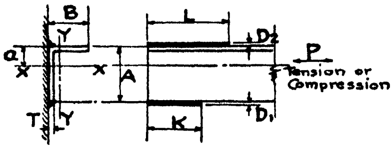
	WEB THICKNESS = t															
	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1"
4	3	6	10	13	17	20	23	27	30	34	37	40	44	47	51	54
5	4	8	12	17	21	25	29	34	38	42	46	51	56	59	63	68
6	5	10	15	20	25	30	35	40	46	51	56	61	66	71	76	81
7	6	12	17	23	29	35	41	47	53	59	65	71	77	83	89	95
8	6	13	20	27	34	40	47	54	61	68	74	81	88	95	102	109
9	7	15	23	30	38	46	53	61	69	76	84	91	99	107	114	122
10	8	17	25	34	42	51	59	68	77	85	93	102	110	119	127	136
11	9	18	28	37	46	56	65	74	84	93	103	112	122	131	140	149
12	10	20	30	40	51	61	71	81	92	102	112	122	133	142	153	163
13	11	22	33	44	55	66	77	88	99	110	121	132	144	154	165	176
14	12	23	35	47	59	71	83	95	107	119	131	143	155	166	178	190
15	12	25	38	51	63	76	89	102	114	127	140	153	166	178	191	204
16	13	27	40	54	68	81	95	109	122	136	149	163	177	190	204	217
17	14	29	43	57	72	86	101	115	130	144	159	173	188	202	217	231
18	15	30	46	61	76	92	107	122	137	153	168	183	199	214	229	245
19	16	32	48	64	80	97	113	129	145	161	178	194	210	226	242	258
20	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272
21	17	35	53	71	89	107	125	143	160	178	196	214	232	250	267	285
22	18	37	56	74	93	112	131	149	168	187	206	224	243	262	280	299
23	19	39	58	78	97	117	137	156	177	196	215	235	254	274	293	312
24	20	40	61	81	102	122	142	163	184	204	225	245	265	285	306	326

NOTE! VALUES ARE BASED ON SHEAR IN BUTT WELD AT 13 600 P.S.I.
VALUES FOR WEBS OVER 1" THICK MAY BE OBTAINED BY COMBINING OTHER VALUES; FOR EXAMPLE VALUE FOR 1" PLUS VALUE FOR 1/8" = VALUE FOR 1 1/8" WEB.
THIS CHART MAY BE USED FOR ANY BUTT WELD IN SHEAR.

PLATE E8

DESIGN TABLES 5.2.3-12

WELDS FOR ATTACHING ANGLES



Eccentricity about Y-Y axis has been neglected. If leg B ≥ A and $D_2 \geq 2T$ the additional stress induced in the weld will not exceed 25% of the computed stress.

Assume $D_1 = D_2 = D$
Weld Size D = Angle thickness T
Allowable shear on leg of fillet weld 9.6k/in²

Then $K = \dfrac{Pa}{9.6AD}$ $L = \dfrac{P(A-a)}{9.6AD}$

Lengths of welds are adjusted to nearest ½", Since no weld should be shorter than four times the weld size it may be necessary to increase the lengths of short welds.

Since for any particular case, length of weld times size of weld is a constant, either may be varied by proportion. $\dfrac{\text{New Length}}{\text{Tabular Length}} = \dfrac{\text{Tabular Size}}{\text{New Size}}$

* Special Sizes

Size of Angle	Lengths of Welds for Various Unit Stresses in Angles.																					
	20k/in²		19k/in²		18k/in²		17k/in²		16k/in²		15k/in²		14k/in²		13k/in²		12k/in²		11k/in²		10k/in²	
	K	L	K	L	K	L	K	L	K	L	K	L	K	L	K	L	K	L	K	L	K	L
8×8	9.0	23.5	8.5	22.0	8.0	21.0	7.5	20.0	7.0	18.5	7.0	17.5	6.0	16.5	5.0	15.0	5.5	14.0	5.0	13.0	4.5	12.0
8×6	9.0	20.0	8.0	19.0	8.0	17.5	7.5	16.5	7.0	15.5	6.5	15.0	6.0	14.0	5.5	13.0	5.0	12.0	5.0	10.5	4.0	10.0
8×4	8.5	15.5	8.0	15.0	7.5	14.0	7.5	13.0	7.0	12.5	6.5	11.5	6.0	11.0	5.5	10.0	5.0	9.5	4.5	8.5	4.0	8.0
7×4	7.5	14.5	7.0	14.0	7.0	13.0	6.5	12.5	6.0	12.0	6.0	11.0	5.5	10.0	5.0	10.0	4.5	9.0	4.5	8.0	4.0	7.0
6×6	6.5	17.5	6.5	17.0	6.0	16.0	6.0	15.0	5.5	14.0	5.5	13.0	5.0	12.0	4.5	11.5	4.0	11.0	4.0	10.5	3.5	8.5
6×4	6.5	14.0	6.5	13.0	6.0	12.5	5.5	11.5	5.0	11.0	5.0	10.0	4.5	9.5	4.5	8.5	4.0	8.0	3.5	7.5	3.0	7.0
* 6×3½	6.5	12.5	6.0	12.0	6.0	11.0	5.5	10.5	5.0	10.5	5.0	9.5	4.5	9.0	4.5	8.0	4.0	7.5	3.5	7.0	3.0	6.5
5×5	5.5	14.5	5.0	14.0	5.0	13.0	5.0	12.5	4.5	11.5	4.0	11.0	4.0	10.0	3.5	9.5	3.5	9.5	3.0	8.0	3.0	7.0
5×3½	5.5	11.5	5.0	11.0	5.0	10.5	4.5	10.0	4.5	9.0	4.0	9.0	4.0	8.0	3.5	7.5	3.5	7.0	3.0	6.5	3.0	5.5
* 5×3	5.5	10.5	5.0	10.0	5.0	9.5	4.5	9.0	4.5	8.5	4.0	8.0	4.0	7.5	3.5	7.0	3.0	6.5	3.0	6.0	3.0	5.0
4×4	4.5	12.0	4.5	11.0	4.0	10.5	4.0	10.0	3.5	9.5	3.5	8.5	3.5	8.0	3.0	7.5	3.0	7.0	2.5	6.5	2.5	5.5
4×3½	4.5	10.5	4.5	10.0	4.0	9.5	4.0	9.0	3.5	8.5	3.5	8.0	3.0	7.5	3.0	6.5	3.0	6.0	2.5	6.0	2.5	5.0
4×3	4.5	9.5	4.5	9.0	4.0	8.5	4.0	8.0	3.5	8.0	3.5	7.0	3.0	7.0	3.0	6.5	3.0	5.5	2.5	5.5	2.5	4.5
3½×3½	4.0	10.0	4.0	9.5	3.5	9.5	3.5	8.5	3.0	8.5	3.0	7.5	3.0	7.0	2.5	6.5	2.5	6.0	2.5	5.5	2.0	5.0
3½×3	4.0	9.0	4.0	8.5	3.5	8.5	3.5	7.5	3.0	7.5	3.0	7.0	3.0	6.5	2.5	6.0	2.5	5.5	2.0	5.0	2.0	4.5
* 3½×2½	4.0	8.0	4.0	7.5	3.5	7.5	3.0	7.0	3.0	7.0	3.0	6.0	3.0	5.5	2.5	5.5	2.5	5.0	2.0	4.5	2.0	4.0
3×3	3.5	8.5	3.5	8.0	3.0	8.0	3.0	7.0	3.0	7.0	2.5	6.5	2.5	6.0	2.0	6.0	2.0	5.5	2.0	4.5	1.5	4.5
3×2½	3.5	7.5	3.5	7.0	3.0	7.0	3.0	6.5	3.0	6.0	2.5	6.0	2.5	5.0	2.0	5.0	2.0	4.5	2.0	4.0	1.5	4.0
* 3×2	3.5	6.5	3.0	7.0	3.0	6.0	3.0	5.5	2.5	5.5	2.5	5.0	2.0	5.0	2.0	4.5	2.0	4.0	2.0	3.5	1.5	3.5
2½×2½	3.0	7.5	3.0	7.0	2.5	6.5	2.5	6.0	2.5	5.5	2.0	5.5	2.0	5.0	2.0	4.5	1.5	4.5	1.5	4.0	1.5	3.5
2½×2	3.0	6.5	3.0	6.0	2.5	6.0	2.5	5.0	2.5	5.0	2.0	5.0	2.0	4.5	2.0	4.0	1.5	4.0	1.5	3.5	1.5	3.0
2×2	2.5	5.5	2.0	5.5	2.0	5.5	2.0	5.0	2.0	4.5	1.5	4.5	2.0	4.0	1.5	4.0	1.5	3.5	1.5	3.0	1.0	3.0
2×1½	2.5	4.5	2.5	4.5	2.0	4.5	2.0	4.0	2.0	4.0	2.0	3.5	1.5	3.5	1.5	3.0	1.5	3.0	1.5	2.5	1.0	2.5
1¾×1¾	2.0	4.0	2.0	4.0	1.5	4.0	1.5	3.5	1.5	3.5	1.5	3.0	1.5	3.0	1.5	2.5	1.5	2.5	1.0	2.5	1.0	2.0
1¾×1¼	1.5	3.5	1.5	3.5	1.5	3.5	1.5	3.0	1.5	3.0	1.5	2.5	1.5	2.0	1.5	2.0	1.0	2.0	1.0	2.0	1.0	2.0
1½×1½	1.5	3.0	1.5	3.0	1.5	2.5	1.5	2.5	1.0	2.5	1.0	2.5	1.0	2.0	1.0	2.0	1.0	2.0	1.0	1.5	1.0	1.5
1¼×1¼	1.0	2.0	1.0	2.0	1.0	2.0	1.0	2.0	1.0	1.5	1.0	1.5	1.0	1.5	1.0	1.5	0.5	1.5	0.5	1.5	0.5	1.5
1×1	1.0	1.0	1.0	1.0	1.0	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.5	0.5	0.5	0.5

K and L for any other unit stresses may be obtained by proportion or interpolation.

PLATE E9

DESIGN TABLES 5.2.3-13

WEB CRIPPLING OF BEAMS ON SEATS

R = Reaction or Load

a = Length of bearing on seat to produce 24,000# crippling stress in beam web.

From A.I.S.C. Spec. (Revised 1936)

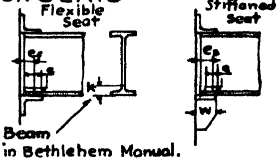
$$a = \frac{R}{24t} - k$$

$e_f = \frac{a}{2} + \frac{1}{2}$" for flexible seats See 5.2.3-9

Distance from bottom of Beam to edge of fillet = k or 0 in Bethlehem Manual.

$e_s = W - \frac{a}{2}$ for stiffened seats See 5.2.3-10

Note e_s may not be less than $\frac{W}{2}$



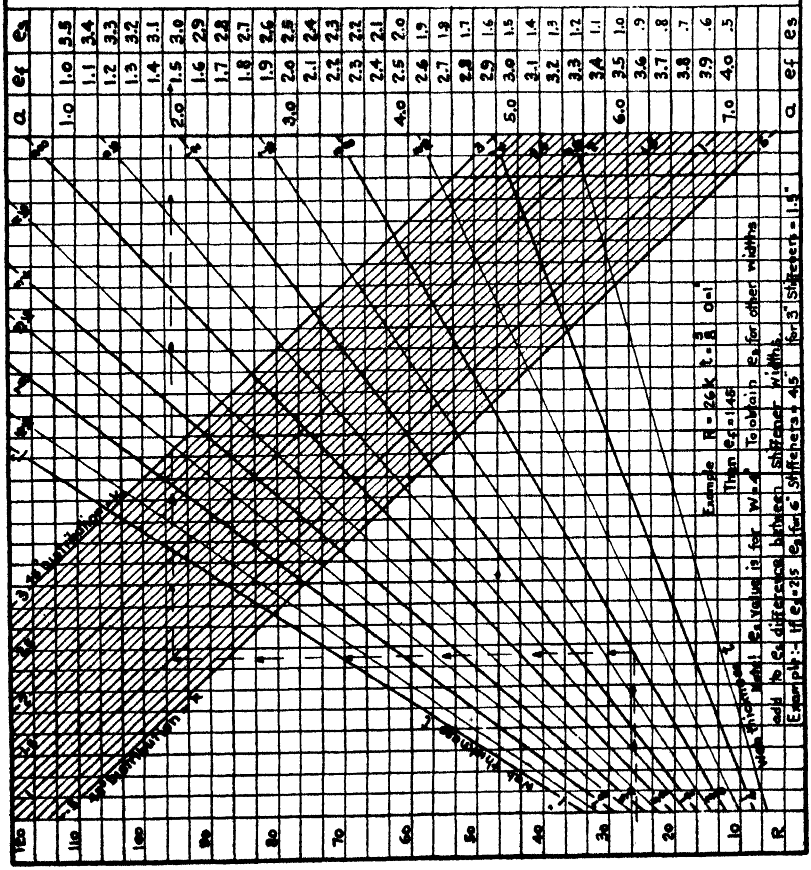
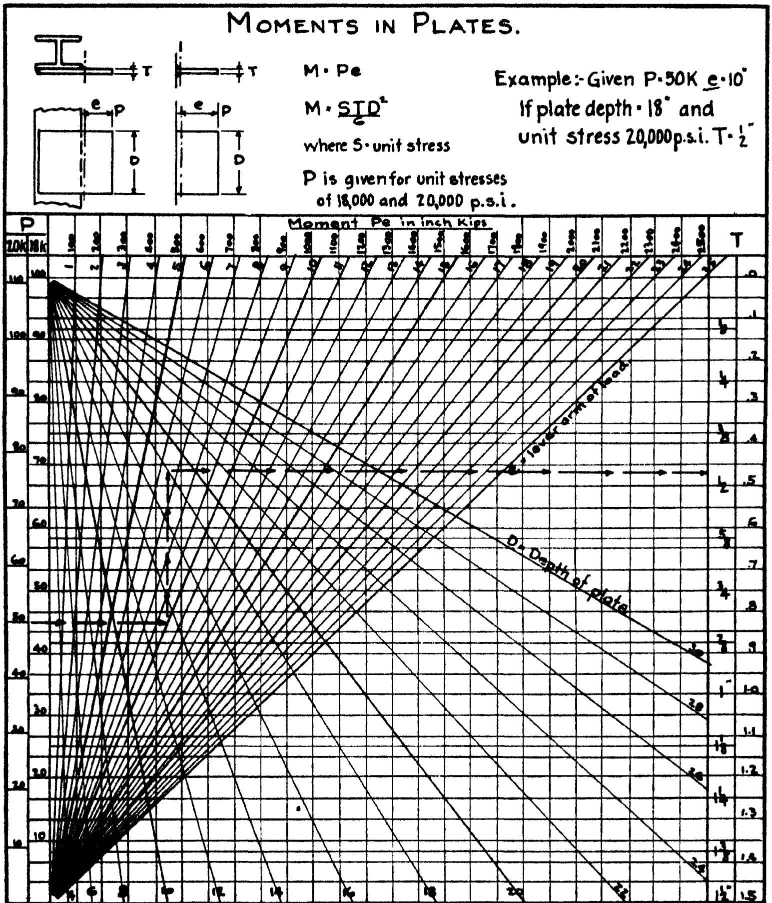
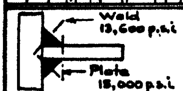


PLATE E10

DESIGN TABLES 5.2.3-14





SHEAR IN PLATES.

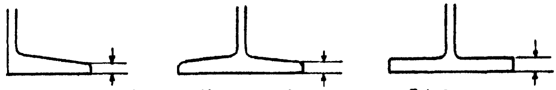
Equivalent thickness of plate to develop a pair of fillet welds

Size of fillet welds	1/8	3/16	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/4	1 1/2
Thickness of plate	3/16	1/4	3/8	7/16	1/2	9/16	11/16	13/16	1	1 1/8	1 3/8	1 5/8	2

PLATE E11

TYPICAL DETAILS 5.2.1-7

FLANGE THICKNESSES FOR WELDING



MAXIMUM WELD SIZE

3/16	1/4	5/16	3/8	1/2	5/8	3/4	7/8	1"
3 C	4 BS 10	4 BS 13	6 BL 16	6 H 30	6 H 41	8 H 58	8 H 67	10 H 100
3 I	5 I	5 BS	6 H 23	8 H 35	8 H 48	10 H 72	10 H 89	10 H 120
4 C	6 I	6 H 20	6 H 27	8 H 40	10 H 60	10 H 77	12 H 92	14 H 136
4 I	6 BL 12	8 BL 15	8 G 24	10 G 41	10 H 66	12 H 85	12 H 99	14 H 142
4 BS 7.5	6 BS 18	8 B 21	8 G 27	10 G 45	12 G 64	14 G 74	12 H 106	14 H 150
5 C	6 B 10	10 I	8 H 31	10 H 49	12 H 72	14 H 84	14 H 119	24 G 160
6 C	7 B 12	10 BL 17	8 H 33	10 H 54	12 H 79	14 H 103	14 H 127	27 G 177
6 BS 15.5	7 C	10 B 23	10 BL 19	12 I 40.8	14 G 53	14 H 111	16 G 114	30 G 190
6 BJ	7 I	12 BL 19	10 B 26	12 I 45	14 G 58	16 G 78	18 G 114	30 G 200
8 BJ	8 C	12 B 28	10 B 29	12 I 50	14 G 61	16 G 96	18 G 124	33 G 210
10 BJ	8 I	14 B 30	10 G 33	12 I 55	14 G 68	16 G 105	21 B 96	33 G 220
12 BJ	8 BL 13	16 B 36	10 G 37	12 B 36	14 G 78	18 B 77	21 B 103	36 B 170
	8 B 17		12 I 31.8	12 G 40	14 H 87	18 B 85	21 G 132	36 B 182
	8 B 19		12 I 35	12 G 45	14 H 95	18 G 105	21 G 142	36 G 230
	9 C		12 BL 22	12 G 53	16 G 64	21 B 89	24 G 150	**1⅛**
	10 C		12 B 32	12 G 58	16 G 71	21 G 122	27 G 154	10 H 112
	10 BL 15		14 B 34	12 H 65	16 G 88	24 I 105.9	27 G 163	12 H 133
	10 B 21		14 B 38	14 B 42	18 B 70	24 I 110	30 B 132	14 H 158
	12 C		15 C	14 G 43	18 G 96	24 I 115	30 G 172	14 H 167
	12 BL 16.5		15 I 42.9	14 G 48	20 I 81.4	24 I 120	30 G 180	30 G 210
	12 B 25		15 I 45	15 I 60.8	20 I 85	24 B 94	33 B 141	36 B 194
			15 I 50	15 I 65	20 I 90	24 G 110	33 B 152	36 G 240
			15 I 55	15 I 70	20 I 95	24 G 120	33 G 200	36 G 250
			16 B 40	15 I 75	20 I 100	24 G 130	36 B 160	**1¼**
			18 B 47	16 B 45	21 B 73	24 G 140		10 H 124
				16 B 50	21 B 82	27 B 106		12 H 147
				16 B 58	21 G 112	27 B 114		14 H 176
				18 C	24 I 79.9	27 G 145		14 H 184
				18 I	24 I 85	30 B 116		33 G 240
				18 B 50	24 I 90	30 B 124		36 G 260
				18 B 55	24 I 95	30 B 132		**1⅜**
				18 B 64	24 I 100	36 B 150		10 H 136
				20 I 65.4	24 B 80			12 H 161
				20 I 70	24 B 87			14 H 193
				20 I 75	24 G 100			14 H 202
				21 B 59	27 B 98			36 G 280
				21 B 63	30 B 108			**1½**
				21 B 68	33 B 125			12 H 176
				24 B 74				14 H 211
				27 B 91				36 G 300

For 14 H Sections with flanges thicker than 1½" See Bethlehem Manual.

PLATE E12

APPENDIX F

LOADS

F-1. Live Loads on Railway Bridges. Both floor systems and trusses of railway bridges are usually designed for the stresses produced by the wheel load concentrations of one or two locomotives followed by a uniform load to represent the weight of the train. The axial loads and spacings of actual locomotives, which are of many types and weights, are generally replaced by simplified standard loadings essentially equivalent to present and future expected concentrations. The 1935 Specifications for Steel Railway Bridges of the American Railway Engineering Association require the use of two 255.6-ton locomotives, with a load of 72,000 lb. on a driving axle, followed by a uniform load of 7200 lb. per foot of track (Cooper *E*-72 loading), or two axle loads of 90,000 lb. each, placed 7 ft. apart, whichever gives the larger stress. In 1894 Theodore Cooper proposed a series of locomotive and train loadings of uniform axle spacing but of varying weight, the heaviest of which equaled $\frac{40}{72}$ of that given above, and this standard is known by his name, the numerical part of the designations indicating the weight on a driving axle, the letter *E* meaning engine. Loads have so increased that a Cooper *E*-90 loading is sometimes used.

In Table 1 is given a moment diagram to facilitate computations of stresses for the Cooper *E*-10. The stresses due to any other Cooper loading may be found by multiplying the *E*-10 stress by the ratio of the axle loads, the effect of an *E*-70 being $\frac{70}{10}$ that of an *E*-10. Any moment recorded above the stepped line is that about the designated wheel of the loads lying to the left of the designated wheel up to and including that one lying on the vertical forming the left division of the column in which the value appears. The moments recorded below the stepped line are those of the loads lying to the right of the designated wheel. These values are used when the engine is brought onto the span from the left. In this case the uniform load is usually placed at the left. However, uniform load may be used on both sides of the locomotives if desired as this condition often occurs in railway operation.

Table 2 shows the wheel which is to be placed at the apex of the influence line for maximum effect if the influence line is a single triangle. Also, with an influence line like that for stress in a diagonal of one of the panels of a parallel chord truss, the table indicates the wheel to place for maximum at the junction of two segments of a three-segment influence line, *provided* that, when so placed, no loads are located on the third segment.

In Table 3 the fourth column gives the total load coming to a floor beam which lies between two stringers of the length shown in the first column.

Tables 4 and 5 may be used for through plate girder spans although labeled for truss bridges.

TABLE 1

COOPER E-10 ENGINE LOADING

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
Axle loads		5.0 k.	10.0	10.0	10.0	10.0	6.5	6.5	6.5	6.5	5.0	10.0	10.0	10.0	10.0	6.5	6.5	6.5	6.5	1.0 k. per lin. ft. uniform load
Wheel Numbers		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
Spacing in feet		8	5	5	5	9	5	6	5	8	8	5	5	5	9	5	6	5	5	
Summations	Kips	5.0	15.0	25.0	35.0	45.0	51.5	58.0	64.5	71.0	76.0	86.0	96.0	106.0	116.0	122.5	129.0	135.5	142.0	142.0
	Feet	0	8	13	18	23	32	37	43	48	56	64	69	74	79	88	93	99	104	109
	Kips	142.0	137.0	127.0	117.0	107.0	97.0	90.5	84.0	77.5	71.0	66.0	56.0	46.0	36.0	26.0	19.5	13.0	6.5	0
	Feet	109	101	96	91	86	77	72	66	61	53	45	40	35	30	21	16	10	5	0
Moments of Wheel Loads About	End of Train	8182.0	7637.0	6627.0	5667.0	4757.0	3897.0	3396.5	2928.5	2499.5	2103.0	1838.0	1388.0	988.0	638.0	338.0	201.5	97.5	32.5	
Wheel Number	18	7472.0	6952.0	5992.0	5082.1	4222.0	3412.0	2944.0	2508.5	2112.0	1748.0	1508.0	1108.0	758.0	458.0	208.0	104.2	32.5		
	17	6794.5	6299.5	5389.2	4529.5	3719.5	2959.5	2524.0	2121.0	1757.0	1425.5	1210.5	860.5	560.5	310.5	110.5	39.0	32.5		
	16	6020.5	5555.6	4705.5	3905.5	3155.6	2455.5	2059.0	1695.0	1370.0	1077.5	892.5	602.5	362.5	172.5	32.5	39.0	110.5		
	15	5408.0	4968.0	4168.0	3418.0	2718.0	2068.0	1704.0	1372.5	1080.0	820.0	660.0	420.0	230.0	90.0	32.5	104.0	208.0		
	14	4364.0	3969.0	3259.0	2599.0	1989.0	1429.0	1123.5	850.5	616.5	415.0	300.0	150.0	50.0	58.5	149.5	279.5	442.0		
	13	3884.0	3464.0	2804.0	2194.0	1634.0	1124.0	851.0	610.5	409.0	240.0	150.0	50.0	50.0	141.0	264.5	427.0	622.0		
	12	3354.0	3009.0	2399.0	1839.0	1329.0	869.0	628.5	420.5	251.5	115.0	50.0	50.0	150.0	273.5	429.5	624.5	852.0		
	11	2924.0	2604.0	2044.0	1534.0	1074.0	664.0	456.0	280.5	144.0	40.0	50.0	150.0	300.0	456.0	644.5	872.0	1132.0		
	10	2316.0	2036.0	1556.0	1126.0	746.0	416.0	260.0	136.5	52.0	80.0	210.0	390.0	620.0	828.0	1068.5	1348.0	1660.0		
	9	1748.0	1508.0	1108.0	758.0	458.0	208.0	104.0	32.5	40.0	200.0	410.0	670.0	980.0	1240.0	1582.5	1864.0	2228.0		
	8	1425.5	1210.5	860.5	560.5	310.5	110.5	39.0	82.5	97.5	307.5	567.5	877.5	1237.5	1580.0	1855.0	2219.0	2615.5		
	7	1077.5	892.5	602.5	362.5	172.5	32.5	39.0	110.5	205.5	475.5	795.5	1165.5	1585.5	1917.0	2281.0	2684.0	3119.5		
	6	820.0	660.0	420.0	230.0	90.0	32.5	104.0	208.0	328.0	648.0	1018.0	1438.0	1908.0	2272.0	2668.5	3104.0	3672.0		
	5	415.0	300.0	150.0	50.0	58.5	149.5	279.5	442.0	607.0	1017.0	1477.0	1987.0	2547.0	2969.5	3424.5	3918.5	4445.0		
	4	240.0	150.0	50.0	50.0	141.0	264.5	427.0	622.0	812.0	1272.0	1782.0	2342.0	2952.0	3407.0	3894.5	4421.0	4980.0		
	3	115.0	50.0	50.0	150.0	273.5	429.5	624.5	852.0	1067.0	1577.0	2137.0	2747.0	3407.0	3894.5	4414.4	4973.5	5565.0		
	2	40.0	50.0	150.0	300.0	456.0	644.5	872.0	1132.0	1372.0	1932.0	2542.0	3202.0	3912.0	4432.0	4984.5	5576.0	6200.0		
	1	80.0	210.0	390.0	620.0	828.0	1068.5	1348.0	1660.0	1940.0	2580.0	3270.0	4010.0	4800.0	5372.0	5976.5	6620.0	7296.0		

TABLE 2

Wheel Giving Maximum Stress

Segments in feet	10	15	20	25	30	35	40	45	50	55	60	65	70	80	90	100	110	120	130	140
500 to 260	2	3	3	4	4	5	5	6	7	7	8	9	10	11	12	13	14	15	17	18
250 to 200	2	3	3	4	4	5	5	6	7	8	8	9	10	11	12	13	14	15	17	18
190 to 150	2	3	3	4	4	5	5	6	7	8	9	9	11	12	12	13	14	15	17	18
140	3	3	3	4	4	5	5	6	7	8	9	10	11	12	12	13	14	15	17	18
130	3	3	3	4	4	5	5	6	7	8	9	10	11	12	12	13	14	15	17	
120	3	3	3	4	4	5	6	6	7	8	9	10	11	12	13	13	14	15		
110	3	3	3	4	4	5	6	7	7	8	9	10	11	12	13	13	14			
100	3	3	3	4	5	5	$\overline{14}$	$\overline{14}$	$\overline{14}$	$\overline{14}$	$\overline{13}$	$\overline{13}$	11	12	13	13				
90	3	3	4	4	5	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	12	13					
80	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	12						
70	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	11							
65	3	3	4	4	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	11								
60	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	11									
55	12	12	12	4	12	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{13}$	11										
50	12	12	12	12	12	$\overline{13}$	$\overline{13}$	$\overline{13}$	12											
45	12	12	12	12	12	$\overline{13}$	$\overline{13}$	13												
40	3	3	3	12	12	$\overline{13}$	13													
35	3	3	4	4	13	13														
30	3	3	4	4	13															
25	3	3	4	4																
20	$\overline{4}$	3	4																	
15	3	3																		
10	3																			

The shorter segment is ahead followed by the longer one except where the wheel is overlined

Cooper E Engine

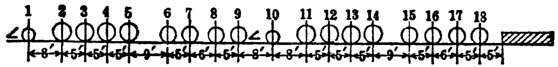


TABLE 3

Maximum Moments, Shears, and Reactions, Cooper E-10 Engine Loading, One Track

Span in feet	Maximum moment, in foot-kips	Maximum shear, in kips	Maximum floor beam load, in kips	Equivalent Uniform Load			Span in feet	Maximum moment, in foot-kips	Maximum shear, in kips	Maximum floor beam load, in kips	Equivalent Uniform Load		
				Moment	Shear	Floor beam					Moment	Shear	Floor beam
7	21.9	12.5	15.7	3.57	3.57	2.24	42	356.7	39.2	56.0	1.62	1.87	1.34
7½	23.5	13.4	16.7	3.33	3.56	2.23	44	385.8	40.3	58.2	1.60	1.83	1.32
8	25.0	14.0	17.5	3.12	3.51	2.19	46	414.9	41.4	60.3	1.57	1.80	1.31
9	28.1	15.3	18.9	2.78	3.39	2.10	48	443.8	42.4	62.4	1.54	1.77	1.30
10	31.2	16.2	20.0	2.50	3.25	2.00	50	475.5	43.5	64.3	1.52	1.74	1.29
11	34.4	17.0	21.8	2.28	3.09	1.98	52	507.6	44.6	66.7	1.50	1.72	1.28
12	40.0	17.7	23.3	2.22	2.95	1.94	54	540.5	45.6	69.0	1.48	1.69	1.28
13	47.5	18.5	24.6	2.25	2.85	1.90	56	576.1	46.5	71.4	1.47	1.66	1.27
14	55.0	19.3	26.1	2.25	2.76	1.86	58	611.6	47.7	74.0	1.46	1.65	1.28
15	62.5	20.0	27.3	2.22	2.67	1.82	60	649.5	48.8	76.6	1.44	1.63	1.28
16	70.0	21.3	28.5	2.19	2.66	1.78	62	688.2	50.0	79.1	1.43	1.61	1.27
17	77.5	22.4	29.4	2.15	2.63	1.73	64	727.7	51.3	81.5	1.42	1.60	1.27
18	85.0	23.3	30.3	2.10	2.59	1.69	66	769.7	52.5	83.9	1.41	1.59	1.27
19	93.3	24.2	31.5	2.07	2.55	1.66	68	811.7	53.9	86.2	1.40	1.59	1.27
20	103.1	25.0	32.8	2.06	2.50	1.64	70	853.7	55.3	88.5	1.39	1.58	1.26
21	112.9	25.7	34.0	2.05	2.45	1.62	72	896.7	56.7	90.7	1.38	1.58	1.26
22	122.8	26.3	35.1	2.03	2.40	1.60	74	939.0	58.1	93.0	1.37	1.57	1.26
23	132.7	27.0	36.1	2.01	2.34	1.57	76	986.0	59.5	95.2	1.36	1.57	1.25
24	142.6	27.7	37.0	1.98	2.31	1.54	78	1 032.7	60.9	97.3	1.36	1.56	1.25
25	152.5	28.4	37.8	1.95	2.27	1.51	80	1 080.0	62.1	99.4	1.35	1.55	1.24
26	162.4	29.1	38.8	1.92	2.24	1.49	82	1 128.3	63.5	101.5	1.34	1.55	1.24
27	172.3	29.6	40.0	1.89	2.20	1.48	84	1 177.7	64.8	103.5	1.34	1.54	1.23
28	182.7	30.2	41.2	1.86	2.16	1.47	86	1 229.7	66.1	105.4	1.33	1.54	1.23
29	194.0	30.8	42.2	1.84	2.12	1.46	88	1 282.0	67.4	107.3	1.32	1.53	1.22
30	205.2	31.5	43.1	1.82	2.10	1.44	90	1 334.7	68.6	109.3	1.32	1.53	1.22
31	216.5	32.2	44.3	1.80	2.08	1.43	92	1 388.3	69.9	111.2	1.31	1.52	1.21
32	227.7	32.9	45.5	1.78	2.05	1.42	94	1 442.7	71.2	113.1	1.31	1.52	1.20
33	239.0	33.5	46.7	1.75	2.03	1.41	96	1 497.3	72.4	115.0	1.30	1.51	1.20
34	250.3	34.1	47.8	1.73	2.00	1.40	98	1 552.7	73.7	116.8	1.29	1.51	1.19
35	261.5	34.6	48.8	1.71	1.98	1.39	100	1 609.7	75.0	118.6	1.29	1.50	1.19
36	274.3	35.3	49.8	1.69	1.96	1.38	125	2 497.7	89.7	140.5	1.28	1.44	1.12
37	287.2	35.9	50.7	1.68	1.94	1.37	150	3 531.0	103.7	162.7	1.25	1.38	1.08
38	300.0	36.5	51.8	1.66	1.92	1.36	175	4 676.3	117.3	185.8	1.22	1.34	1.06
39	313.3	37.2	52.9	1.65	1.90	1.36	200	5 939.0	130.5	209.5	1.19	1.31	1.05
40	327.8	37.7	54.0	1.64	1.88	1.35	250	8 796.3	156.6	257.6	1.13	1.25	1.03

TABLE 4

MAXIMUM MOMENTS, IN FOOT-KIPS, FOR TRUSS BRIDGES, COOPER E-10 ENGINE LOADING, ONE TRACK

Number of panels	Panel points	Panel Lengths										
		10′ 0″	12′ 0″	15′ 0″	18′ 0″	20′ 0″	22′ 0″	25′ 0″	28′ 0″	30′ 0″	35′ 0″	40′ 0″
3	1/3	186	248	360	491	586	....	....	....	....	....	....
4	1/4	253	344	502	684	824	996	1 265	1 566	1 774	....	....
	2/4	328	444	648	896	1 080	1 282	1 610	1 999	2 298	....	....
5	1/5	318	428	632	897	1 092	1 311	1 660	2 037	2 304	3 047	....
	2/5	460	630	924	1 285	1 577	1 907	2 437	2 997	3 393	4 477	....
6	1/6	372	512	786	1 108	1 345	1 603	2 024	2.483	2 810	3 715	4 730
	2/6	586	799	1 196	1 723	2 104	2 500	3 118	3 832	4 352	5 804	7 454
	3/6	647	896	1 335	1 862	2 298	2 768	3 528	4 345	4 918	6 459	8 164
7	1/7	432	612	933	1 307	1 582	1 881	2 374	2 914	3 300	4 368	5 572
	2/7	703	976	1 504	2 113	2 530	3 012	3 812	4 697	5 332	7 103	9 136
	3/7	834	1 159	1 761	2 515	3 058	3 632	4 578	5 645	6 394	8 429	10 700
8	1/8	502	706	1 072	1 496	1 810	2 149	2 715	3 335	3 780	5 010	6 404
	2/8	824	1 173	1 774	2 472	2 983	3 556	4 498	5 540	6 292	8 384	10 798
	3/8	1 012	1 425	2 213	3 080	3 694	4 397	5 604	6 914	7 840	10 368	13 201
	4/8	1 080	1 497	2 299	3 265	3 977	4 722	5 928	7 229	8 162	10 760	13 744
9	1/9	567	799	1 205	1 679	2 032	2 412	3 049	3 750	4 253	5 646	7 228
	2/9	962	1 360	2 043	2 843	3 431	4 086	5 170	6 372	7 240	9 658	12 447
	3/9	1 194	1 723	2 602	3 584	4 352	5 191	6 611	8 162	9 267	12 284	15 681
	4/9	1 317	1 870	2 882	4 012	4 869	5 789	7 282	8 904	10 068	13 316	17 055
10	1/10	632	885	1 333	1 858	2 249	2 672	3 379	4 160	4 709	6 277	8 048
	2/10	1 093	1 541	2 304	3 205	3 871	4 608	5 833	7 195	8 180	10 920	14 085
	3/10	1 400	1 993	2 946	4 125	5 008	5 966	7 603	9 397	10 678	14 183	18 142
	4/10	1 577	2 256	3 393	4 721	5 743	6 837	8 616	10 556	11 952	15 850	20 340
	5/10	1 610	2 298	3 528	4 918	5 928	7 007	8 780	10 760	12 200	16 263	20 950

TABLE 5

MAXIMUM SHEARS, IN KIPS, FOR TRUSS BRIDGES, COOPER E–10 ENGINE LOADING, ONE TRACK

Number of panels	Panel point	Panel Lengths										
		10′ 0″	12′ 0″	15′ 0″	18′ 0″	20′ 0″	22′ 0″	25′ 0″	28′ 0″	30′ 0″	35′ 0″	40′ 0″
3	$\frac{1}{3}$	18.6	20.6	24.0	27.3	....	....	....	....	....	....	....
	$\frac{2}{3}$	4.0	....	....	....	....	....	....	....	....	....	....
4	$\frac{1}{4}$	25.2	28.6	33.4	37.8	41.2	45.3	50.6	55.7	59.2	....	....
	$\frac{2}{4}$	12.0	13.8	16.2	18.9	20.5	22.0	23.9	25.8	27.3	....	....
	$\frac{3}{4}$	2.0	3.2	4.3	5.4	6.0	6.5	7.1	7.7	8.1	....	....
5	$\frac{1}{5}$	31.8	35.7	42.2	49.8	54.6	59.6	66.4	72.8	76.8	87.0	....
	$\frac{2}{5}$	18.7	21.5	25.4	29.0	31.6	34.4	38.6	42.8	45.6	51.8	....
	$\frac{3}{5}$	8.8	10.4	12.5	14.3	15.5	16.6	18.2	19.9	21.1	24.3	....
6	$\frac{1}{6}$	37.4	42.7	52.4	61.5	67.2	72.9	81.9	88.7	93.7	106.2	118.2
	$\frac{2}{6}$	25.2	28.8	33.8	40.4	44.4	48.3	53.8	59.2	62.7	71.0	78.8
	$\frac{3}{6}$	15.0	17.4	20.7	23.4	25.4	27.9	31.5	34.8	37.0	42.0	47.0
	$\frac{4}{6}$	6.6	8.1	9.9	11.6	12.6	13.6	14.8	16.0	16.9	19.7	22.2
7	$\frac{1}{7}$	43.2	51.0	62.2	72.6	79.1	85.5	95.0	104.1	110.0	124.8	139.3
	$\frac{2}{7}$	31.3	35.6	43.8	51.8	56.8	61.5	68.1	74.8	79.2	89.6	99.9
	$\frac{3}{7}$	21.0	23.8	28.4	33.7	37.2	40.6	45.4	49.9	52.7	59.8	66.7
	$\frac{4}{7}$	12.2	14.4	17.4	19.8	21.4	23.2	26.3	29.2	31.1	35.5	39.5
	$\frac{5}{7}$	....	6.4	8.1	9.6	10.5	11.4	12.5	13.5	14.2	16.4	18.6
8	$\frac{1}{8}$	50.2	58.9	71.5	83.1	90.5	97.7	108.6	119.2	126.0	143.2	160.1
	$\frac{2}{8}$	37.0	43.4	53.5	62.8	68.5	74.0	82.0	90.0	95.3	108.0	120.6
	$\frac{3}{8}$	26.9	30.7	37.6	44.6	49.0	53.2	59.0	64.7	68.3	77.6	86.5
	$\frac{4}{8}$	17.8	20.8	24.6	28.9	31.9	34.9	39.2	42.2	45.6	51.8	57.6
	$\frac{5}{8}$	10.2	12.2	14.8	17.0	18.4	20.0	22.6	25.0	26.8	30.6	34.2
	$\frac{6}{8}$	....	5.2	6.8	8.1	9.0	9.8	10.8	11.6	12.3	14.0	15.9
9	$\frac{1}{9}$	56.8	66.6	80.3	93.3	101.6	109.6	122.0	134.0	141.8	161.3	180.7
	$\frac{2}{9}$	43.3	51.3	62.7	73.2	79.8	86.2	95.5	105.0	111.1	126.2	141.1
	$\frac{3}{9}$	32.5	38.2	46.7	55.1	60.2	65.2	72.3	79.2	83.8	95.2	106.3
	$\frac{4}{9}$	23.4	27.0	33.1	39.0	42.9	46.6	52.0	57.0	60.2	68.2	76.2
	$\frac{5}{9}$	15.4	18.1	21.5	25.4	28.0	30.6	34.3	37.9	40.2	45.6	50.8
	$\frac{6}{9}$	....	10.5	12.9	14.9	16.2	17.6	19.8	22.0	23.4	26.8	30.1
	$\frac{7}{9}$	....	....	....	....	....	....	....	...	....	....	14.0
10	$\frac{1}{10}$	63.2	73.8	88.9	103.3	112.4	121.5	135.2	148.6	157.4	179.3	201.2
	$\frac{2}{10}$	50.0	58.9	71.5	83.3	90.8	98.2	108.8	119.7	126.8	144.2	161.6
	$\frac{3}{10}$	38.6	45.8	55.7	65.2	71.2	77.1	85.5	93.7	99.2	112.9	126.2
	$\frac{4}{10}$	28.7	34.0	41.8	48.9	53.6	58.2	64.6	70.9	75.0	85.0	95.1
	$\frac{5}{10}$	20.7	23.9	29.5	34.8	38.2	41.4	46.2	50.9	53.9	61.1	68.1
	$\frac{6}{10}$	13.5	16.0	19.1	22.7	25.0	27.3	30.6	33.7	35.8	40.7	45.4
	$\frac{7}{10}$	7.4	9.1	11.3	13.3	14.4	15.6	17.7	19.7	20.9	23.9	26.8
	$\frac{8}{10}$	3.1	3.8	4.9	6.1	6.7	7.4	8.3	9.0	9.6	11.0	12.5

F-2. Roof Loads. The external forces which may act upon a roof include: (*a*) a uniform live load of from 20 to 30 lb. per sq. ft. on flat roofs, to cover the effect of workmen and other human occupancy of the area; (*b*) the weight of snow and ice, in amount varying with climate and the steepness of the roof surface; and (*c*) wind, usually assumed a uniform normal pressure over the whole exposed surface.

The following table is reproduced by permission from KIDDER's Architects' and Builders' Pocket Book (page 1052, 17th Ed.).

ALLOWANCE FOR SNOW IN POUNDS PER SQUARE FOOT OF ROOF SURFACE

Location	Pitch of Roof[1]							
	$\frac{1}{2}$		$\frac{1}{3}$		$\frac{1}{4}$		$\frac{1}{5}$	$\frac{1}{6}$ or less
	*	†	*	†	*	†		
Southern States and Pacific Slope	0	0	0	5	0	5	5	5
Central States	0	5	7	10	15	20	22	30
Rocky Mountain States	0	10	10	15	20	25	27	35
New England States	0	10	10	15	20	25	35	40
Northwest States	0	12	12	18	25	30	37	45

Columns headed by an asterisk (*) are for slate, tile, or metal; those headed by a dagger (†) are for shingles.

When snow guards are placed on a roof the same allowance is made for half-pitch as for a one-third pitch.

The pressure of wind on a vertical normal surface is specified by building codes usually at values of 30 lb. per sq. ft. or less. As would be expected the pressure on sloping surfaces is less than on a vertical. This reduced normal pressure is usually computed by the following empirical formula of DUCHEMIN (1829),

$$P_n = P\frac{2\sin\alpha}{1+\sin^2\alpha}$$

where P = the intensity on a vertical surface, normal to the wind, P_n = intensity of normal pressure on a sloping surface with horizontal trace perpendicular to the wind, α = angle of the surface with the horizontal. A simpler formula, much used, is $P_n = P\alpha^\circ/45^\circ$.

Aeronautical research has shown that the actual wind pressure on a structure[2] is quite different from that assumed, principally in the presence of negative pressure on the leeward side.

In addition to the dead load these combinations of live load may be considered for long-span roofs: snow over all, snow on one side and wind on the other, wind on one side, wind on one side and ice (sleet) over the

[1] The pitch of a roof is, for the usual symmetric roof truss, the ratio of the rise to the span. For example, a truss of 60 feet span with peak 15 feet above the horizontal bottom chord is a 1/4 pitch truss: similarly, a truss or roof with a 45° slope is 1/2 pitch.

[2] "What Aerodynamics Can Teach the Civil Engineer," W. W. PAGON, *Engineering News-Record*, March 15, 1934.

whole roof. For moderate-span roofs it is customary to replace all these load combinations with a single uniformly distributed load of sufficient intensity to give stresses equal to the maximum found by using the more precise loading. The following table gives recommended minimum values of this combined load for spans up to 100 ft.

ALLOWANCE FOR WIND AND SNOW COMBINED IN POUNDS PER SQUARE FOOT OF ROOF SURFACE

Location	Pitch of Roof					
	60°	45°	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
Northwest States	30	30	25	30	37	45
New England States	30	30	25	25	35	40
Rocky Mountain States	30	30	25	25	27	35
Central States	30	30	25	25	22	30
Southern and Pacific States	30	30	25	25	22	20

From Kidder's Architects' and Builders' Pocket Book, p. 1052, 17th Edition.

The following tables give typical weights of roof trusses for use in preliminary design and estimate.

WEIGHTS OF WOODEN TRUSSES PER SQUARE FOOT OF ROOF SURFACE*

Span	$\frac{1}{2}$ Pitch	$\frac{1}{3}$ Pitch	$\frac{1}{4}$ Pitch	Flat
Ft.	Lb.	Lb.	Lb.	Lb.
Up to 36	3	$3\frac{1}{2}$	$3\frac{3}{4}$	4
36 to 50	$3\frac{1}{4}$	$3\frac{3}{4}$	4	$4\frac{1}{2}$
50 to 60	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$
60 to 70	$3\frac{3}{4}$	$4\frac{1}{2}$	$4\frac{3}{4}$	$5\frac{1}{4}$
70 to 80	$4\frac{1}{4}$	5	$5\frac{1}{2}$	6
80 to 90	5	6	$6\frac{1}{2}$	7
90 to 100	$5\frac{3}{4}$	$6\frac{3}{4}$	7	8
100 to 110	$6\frac{1}{2}$	$7\frac{1}{2}$	8	9
110 to 120	7	$8\frac{1}{2}$	9	10

* For scissors trusses, increase one-third.

From Kidder's Architects' and Builders' Pocket Book, p. 1051, 17th Edition.

APPENDIX G

EXCERPT FROM "SIMPLIFICATION OF GRADING RULES AND CLASSIFICATION OF TIMBER FOR RAILWAY USES." *Proceedings*, AMERICAN RAILWAY ENGINEERING ASSOCIATION, Volume 37

GRADING STRUCTURAL TIMBERS ON THE BASIS OF UNIT WORKING STRESSES — 1936

FOREWORD

1. The following specifications for stress-grades of structural timber conform to the principles of strength-grading as presented in U. S. Department of Agriculture Miscellaneous Publication 185, "Guide to the Grading of Structural Timbers and the Determination of Working Stresses," and the stress-grades are in conformity with tests of the Forest Products Laboratory.

2. These specifications represent complete grading rules, capable of use in both engineering design and timber purchase, and incorporate all factors affecting the strength and utility of structural timbers and the method of grading them.

3. Existing commercial grading rules of the regional lumber manufacturers' associations are in conformity generally with the structural grading hereinafter presented, and provide grades of equal or higher working stresses. The following specifications may, therefore, be used for mill orders, or for selection or appraisal of stock on hand in either manufacturers', distributors' or users' storage yards.

4. The designations 1800 lb f, 1200 lb c, etc., by which the various stress-grades are identified in these specifications represent safe unit working stresses, applicable to material when used in locations continuously dry or covered: (*a*) for extreme fiber in bending (f = fiber stress) in the case of joist, plank, beams, and stringers, or for longitudinal tension, for which the same stresses as in bending may be used, with grading modified as provided below; and (*b*) for compression parallel to grain (c = compression stress) for short columns in the case of posts and timbers. These stresses may be used without allowance for impact up to impact of 100 per cent of loads figured. Grade limitations applicable to the center portion of pieces in bending should be applied throughout the entire length of pieces subject to longitudinal tension. The stresses for joist and plank assumed that strength increased by seasoning, and were increased accordingly; hence the stresses in extreme fiber should be reduced as provided below when the joist and plank are used where they will not be continuously dry. For timber continuously submerged, except joist and plank, the stresses for use continuously dry or covered apply except in compression perpendicular to grain,

which should not exceed 70 per cent of those stresses since they have been increased for surface drying and consequent increase in hardness and bearing value. For joist and plank continuously submerged, stresses determined from the basic stresses without increase in strength from seasoning should be used, except in compression perpendicular to grain, which should be taken at 70 per cent of those stresses.

5. The amount by which the stresses for all structural grades and items should be reduced, or the size of the timbers increased, to adapt them to exposed conditions of use, is dependent upon the extent to which the exposure favors decay, required life of the structure or part, frequency and thoroughness of inspection, original cost and cost of replacements, proportion of sapwood and durability of heartwood of the species if untreated, and character and efficiency of the treatment if treated. This is left to the judgment of the engineer. The following percentages of the stress-grade values continuously dry or covered (100 per cent), recommended by the Forest Products Laboratory, may be used as a guide:

Kind of Stress for Treated or Untreated Material	*Occasionally Wet but Quickly Dried Per Cent*
Extreme fiber in bending	87½
Compression perpendicular to grain	70
Compression parallel to grain	92
Horizontal shear	100
Modulus of elasticity	100

When joist and plank are used in exposures " Occasionally wet but quickly dried," the above percentages apply to the original values not increased for seasoning.

6. In locations of more extreme exposure than " Occasionally wet but quickly dried " and where serious depreciation is more likely to occur, a further reduction in the working stresses for extreme fiber and compression may be made, in the judgment of the engineer.

PHYSICAL REQUIREMENTS

General Quality. 2. Except as hereinafter provided, all structural timber shall be free from defects which may impair strength or durability, such as decay, wane, cross grain, holes, knots, shakes, checks or split, any of which is greater in extent or size than provided in tables following.[1]

DESIGN

Dimensions. 40. The standard sizes of structural timbers are as shown in following table.

	Joists and Planks	*Beams and Stringers*	*Posts and Timbers*
Nominal thickness.	2″, 3″ and 4″	5″, 6″ and up in multiples of 2″	5″, 6″ and up in multiples of 2″

[1] Specifications from which this matter is taken contain elaborate rules which cannot be reproduced here for want of space. These rules give for all grades of material the permissible limits for shakes, checks, splits, wane, knots, etc.

	Joists and Planks	*Beams and Stringer*	*Posts and Timbers*
Nominal width.....	4″ and up in multiples of 2″	8″ and up in multiples of 2″	5″, 6″ and up in multiples of 2″
Nominal length....	6′, 8′, 9′ and 10′ to 40′ in multiples of 2′	6′, 8′, 9′, and 10′ and up in multiples of 2′	6′, 8′, 9′, and 10′ and up in multiples of 2′

Sizes in fractional or odd feet or inches other than those above are special cutting.

Tolerances and Surfacing Allowances. 41. The actual dimensions of structural timbers when green shall conform to the following tolerances and surfacing allowances.

	Nominal Thickness	Minimum* Rough Thickness	Surfaced Thickness S1S or S2S	Nominal Width	Minimum* Rough Width	Surfaced Width S1S or S2S
Joists, planks and other framing members with load applied to either wide face or narrow face.	2″	1/8″ off	3/8″ off	4″	3/16″ off	3/8″ off
	3″	3/16″ off	do	6″	do	do
	4″	do	do	8″ and wider	1/4″ off	1/2″ off
Beams, stringers and other heavy members with load applied to narrow face.	5″, 6″	do	1/2″ off		do	do
	8″ and thicker	1/4″ off	do	8″ and wider		
Posts, timbers and other members to carry longitudinal loads.	5″, 6″	3/16″ off	do	5″, 6″	3/16″ off	do
	8″ and thicker	1/4″ off	do	8″ and wider	1/4″ off	do

* Permissible in not to exceed 20 per cent of the pieces in any one shipment.

JOIST AND PLANK
STANDARD STRESS-GRADES AND WORKING STRESSES

Grades and Species	Fiber Stress in Bending or Tension: Continuously Dry	Fiber Stress in Bending or Tension: *Continuously Submerged	Maximum Horizontal Shear	Compression Perpendicular to Grain	Modulus of Elasticity
Softwoods					
1800 lb. f. Dense Douglas Fir (Coast and Inland)	1800	1590	120	380	1,600,000
1800 lb. f. Dense Longleaf and Shortleaf Southern Pine	1800	1590	120	380	1,600,000
1600 lb. f. Close-Grained Douglas Fir (Coast)	1600	1420	100	345	1,600,000
1600 lb. f. Dense Longleaf or Shortleaf Southern Pine	1600	1455	120	380	1,600,000
1600 lb. f. Close-Grained Redwood	1600	1350	80	267	1,200,000
1400 lb. f. Southern Cypress	1400	1220	120	300	1,200,000
1400 lb. f. Dense Longleaf Southern Pine	1400	1320	100	380	1,600,000
1400 lb. f. Close-Grained Redwood	1400	1220	80	267	1,200,000
1200 lb. f. Port Orford Cedar	1200	1045	100	250	1,200,000
1200 lb. f. Douglas Fir (Coast)	1200	1135	100	325	1,600,000
1200 lb. f. Dense Shortleaf Southern Pine	1200	1190	100	380	1,600,000
1200 lb. f. Close-Grained Redwood	1200	1085	70	267	1,200,000
1100 lb. f. Port Orford Cedar	1100	980	80	250	1,200,000
1100 lb. f. Southern Cypress	1100	1020	100	300	1,200,000
1100 lb. f. Eastern Hemlock	1100	980	70	300	1,100,000
1000 lb. f. Western Red Cedar	1000	865	100	200	1,000,000
Hardwoods					
1800 lb. f. White Ash, Red and White Oak	1800	1510	120	500	1,500,000
1800 lb. f. Beech, Birch, Hard Maple	1800	1535	120	500	1,600,000
1800 lb. f. Rock Elm	1800	1535	120	500	1,300,000
1600 lb. f. White Ash, Red and White Oak	1600	1380	120	500	1,500,000
1600 lb. f. Beech, Birch, Hard Maple	1600	1400	120	500	1,600,000
1600 lb. f. Rock Elm	1600	1400	120	500	1,300,000
1400 lb. f. White Ash, Red and White Oak	1400	1245	120	500	1,500,000
1400 lb. f. Beech, Birch, Hard Maple	1400	1265	120	500	1,600,000
1400 lb. f. Rock Elm	1400	1265	120	500	1,300,000
1400 lb. f. Soft Elm	1400	1175	100	250	1,200,000
1400 lb. f. Black and Red Gum, Tupelo	1400	1175	100	300	1,200,000
1200 lb. f. White Ash, Red and White Oak	1200	1110	100	500	1,500,000
1200 lb. f. Beech, Birch, Hard Maple	1200	1135	100	500	1,600,000
1200 lb. f. Chestnut	1200	1010	100	300	1,000,000
1200 lb. f. Rock Elm	1200	1135	100	500	1,300,000
1200 lb. f. Soft Elm	1200	1045	100	250	1,200,000
1200 lb. f. Black and Red Gum, Tupelo	1200	1045	100	300	1,200,000
1000 lb. f. Chestnut	1000	880	100	300	1,000,000
1000 lb. f. Soft Elm	1000	910	100	250	1,200,000
1000 lb. f. Black and Red Gum, Tupelo	1000	910	100	300	1,200,000

* Values under the heading "Continuously Submerged" should be used in application of the percentage of reduction in fiber stress in bending or tension for conditions of exposure which will cause depreciation in strength (paragraph 5, Foreword).

BEAMS AND STRINGERS
STANDARD STRESS–GRADES AND WORKING STRESSES

Grades and Species	Fiber Stress in Bending or Tension	Maximum Horizontal Shear	Compression Perpendicular to Grain	Modulus of Elasticity
Softwoods				
1800 lb. f. Dense Douglas Fir (Coast and Inland)	1800	120	380	1,600,000
1800 lb. f. Dense Longleaf and Shortleaf Southern Pine	1800	120	380	1,600,000
1600 lb. f. Close-Grained Douglas Fir (Coast)	1600	100	345	1,600,000
1600 lb. f. Dense Longleaf and Shortleaf Southern Pine	1600	120	380	1,600,000
1600 lb. f. Close-Grained Redwood	1600	80	267	1,200,000
1400 lb. f. Southern Cypress	1400	120	300	1,200,000
1400 lb. f. Dense Longleaf Southern Pine	1400	100	380	1,600,000
1400 lb. f. Close-Grained Redwood	1400	80	267	1,200,000
1200 lb. f. Dense Shortleaf Southern Pine	1200	100	380	1,600,000
1200 lb. f. Close-Grained Redwood	1200	70	267	1,200,000
1100 lb. f. Port Orford Cedar	1100	80	250	1,200,000
1100 lb. f. Southern Cypress	1100	100	300	1,200,000
1100 lb. f. Eastern Hemlock	1100	70	300	1,100,000
1000 lb. f. Western Red Cedar	1000	100	200	1,000,000
Hardwoods				
1600 lb. f. White Ash, Red and White Oak	1600	120	500	1,500,000
1600 lb. f. Beech, Birch, Hard Maple	1600	120	500	1,600,000
1600 lb. f. Rock Elm	1600	120	500	1,300,000
1400 lb. f. White Ash, Red and White Oak	1400	120	500	1,500,000
1400 lb. f. Beech, Birch, Hard Maple	1400	120	500	1,600,000
1400 lb. f. Rock Elm	1400	120	500	1,300,000
1200 lb. f. White Ash, Red and White Oak	1200	100	500	1,500,000
1200 lb. f. Beech, Birch, Hard Maple	1200	100	500	1,600,000
1200 lb. f. Rock Elm	1200	100	500	1,300,000
1200 lb. f. Soft Elm	1200	100	250	1,200,000
1200 lb. f. Black Gum, Red Gum, Tupelo	1200	100	300	1,200,000
1000 lb. f. Chestnut	1000	100	300	1,000,000
1000 lb. f. Soft Elm	1000	100	250	1,200,000
1000 lb. f. Black Gum, Red Gum, Tupelo	1000	100	300	1,200,000

POST AND TIMBERS

STANDARD STRESS-GRADES AND WORKING STRESSES

Grades and Species	Compression Parallel to Grain, Short Columns	Compression Perpendicular to Grain	Modulus of Elasticity
Softwoods			
1300 lb. c. Dense Douglas Fir (Coast and Inland)	1300	380	1,600,000
1300 lb. c. Dense Longleaf and Shortleaf Southern Pine	1300	380	1,600,000
1200 lb. c. Southern Cypress	1200	300	1,200,000
1200 lb. c. Close-Grained Douglas Fir (Coast)	1200	345	1,600,000
1200 lb. c. Dense Longleaf and Shortleaf Southern Pine	1200	380	1,600,000
1200 lb. c. Close-Grained Redwood	1200	267	1,200,000
1100 lb. c. Douglas Fir (Coast)	1100	325	1,600,000
1100 lb. c. Close-Grained Redwood	1100	267	1,200,000
1000 lb. c. Port Orford Cedar	1000	250	1,200,000
1000 lb. c. Southern Cypress	1000	300	1,200,000
1000 lb. c. Dense Longleaf Southern Pine	1000	380	1,600,000
1000 lb. c. Close-Grained Redwood	1000	267	1,200,000
900 lb. c. Port Orford Cedar	900	250	1,200,000
900 lb. c. Dense Shortleaf Southern Pine	900	380	1,600,000
800 lb. c. Western Red Cedar	800	200	1,000,000
Hardwoods			
1300 lb. c. Beech, Birch, Hard Maple	1300	500	1,600,000
1300 lb. c. Rock Elm	1300	500	1,300,000
1200 lb. c. White Ash	1200	500	1,500,000
1200 lb. c. Beech, Birch, Hard Maple	1200	500	1,600,000
1200 lb. c. Rock Elm	1200	500	1,300,000
1100 lb. c. Red and White Oak	1100	500	1,500,000
1000 lb. c. White Ash	1000	500	1,500,000
1000 lb. c. Beech, Birch, Hard Maple	1000	500	1,600,000
1000 lb. c. Rock Elm	1000	500	1,300,000
1000 lb. c. Red and White Oak	1000	500	1,500,000
900 lb. c. Chestnut	900	300	1,000,000
900 lb. c. Soft Elm	900	250	1,200,000
900 lb. c. Black Gum, Red Gum, Tupelo	900	300	1,200,000

NOTES ON THE USE OF STRESS-GRADES

1. Working values may be used without allowance for impact up to impact of 100 per cent of loads figured. The ability of timbers to support loads is very dependent on the duration of the stress. Tests have demonstrated that the load required to break timbers in several years is about 9/16 of that required to break them as in ordinary laboratory tests. When the time is shortened still further, as in impact loading, the load required to break a timber is correspondingly increased. Approximately, this increase is 10 per cent when the time is reduced to 1/10 of the previous time. Working stresses for 5-minute loading may be increased 50 per cent over those for long-time loading.

2. Working values for horizontal shear are maximum values. The maximum unit horizontal shear at any point in the length of a beam is usually

calculated as being $\frac{3}{2}$ of the average vertical shearing stress at that point. This assumption gives a very simple formula and, though with checked beams loaded near the supports it is greatly in error, its continued use is recommended with certain assumptions as to the placement of live loads and the neglecting of certain loads near the supports aimed at correcting the most outstanding errors in the formula without complicating the calculations.

3. However, there is given a formula for use in conjunction with the ordinary shear formula to be applied whenever the simple assumptions as to loads and their placement are not sufficiently accurate.

4. The following procedure is recommended for calculating the horizontal shear on the neutral plane in checked beams:

1. Use the following shear formula:

$$s = \frac{3\,R}{2\,bh}$$

where s = maximum unit shear per square inch,
R = reaction in pounds,
b = breadth of beam in inches,
h = height of beam in inches.

2. Use the customary allowable unit shear stress.

3. In calculating the reaction for use in the formula:

(*a*) Take into account any relief to the beam under consideration resulting from load being distributed to adjacent parallel beams by flooring or other members of the construction.
(*b*) Neglect all loads within the height of the beam from both supports.
(*c*) If there are any moving loads, place the largest one at three times the height of the beam from the support.
(*d*) Treat all other loads in the usual manner.

5. If a timber does not qualify under the above recommendations, which under certain conditions may be over-conservative, the reactions for the concentrated loads should be determined by the following equation:

$$r = \frac{10\,P\,(L - a)\left(\frac{a}{h}\right)^2}{9\,L\left[2 + \left(\frac{a}{h}\right)^2\right]}$$

where r = reaction to be used as due to a load P,
L = span in inches,
a = distance in inches from reaction to load P,
h = height of beam in inches.

6. Shear stresses for joint details may be taken as 50 per cent greater than the values for horizontal shear given in the tables.

7. Timber acquires a permanent set under long-continued loading. This set with a fully loaded beam is about equal to the deflection, using the modu-

lus of elasticity as given in the tables. In calculating ultimate deflection under long-continued loading, this factor should be recognized.

8. The working stresses for compression parallel to grain are for use on posts, struts, etc., with unsupported length not greater than 10 times their least dimension. They are also for use in end bearing on compression members, as a short column or strut is more likely to fail at the end than at any other point in its length, and the variations in moisture content are greater at that point.

9. For bearing stresses on surfaces at an angle to the direction of the grain, the following (Hankinson) formula has been found to be the best for general use in timber framing:

$$N = \frac{PQ}{P \sin^2 \theta + Q \cos^2 \theta}$$

where N = unit compressive stress in a direction at inclination θ with the direction of the grain,
P = unit stress in compression parallel to the grain,
Q = unit stress in compression perpendicular to the grain.

When the stress acts parallel to the grain, θ is zero.
When the stress acts perpendicular to the grain, θ is 90°.

10. For columns of intermediate length, the Forest Products Laboratory finds that a fourth-power parabola, tangent to the Euler curve, is a conservative representation of the law controlling the strength. That is, from the short block to the long column in which the strength is dependent on stiffness, there is a falling off in ultimate strength which follows a smooth curve, very flat at first but curving sharply to become tangent to the Euler curve at two-thirds of the ultimate crushing strength.

11. For columns from

$$\frac{P}{A} = C \quad \text{to} \quad \frac{P}{A} = \frac{2}{3}C:$$

$$\frac{P}{A} = C\left[1 - \frac{1}{3}\left(\frac{L}{Kd}\right)^4\right]$$

where P = total load in pounds,
A = area in square inches,
P/A = unit compressive stress,
C = safe stress in compression parallel to grain for short columns,
L = unsupported length in inches,
d = least dimension in inches,
E = modulus of elasticity,
K = the L/d at the point of tangency of the parabolic and Euler curves, at which $\frac{P}{A} = \frac{2}{3}C$.

The value of K for any species and grade is $\frac{\pi}{2}\sqrt{\frac{E}{6C}}$.

12. The influence of defects on the compressive strength of columns of constant cross section decreases as the length increases. When L/d equals the value of K for the species and grade, strength-reducing factors allow-

able in the grade have little influence on the strength as a column. Beyond this length the investigation of the strength of columns by the Laboratory indicated that the Euler formula is quite accurate for long wood columns with pin-end connections and that the maximum load is dependent upon stiffness. In such columns, a factor of safety of 3 should be applied to values of modulus of elasticity in order to obtain safe loading.

13. The Laboratory does not, with the present data and under ordinary conditions, find justification for increasing the stresses on square-end columns over those for carefully centered pin-end columns. Tests to determine the influence of end conditions are still being made, and it is probable that under special conditions higher stresses can be used.

14. For long columns, including factor of safety of 3:

$$\frac{P}{A} = \frac{\pi^2}{36}\frac{E}{(L/d)^2} = \frac{0.274\,E}{(L/d)^2}$$

15. Columns should be limited in slenderness to $L/d = 50$.

16. Post and timber grades may be applied to material smaller than sizes given by limiting the size of knots to the proportion of width of face permitted on sizes given.

17. For direct tension the same values as for extreme fiber stress in bending may be used. Straight-grained wood has greater resistance to tension than to any other kind of stress, and it has been difficult to design joints that will develop the full tensile strength. The availability of ring-connectors now largely eliminates this difficulty.

18. Grades of joists or beams may be used for members in longitudinal tension, such as bottom chords or tension members of trusses, but grade limitations which apply to the center portions of joists and beams should be applied throughout the entire length of pieces subjected to longitudinal tension.

19. The provisions of the joist and plank grades are such that material graded on them may be used on edge as joist or rafters, or flat, as scaffold plank or factory flooring, and working stresses for these grades may be applied to such material used with wide faces either vertical or horizontal. Joist and plank grades apply to material not thicker than 4 in. Material thicker than 4 in., for use in bending, should be graded on beam and stringer grades. In such material with loads applied to the wide face, the knot limitations for this face are those for the narrow face as given in the rules.

20. Material to be used for such purposes as caps, bridge ties, etc., where strength in bending is a factor, should be specified in beam and stringer grades, although of shape more commonly considered as of timber grades, as the method of measuring knots in post and timber grades makes it impracticable to assign bending stresses to them. Caps and bridge ties are often square or have horizontal faces wider than the vertical faces, in contrast to beams and stringers, in which the narrow faces are horizontal faces and the wide faces are vertical, and care should be exercised that knot limitations are applied to the proper faces.

21. In railway stringers of two span lengths, it should be specified that grade factors throughout the center two-thirds be limited as in the middle third or middle half of a single-span stringer, for the maximum moment will be over the center support, and although the full positive moment would

not be developed in either span as long as there was resistance to negative moment over the center support, there might be circumstances in which full positive moment of resistance at the centers of the two spans would be desirable.

22. In material used over three or more spans, or subject to varying bending moments, such as full-length sills of wood under-framed cars, grade limitations should be applied throughout the entire length as in the middle third or middle half of joist and plank and beams and stringers.

23. The strength of timbers and posts in round form is greater than would be expected from the ordinary engineering formulas. The strength, stiffness, and shearing value in bending of round timbers of any species may be assumed to be identical with that of square timbers of the same grade and cross-sectional area. Tapered timbers should be assumed as of uniform diameter, the point of measurement being one-third the span from the small end, but the diameter should not be assumed to be more than one and one-half times the end diameter.

24. The strength of round columns may be considered the same as that of square columns of the same cross-sectional area. In long tapered columns the strength may be assumed as identical with that of a square column of the same length, and of cross-sectional area equal to that of the round timber measured at a point one-third its length from the small end. The stress at the small end must not exceed the allowable stress for short columns.

25. Detailed studies on the strength of timber in round form are available in Reports 180 and 181 of the National Advisory Committee for Aeronautics on " The Influence of the Form of a Wooden Beam on Its Stiffness and Strength."

26. In determining working stresses, the Forest Products Laboratory has considered both elastic limit and breaking strength. Elastic limit, however, is more variable and less definite than ultimate strength, and the latter is taken as the more dependable basis for the determination of safe working stresses.

27. The factor of safety at a given working stress varies materially with the duration of the stress. At the recommended working stresses, the average timber in buildings has a factor of safety of 6[1] on impact loading, 4 under 5-minute loads and $2\frac{1}{4}$ under long-time loading, with a minimum factor of safety of 2 on 75 per cent of the pieces under long-time loading, while about 1 piece in 100, of very light weight and with maximum defects for the grade would be expected to break at $1\frac{1}{2}$ times the recommended stress under loading of approximately 10 years' duration. The factor of safety on new timbers in bridge work is about $\frac{1}{7}$ greater than the above values.

[1] If impact stresses are neglected when less than 100 per cent of the load producing them, the factor of safety for such loads would be reduced from 6 to a minimum of 3.

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